Instituto Nacional de Matemática Pura e Aplicada

FINANCIAL CRISIS INTERVENTIONS IN GENERAL EQUILIBRIUM MODELS

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À minha família

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RESUMO

Esta tese apresenta dois trabalhos sobre intervenções em períodos de crise. O primeiro apresenta uma nova política envolvendo o relaxamento do requerimento de colateral e uma participação do governo dividindo o comprometimento dos agentes nos ativos financeiros. Resultados numéricos são apresentados para mostrar que esta política pode conduzir a uma melhoria de Pareto inclusive em economias que não podem ser melhoradas, no sentido de Pareto, com a política monetária não-convencional. No segundo trabalho é realizada uma análise numérica dos efeitos de duas intervenções de crise, a política monetária não convencional e a nova política apresentada, num contexto de crenças heterogêneas. Os resultados numéricos indicam que o otimismo relativo é importante para determinar a restrição dos agentes e que a política monetária não-convencional é potencializada quando o agente pobre é otimista. Além disto, os resultados sugerem que a política proposta no primeiro trabalho é preferível quando o agente pobre é relativamente pessimista e que a política não-convencional é preferível quando ele é relativamente otimista.

Palavras-chave: equilíbrio geral · colateral · crise · melhoria de Pareto · crenças

ABSTRACT

This thesis presents two works on interventions in period of crisis. The first one presents a new policy involving a relaxement of the collateral requirement and a government participation sharing the commitment of the agents in the financial assets. Numerical results show that this policy can lead to a Pareto improvement in some economies in which the unconventional monetary policy does not lead to. In the second work a numerical analysis is developed on the effects of two crisis interventions, the unconventional monetary policy and the new policy presented, in a context of heterogeneous beliefs. The numerical results indicates that the relative optimism is important to determine the constraints of the agents and that the unconventional monetary policy is potentialized with the relative optimism of the poor agent. Furthermore, the results suggests that the policy proposed in the first work is preferable when the poor agent is relatively pessimistic and the unconventional monetary policy is preferable when he is relatively optimistic.

Keywords: general equilibrium · collateral · crisis · Pareto improvement · beliefs

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Chapter 1

Introduction

1.1 Preliminaries

The 2007-2009 US financial crisis was not the first one in human history. Dozens occured before it like the 17th century Dutch tulip mania bubble. The systematic repetition of such events of great magnitude and social impact reveals that the human understanding of the underlying forces that causes it is incomplete and unsatisfactory. It shows that the society has not yet developed a proper intelectual construction that can be used to prevent it or to mitigate its social consequences.

Along the last three centuries a great effort has been done by the society in the attempt to adress this issue. The economic literature on financial crisis demonstrates that this topic has intrigued and called the attention of many great economists. Several branches of discussions were developed along the years providing, for example, works organizing the past experience and a few theoretical framework to analyze the financial crisis and its economical conexion or consequences. The insights given by these researches are frequently used for normative proposals on government actions or agreements. This introduction will give a brief sketch of some historical events that are related to the appearence of the concept of *regulation* or *crisis policy* in the financial sector, which historically contextualize the present work.

A good way to understand the evolution of the various intelectual visions about the regulation of the financial and monetary system, the great variety of political efforts to implement it and as well its difficulties, is looking to the recent history of cooperation of the central banks. With this history it is possible to note how operational and technological changes in the financial and monetary system, as well changes on the dominant concepts, deeply affected the goals and instruments used as means of cooperation by the central banks.

In this thesis the attention is restricted to the analysis of formal models with a regulatory parameter in order to understand how changes in this regulatory parameter affects the economy, especially on the welfare of the agents. It will be presented a general equilibrium model with new policy for moments of crisis that captures a class of phenomena that the current literature has not yet directly modeled, studied or discussed. This class of phenomena is the one in which the central authority alleviate the collateral requirement of the financial assets assuring the payment of a fraction of the amount relaxed in the next period. This could be done in a moment of crisis to rebuild the economy or to avoid a potentially worsening of the crisis. This is the case of the financial institutions considered too big to fail or highly connected with great systemic spread of its losses, in which the central authority institution gives them a temporary relief in some constraint, like the collateral requirement, trying to avoid a greater social cost that could come in case of bankruptcy. Intuitively one can think that under some circumstances of the economy, like the endowment distribution, this changes can make the agents better off or worse off. The results achieved with this new regulation will be compared with the results of the unconventional monetary policy, used with great importance in the last US financial crisis, presented by Araujo et al. (2015). Finally, it will be numerically analyzed the effects of these two policies in economies with heterogeneous expectations.

Formally, the thesis is organized in the following way. This first chapter of preliminaries is divided into a historical perspective, where it is shown the evolution of the commom thinking on crisis, intervention and cooperation over the last century, and a review of the literature, to show how the recent specialized literature analyzes regulation in financial crisis contexts. The rest of the work has two parts: chapter 2 presents and analyzes the new policy mentioned; chapter 3 analyzes this new policy and the unconventional monetary policy in a context of heterogeneous expectation.

1.2 Historical Perspective[1]

In any historical analysis a starting point must be defined. The classical gold standard in the period (1873-1913) was somewhat arbitrarily defined as the starting point sufficient for the purposes of this work. From there to here the history of the international financial coordination can be divided into four periods: classical gold standard (1873-1913), inter-war period (1914-1945), Bretton Woods (1945-1973) and post-Bretton Woods (1972-present days).

In this subsection, the expression "financial stability" is related to the risk of a generalized spread of the default on financial institutions and "monetary stability" is related to price stability or exchange rate stability, depending on the period.

Roughly speaking, the first hundred years, from 1873 to 1973, were dominated by the belief that a fixed rate of exchange was a global priority since it would favor a price stability. Thus, most of the efforts of the international coordination was applied to maintain the fixed rate system or to restore it when it was lost. Cooperation between the countries was limited to emergency liquidity help in order assure convertibility.

In the case of the classical gold standard, the convertibility was in gold. Cooperation was somewhat limited because governments were reluctant to use its gold reserves to help a potential non-friendly country and also because international finance was part of a government policy, not a mainly technical issue. In fact, around 1900 there were only about eighteen central banks in the world. With the free capital and labor mobility that prevailed at that time, the balance of payments of the countries were naturally corrected and the fixed exchange rate system survived without major disrpution for several years.

During the First World War the gold standard was abandoned, as well as the fixed exchange rates, due to the war efforts made by several important economies. At the end of the war many countries faced high inflation and the gold convertibility came back to discussion because of the belief that it would favor price stabilization. The objective and instruments of cooperation was similar to the previous period. However, the inflationary experience in Europe and the high gold reserves of the US helped to develop a notion of monetary estability which was much more associated to price stability than to convertibility.

In the thirties the major global event was the great depression. The first institutional mean of international cooperation between central banks, the Bank of International Settlements (BIS), was created in 1930. One of the main reasons was the attempt to commercialize the german reparation payments of the First World War in a way that part of the german debit could be issued as long term asset to be subscribed by international private banks. Nevertheless, with the global spread of the great depression along the thirties, the sucession of several bank crisis and the increasing of political friction between countries, the gold standard slowly disintegrated and the international cooperation has become an autarchy system. During the Second World War, seeing the important role the BIS could play in a post-war reconstruction, the central banks

[1] This section is based on Bordo (1993) and Borio and Toniolo (2008).

made an effort to the BIS behave itself with neutrality in order to keep functioning. The BIS was the only active international organization during the war.

At the end of the war the countries tried to renew the gold standard. They wanted to set a new framework trying to balance the advantages and disdvantages of both systems: classical gold standard and floating rates. The exchange rate stability was seen as an advantage of the gold standard and the freedom to pursue domestic macroeconomic goals an advantage of the floating rates. The rigidity that makes difficult to absorb the international transmission of the economic cycle was the disadvantage of the fixed exchange rate and the destabilizing speculation or the competitive devaluation (called "beggar-thy-neighbor") was the disadvantage of the floating rate. As a result, the Bretton Woods Agreement in 1944 defined an adjustable system in which the dollar was convertible to gold but all other currencies would be convertible to dollar. The fixed rates of the other currencies in dollar could be adjusted only in the case of a fundamental disequilibrium.

After the Second War, still with the exchange controls, the European countries were concerned with the possibility that a shortage of dollar to import US products would imply in a deflation pressure to maintain the convertibility. The Marshall Plan, between 1948-1952, and the devaluation of tewnty four countries' currencies around 1950 solved this question and the deflation was no longer a threat. As a consequence, the US incurred in a deficit in its international balance of payments but this was not a problem at the begining because it accumulated most of the world's monetary gold reserves during the war. The major academic and political discussion during 1959-1967 was about three major problems of Bretton Woods: adjustment, liquidity and confidance. However, the continued US balance of payments deficit and the constant decline of confidance in the dollar made US stop trading gold in the open market in 1968. This turned, in practice, the gold standard to a *de facto* dollar standard. The collapse of Bretton Woods came in 1973 after a US monetary expansion that exacerbated worldwide inflation thus breaking the implicit rules of the dollar standard by not maintaining price stability. In the Bretton Woods period, in view of the high domestic control over international trading, the financial crisis were not frequent or huge, so the financial stability was not a relevant topic in international discussion.

In the post-Bretton Woods period the world saw a return of the floating exchange rates and more mobility of capital, defined much more by market forces. In the monetary aspect the coordinations were reduced, limited to great misalignement to the dollar and focused on the price stability. The understanding was that more cooperation would take the power of central banks to use monetary expansion to adjuts global imbalances. In this way, the cooperation acted much more in the institution design of the financial sector than offering liquidity, which was a role absorbed by the International Monetary Fund (IMF). The high capital mobility, together with the technological inovations in the financial sector, increased the frequency and gravity of financial instabilities. This made the BIS, from 1970 onwards, develop and strenghten a regulation with prudential purposes and also improve the infrastructure of the settlement system. Slowly it was developed a central comprehension that the financial infrastructure of individual countries could have an important effect on financial instability, both domestically and internationally.

In the financial cooperation the BIS acted both in crisis management and crisis prevention. The operability of the crisis management was generally through the prefinance of disbursements from the IMF to the country with trouble. The last BIS-coordinated package was granted to Brasil in 1998 to supplement IMF lending. The crisis prevention was concentrated in three elements: financial institutions, payment/settlement systems and market functioning.

Through the last decades three important BIS-comissions were created, giving major contribution to the current shape of the international financial design, architecture and regulation. They are the Basel Committee on Banking Supervision (BCBS), the Committee on Payment and Settlements Systems (CPSS) and the Committee on the Global Financial System (CGFS).

The origin of the CGFS is the Euro-Currency Standing Committee, created in 1971. This committee aimed to study the implications of the financial innovation and seeked to improve the flow of information between the different financial markets. The scope was initially the european countries but gradually incorporated other countries. In 2000 it changed its name to CGFS to reflect the more internationalized scope.

The BCBS and the CPSS were originated after the failure of the Bankhaus Herstatt, in Germany 1974. The liquidation of the Bankhaus Herstatt had major cross-jurisdiction effects due to liquidity problems in the foreing exchange markets. In that case several foreing banks delivered Deutsche Marks to the german bank but did not receive the corresponding US Dollars in New York. This episode called the international attention to the importance of a reliable liquidity infrastructure to the financial stability. In response, the BCBS and the CPSS were created by the international community.

The first landmark of good practices and standards on the supervision and regulation of the financial sector, proposed by the BCBS, was consolidated in the Basel I Agreement, in 1988. The Basel I Agreement was revised twice, first in the Basel II Agreement, in 2004, and second in the Basel III Agreement, in 2010. All three Basel Accords are based, essentially, in balance sheet accounting criteria complemented with some other practices that are recomended. They aim to provide relations between some of the accounts of the balance sheet that should be respected by the financial institutions. The claim is that these relations could bring more safety and stability to the financial systems. One of the main concepts behind the Basel Accords is the establishment of a “proper” amount of “good” capital in each financial institution in order to absorb losses in moments of bust. Since the accords are made over parameters of the balance sheet, the definition of “proper” amount and what is “good” capital is made in balance sheet terms. These definitions slightly changes between the three Basel Accords.

Another landmark proposed by the BCBS was the Core Principles for Effective Banking Supervision, in 1997, after the mexican crisis of 1995. In 1999, after the asian crisis in 1997, when the Financial Stability Forum (FSF) was created by the G7 to propose a new financial architecture, the Core Principles of the BCBS became part of the twelve codes and standards proposed by the FSF. The Basel Committee truly stablished an international reference in terms of cooperation effort of regulatory authorities in the financial field.

An important thing to mention is that all agreements made in the BIS organization do not have the power of an international treaty subscribed by the nations leaders, like the World Trade Organization (WTO). Actualy, they are informal agreements that are voluntarily implemented by the participants through national law or national regulatory schemes. This is called “soft law”.

Recently the expressions “macroprudential” and “microprudential” became popular and emphasize the new mindset of the prudential regulation, including the one that has led to the Basel III.[2] In recent years, the recognition that the financial sector is especially vulnerable to vicious circles or moments of euphoria has established a difference between the risks that affects the operationality of a single institution and the risks of the banking sector, which comprehends those individual behaviour of financial institutions that produces externality risk to the whole financial sector. In Clement (2010) it is shown that the first time it appeared in BIS records was in 1979 with similar understanding of the current one.

> ...‘macroprudential’ approach considers problems that bear upon the market as a whole as distinct from an individual bank, and which may not be obvious at the micro-prudential level (Clement (2010))

[2]See Brunnermeier et al. (2009) for a more precise distinction between the terms macroprudential and micro-prudential.

1.3 Review of the Recent Literature

In recent years the banking regulation has been studied through several different approaches and methodologies. The recent literature can be partitioned in various ways, depending on the criteria used. Here it will be arbitrarily divided into four groups: conceptual description, numerical analysis, macroeconomic modeling and general equilibrium modeling.

One important reference of the conceptual description approach is Brunnermeier et al. (2009). In this report the authors offer an extensive description of the anatomy of the crisis and also the conceptual understanding of its dynamics. They argue that the financial sector, unlike other sectors of the economy, has particular negative externalities in case of financial collapse, costing more to the entire society than to the financial institutions. They describe the self-amplifying mechanisms of a financial crisis, the loss and margin spiral, which tend to make a crisis worse since it has started. The loss spiral is mainly associated with the procyclical marked to market system, in which a price decline can instantaneously generate balance sheet losses. The margin spiral focuses on the procyclicality of leverage. In this case, a drop in the price makes the risk measures goes up and the margins to be much harder, forcing the financial institution to deleverage, which in turn makes the market more iliquidity and the conditions for a new round of price decline is created. Finally they conclude with some recommendations to mitigate the crisis, such as some capital requirements criteria and also liquidity requirements. In this group of conceptual description can also be cited Hanson et al. (2011) and Borio (2003). The first one argues that previously to the 2008 crisis the regulation was microprudentally focused and the crisis showed that general equilibrium effects must be considered in the definition of the financial regulation. The concept of macroprudential naturally considers these general equilibrium effects and therefore the regulation must be more macroprudentially oriented. The latter reference of Claudio Borio carefully define the concepts of microprudential and macroprudential and makes a long discussion relating these concepts with the nature of the financial stability and the desirable policy efforts. This paper was published in 2003, much previous to the 2008 crisis, which possibly makes Borio a forerunner of the language used in recent discussion on prudential regulation. This work is cited in almost all post-2008-crisis papers in prudential regulation.

About the second group of papers, Repullo et al. (2010) can be cited as an example. In this paper the authors analyze the pro-cyclicality of the Basel II Accord capital requirement, motivated by the 2008 crisis. They confirm the pro-cyclicality of Basel II, especially if compared with Basel I, by estimating the spanish probability of default (PD) from 1987 to 2008, using Basel II formulas to compute the capital requirement per unit of loan. They then analyze two different procedures proposed to mitigate this ciclicality. One of them is smoothing input of Basel II formula by smoothing the PD series, and the second one is smoothing the output of Basel II formula by smoothing directly the capital requirement. They found that the latter would be better in terms of simplicity, transparency and low cost of implementation. Some other papers focused on empirical analysis are summarized in Galati and Moessner (2013). They are focused on the quantification of the financial instability or systemic risk, the assessment of the systemic importance of individual institutions and the analysis of the effectiveness of the macroprudential tools.

Galati and Moessner (2013) is a good review of the literature on prudential regulation, in its various approaches. They present various papers on the conceptual description, some papers on the numerical analysis, as mentioned in the previous paragraph, and several papers in the macroeconomic modeling. They argue that the 2008 crisis triggered a new challenge for macro models, which would be modeling the financial system in a way that their effects to the macroeconomy could be analyzed clearly and effectively. Thus, the research on a theoretical basis for macroprudential approach is still at the beginning and there is no consensual "workhorse" family of models to use. Not even the definitions and the relationship between microprudential policy and macroprudential policy are completely pacified. The main family of models used in the macroeconomic modeling is the dynamic stochastic general equilibrium (DSGE). One branch of the recent literature analyzes monetary policy with financial friction in the credit constraints of non-financial borrower, and another branch studies the role of bank capital in monetary transmission mechanism with frictions in financial intermediaries.

In the modeling with general equilibrium there is also a lack of consensus around a basic formal framework to analyze regulatory issues or policies for banking crisis. The next set of papers shows the best recent efforts in analyzing regulation or policies through general equilbrium models. Differently from the groups previously described, more details on this literature will be given since it is the broad context of the work presented here.

Geanakoplos and Zame (2014) present a new general equilibrium model with financial markets and collateral. The model has a finite number of agents, two periods and finite number of states of nature. The existence of equilibrium is proved and several related issues are explored such as effiency and regulation. They show that the equilibrium may be inefficient even if the markets are complete, because some assets may not be transacted, and it is equivalent to an Arrow Debreu equilibrium when it is efficient. The assets traded by the agents are endogenously chosen by the agents. Theoretically, a huge amount of assets, different from each other only in the collateral requirement and equal in everything else, is available to the agents and they are able to endogenously choose the collateral requirement levels of the assets they will trade in equilibrium. Typically only a small subset of the assets available will be effectively traded. The kind of regulation they analyze is on the collateral requirement, i.e., on the assets available to trade. They present conditions that guarantee the *constrained pareto eficciency*[3] of the endogenously chosen assets. This result is different from the one in Geanakoplos and Polemarchakis (1986), for general equilibrium model with incomplete markets, which proves that the equilibria are generic constrained-suboptimal. Thus, an analogous result would be impossible to the case of the general equilibrium model with financial markets and collateral.

Araujo et al. (2012) used the same theoretical framework of Geanakoplos and Zame (2014) and analyzed the effects of the regulation on the durable or on the collateral-requirements in the welfare of the agents. They show that the constrained efficiency result of Geanakoplos and Zame (2014) also applies for economies in which all agents have homothetic preferences. Through numerical examples they showed that in some cases the equilibria are Pareto-ranked in the collateral requirement. Their main numerical result is that, even in the case of economies with heterogeneous utilities, which is outside the conditions of the theorem of constrained-efficiency, a Pareto improvement was not found when changing the collateral requirement.

Araujo et al. (2015) present a modification of the basic general equilibrium model with endogenous collateral of Araujo et al. (2012) and introduce money in order to analyze the unconventional monetary policy used in the 2008 crisis. The paper explores the interaction between the conventional monetary policy, defined by the interest rate, and the unconventional monetary policy, which is modeled as the amount of durable of the economy purchased by the Central Bank. The many possible effects on the welfare of the agents is also analyzed. From the theoretical point of view, the economy has two periods, two states and endogenously chosen assets in equilibrium. They show that the assets are *inessential* in the sense that the same prices and commodities allocations of equilibrium could be recovered even if the assets markets were closed. Fostel and Geanakoplos (2015) also has a similar result in their binomial economy, which

[3] Geanakoplos and Polemarchakis (1986) define the negation of *constrained efficiency* in the following way: "We say that the asset allocation at a competitive equilibirum is *constrained suboptimal* if a reallocation of assets alone can lead to a Pareto improvement when prices and allocations in the commodity spot markets adjust to maintain equilibrium". This definition says that an equilibrium is not constrained efficient when one can find another equilibrium in the span of the returns of the assets that Pareto dominates the original equilibrium. This is especially important in the case of general equilibrium models with incomplete markets because the span of the returns of the assets is tipically a proper subset of the entire space. Thus, it makes sense to analyze the Pareto efficiency restricted to the set of allocations allowed by the wealth transferences made inside the span of the returns.

is a model much closer to the original model in Geanakoplos and Zame (2014). In both papers, however, the inessentiability result was possible due to a special feature of the collateral in their economies: they yeld no utility to the agents. In Araujo et al. (2015) the durable is used as collateral but only the service of the durable affects the utility of the agents and in Fostel and Geanakoplos (2015) the collateral is a financial asset, which gives no utility to the agents.

Another group of researchers, led by Charles Goodhart, Dimitrios Tsomocos, Anil Kashyap and Alexandros Vardoulakis, is pushing a research program in the last years that aims to construct a general equilibrium model to analyze not only the kind of regulation of Geanakoplos and Zame, in the collateral requirement, but also several others types of regulation, like those proposed by the Basel Accords for the financial markets.

Tsomocos (2003) introduces money and default in a general equilibrium with incomplete markets that allows for competitive banking and financial instability analysis. He also introduces a capital requirement constraint in order to model some of the Basel II Accord recommendations. The regulatory parameters are the capital-adequacy ratio, the bankruptcy penalty in case of default and the risk weight of bank assets that are used to calculate the capital requirements. He found that the monetary, fiscal and regulatory policy have real effects in the economy. This non-neutrality is due to the real and nominal determinacy of the equilibrium. The default modelled in this paper, however, is not done with the collateral, as in Geanakoplos and Zame (2014). Collateral is not present in this economy.

Pederzoli et al. (2010) use a general equilibrium model with heterogeneous agents, endogenous default and limited participation in the financial markets in order to analyze the relation between the procyclicality and different possible rating systems. Again, the default in this economy is not modelled using collateral and there is a default penalty in the utility function.

Goodhart et al. (2012) uses a general equilibrium model with heterogeneous agents in order to analyze five different regulatory policies that have been proposed to mitigate the effects of a crisis in the financial sector: loan to value limits, capital requirement for banks, liquidity ratio, dynamic loan loss provisioning for banks and margin requirement on repurchase agreement. They have been proposed mainly by the Basel agreements. The model has an intended interpretation of default, collateral, fire sales and credit that allows to analyze the main effects of a financial crisis. Those interpretations give some intuition about the relationship of these different tools and determine if they are substitutes of complements. The default here is modelled in a different way from previous papers from the same authors and specially from Geanakoplos and Zame (2014). It relies much more on an interpretation over the value of the decision variables than in an explicit financial structure that requires exogenous (or endogenous) fixed quantity of collateral that should be posed in the initial state. Here the agent defaults when the value of the durable chosen by the agent is less than the value of the loan. In Goodhart et al. (2013) the authors analyze a specific parametrization of this model. They show that the control of the fire-sale risk is important to the financial stability and also clarify the combination of those various regulatory instruments that is most effective to it. However, they also showed that one can easily combine regulation with some adverse effects to the economy. And in Kashyap et al. (2011) the authors do a brief and general discussion about the model in Goodhart et al. (2012).

In Kashyap et al. (2014) they use a variant of the banking model proposed by Diamong-Dybvig in 1983 in the paper "Bank Runs, Deposit Insurance and Liquidity" to analyze how capital regulation, liquidity regulation, deposit insurance, loan to value limits and dividend taxes can be used to offset the possibility of a run and the limited liability faced by the bank and enterpreneur. One of their results is that the Pareto improvement can be achieved with regulation and correcting those two frictions is possible only with more than one kind of regulation. As in the other papers from this group, the default is not modelled with a collateral and there is a default penalty in the utility function of the agents.

The paper Geanankoplos (2009) analyzes the role of optimistic agents in a general equilibrium model with collateral and two or three periods. He models crisis as the occurance of a bad state in the subsequent period and associates the behavior of the agents in a crisis cycle with the level of leverage of the assets. He claims that the crisis cycle is actually caused by what he calls *leverage cycle*. Recently, the unpublished work of Tsomocos and Yan (2016) use a variant of Geanakoplos' model with three period and bayesian update to analyze Pareto improvement in a context of optimism. These papers shows that it is still of interest in the literature the analysis of the relation between heterogeneous beliefs of the agents, crisis and regulatory means to mitigate it.

1.4 Objective

The objective of this thesis is first to propose a new kind of policy involving the collateral requirement relaxation and a government compensation that can be used in moments of crisis, and second to numerically analyze the relationship between the effects of crisis policies and the beliefs of the agents. For the first purpose it will be formally defined a general equilibrium model with financial markets, collateral and two regulatory parameter, one affecting the collateral requirement of the assets today, and the other representing a government compensation tomorrow. For the second project it will be numerically compared two policies: the unconventional monetary policy of Araujo et al. (2015) and the new policy proposed here.

The inspiration for these works comes from different sources. The basic structure of a general equilibrium with financial markets and collateral comes from Geanakoplos and Zame (2014), Araujo et al. (2012) and especially Araujo et al. (2015). The ideia of a regulation/policy to be used in crisis approaches this work to Araujo et al. (2015) and Geanankoplos (2009). The relationship between crisis regulation/policy and agents with heterogeneous expectation also comes from Geanankoplos (2009). Finally, the methodology to produce and analyze the results, in both works, follows closely Araujo et al. (2015) and the computation of the equilibria uses the technique proposed in Schommer (2013).

The arguments for the choice of a general equilibrium as a strategy of modeling is in tune with the arguments posed by Tsomocos and his group. The general equilibrium model captures all the indirect effects and feedback mechanisms going on in the economy. The behavior of the agents and their chosen actions are completely reflected in the prices. Thus, welfare analysis and distributional questions are more rigorous and in some sense more robust than other approaches.

Chapter 2

Collateral Requirement Regulation in a General Equilibrium Model

2.1 Introduction

In this chapter a new kind of policy designed to be used in crisis moments will be presented and analyzed. In Geanankoplos (2009) there is a suggestion on how the government should act in moments of crisis to rescue the economy:

> "To reverse the crash once it has happened requires reversing the three causes. [...] Second, leverage must be restored to reasonable levels. One way to accomplish this is for the central bank to lend directly to investors at more generous collateral levels than the private markets are willing to provide." (Geanakoplos, 2009,p. 4)

The regulation proposed here is in tune with Geanakoplo's ideas and involves the relaxation of the collateral requirement of the assets in the first period and a central authority acting in the second period to recover, to some extent, the payoff of the original asset. It is also related to the idea of a recourse loan, in which the part not honored of the claim can be pursued and recovered by institutional means, like legal court, bankruptcy process, etc.[1]

There is some empirical evidence of policies designed in a somewhat similar way it is modeled here. In Brazil, the Fundo Garantidor de Crédito (FGC) is a non-profitable privately managed institution, created by financial institutions, to protect the depositors (lenders) in case of bank intervention or bankruptcy. The fund is raised by the associated institutions with monthly contributions of small fractions of the corresponding obligation. Hence, the fund is raised from the borrowers to assure part of the promises made to the lenders in case of default or bankruptcy. The brazilian recent real estate program Minha Casa, Minha Vida also has a modality in which the bank finance a fraction of a house and the government subsidizes the rest of it, which in practice works as if the government were relaxing the collateral requirement of the home buyer and assuring the full payment of the asset to the lender. Finally, in the 2007-2009 financial bust, the Iceland government got US$ 5.1 billion in soverein debt with the IMF to guarantee all domestic deposits in Iceland banks and did not used this value to fund the banks.

The methodology chosen was to use the same basic theoretical framework of Araujo et al. (2015), because the unconventional monetary policy has become the symbol of a change in the mindset on crisis intervention, which has become much more open to unconventional policies as a mean to complement and potentialize the effects of the usual conventional monetary policy. This choice has the advantage of keeping both types of policies on the same platform so that

[1] There is a large literature on recoure or non-recourse loans. For example see Poblete-Cazenave and Torres-Martínez (2013), which proves existence of equilibrium in limited-recourse collateralized loans economies.

the language is similar and the results more comparable. Thus, the cases wether one is more effective/preferable than the other could be seen more clearly.

It will be shown through numerical results that if the government, in moments of crisis, replaces the original endogenous assets traded with new ones with more advantageous collateral requirement then there will be Pareto improvement only if the government enters in the second period to assure some positive compensation for the additional default caused by the decrease of the collateral requirement levels.

Contrary to the commom intuition, the collateral requirement relaxation without any additional government intervention in the second period surprisingly leads to a more constrained agents in the set of feasible transferences. This happens in spite of the fact that the relaxation of the collateral requirement indeed increases the leverage of the assets, as believed Geanakoplos. This result offers a new perspective to the result in Araujo et al. (2012), in which they numerically show that even for economies with heterogeneous utilities there is no Pareto improvement when changing the endogenous collateral requirement. Although all the analysis here is developed with homogeneous homothetic preferences, the result showing the decrease of the feasible set of transferences does not rely on the utility of the agents, it depends only on the financial market structure of the economy.

The Pareto improvement obtained when the government acts in the second period occurs both in economies short sale constrained and leverage constrained[2] on the transferences. The possibility of a Pareto improving in short sale constrained economies is a major step up in comparison with the unconventional monetary policy. Indeed, Araujo et al. (2015) show that the unconventional monetary policy in fact tightens the short sale constraint of the agents and a Pareto improvement never was found in a short sale constrained economy in their numerical examples. The policy proposed here may allow for a relaxement in the short sale constraint and this means that the collateral constraints of the agents are crucial to understand the effects of this policy.

The results found and exposed in this chapter put the policy proposed as a relevant complementary policy to be used in crisis, especially in comparison with the unconventional monetary policy.

The remainder of the chapter is organized in the following way. Section 2.2 presents the new collateral requirement regulation with compensation and its basic theoretical properties; section 2.3 is the place of the numerical results and the conclusions are presented in section 2.4. The appendix has the equations used to produce the numerical results.

2.2 Model

2.2.1 Collateral requirement regulation with compensation

The main theoretical structure of the basic framework mentioned in the introduction is the general equilibrium model with financial markets and endogenous collateral from Araujo et al. (2015). The main novelty of the collateral model, in comparison with the standard and well known general equilibrium model with incomplete markets, is the collateral associated with each financial contract. The intuition is that the institutions behind the financial contracts, like the legal enforcement structure, impose that the agents should back their promises with a collateral when issuing an asset. In the general equilibrium with incomplete markets model the promises made by the agents when issuing an asset should be fulfilled. In this case it is supposed that the institutions and the law enforcement are perfect, so they succeed to oblige all agents to

[2] Intuitively, an agent is short sale constrained when he brings the maximum possible amount of wealth to the bad state of the second period and the minimum amount to the good state. And he is leverage constrained when he chooses the least possible amount of wealth to the bad state. The precise definition will be given later.

deliver exactly what was promised. In the collateral model this unreal hypothesis of perfect law enforcement is relaxed and the agents are allowed to default. If a promise is made today, tomorrow the agent is allowed to choose what is better to deliver: the promise made or the collateral. The agent always chooses to deliver what is worth least, so the real delivery of the asset in the next period is the minimum between the value of what was promised and the value of the collateral used to back it. Conceptually, this is a literature of lack of commitment or commitment limited to the collateral.

The process of backing the promises with a collateral imposes a friction in the financial markets, which is translated into the model as a new constraint in the household's budget set, called collateral constraint. It says that the agent must hold at least the amount of collateral associated with the assets issued by him in the first period. This new constraint has some intuitive implications on the inefficiency of the equilibrium. Indeed, in a general equilibrium model with incomplete markets some desirable allocations of the economy may demand a transfer of wealth not allowed by the financial structure, i.e., an allocation outside the span of the return of the assets. In the presence of collateralized promises, the scarcity of the collateral may impose additional restriction on the possible allocations on the economy in the sense that some allocations *inside* the span may not be reachable due to a high amount of collateral demanded to back the corresponding financial position.

Formally, the basic framework is the general equilibrium with collateral *and* money. It has two periods with one state in the first period and $S \in \mathbb{N}$ states in the second period. The symbol $S^* = S + 1$ will denote all states in the economy. There are $H \in \mathbb{N}$ agents, $L \in \mathbb{N}$ goods and $J \in \mathbb{N}$ assets in the economy. The utility functions of the agents defines their preferences.

In the analysis developed here, as well as in Araujo et al. (2015), the number of good will be restricted to three, being good 1 an ordinary perishable good, such as food, good 3 the durable and good 2 is the service of a durable. The agents obtain utility only from goods 1 and 2. The durable is used either as a collateral to the assets issued or to enjoy its service. In this sense, the durable good in this economy acts like a risky asset, paying its value $p_{s3}x_{03}$ in each state of the second period, which tipically will be different in each state. An example of a durable in this model would be a house. The owner of a house can rent it to some other person. The agent actually enjoying the house and taking utility from it is the one paying the rent, not necessarily the owner. Even if one lives in its own house, it is theoretically possible to split this relation making him pay a rent to himself. Once the agent chooses the amount of durable x^h_{s3} in state s, he is able to rent it and therefore receive the value $-p_{s2}x^h_{s3}$.

A reasonable hypothesis on the endowments is $e^h_{s2} = 0$ for all s and h, because an endowment of service would be economically meaningless. Furthermore, in this model there will be no endowment of durable in the second period, that is, $e^h_{s3} = 0$ for $s \in S$ and for all h. The durable in the second period will be only those carried from $s = 0$. From the first order conditions (FOC) and the market clearing it is true that $p_{s2} = p_{s3}$ for $s \in S$ in equilibrium.[3]

One of the distinctive feature of this model is the presence of a risk-free bond paying $1 + i$ non-contingent in the second period for each unit of bond hold, which can be interpreted as an equivalent of money. At state zero the agents have an endowment m^h of money and chooses μ^h. Any agent holding μ^h units of money at $s = 0$ will receive $(1 + i)\mu^h$ at any state s of

[3]In the next paragraphs the differentiability hypothesis of $u^h(\cdot)$ will be stated, as well as the strict convexity and strong monotonicity. In this setting, for each $s \in S$, the market clearing conditions and the first order condition of x^h_{s3} in the first period $(s = 1, 2)$ imply that $\mu^h_s(p_{s3} - p_{s2}) = 0$ for at least one agent h. Since $\mu^h_s > 0$, then $p_{s3} = p_{s2}$. Recall that μ^h_s is the marginal variation of the value function $v^h(c)$ of the agent's optimization problem (indirect utility function) due to a marginal relaxation of the s budget constraint. This is a well known result in convex optimization, e.g. Borwein and Lewis (2006), page 47. In other words, $\mu^h_s \in \partial v^h(c)$, where ∂ is the supergradient of the value function $v^h(c)$ and c is a vector of constants of the constraints. Since $u^h(\cdot)$ is strongly monotone, a marginal relaxation of the state s budget constraint would lead to an increase in $v(c)$, hence $\mu^h_s > 0$.

period 1. The Central Bank in this model plays only two roles. First it is responsable to redeem the bonds at the second period to clean the money of the economy. This operation is done through the taxation θ^h across the households. Second, it determines the interest rate as a conventional monetary policy and the holds the value of the money in the second period as part of its monetary policy. In the second period the Central Bank guarantee that one unit of money will value one unit of perishable, that is, $p_{s1} = 1$, $s \in S$.[4]

The financial market of this economy follows the standard general equilibrium model with incomplete markets and collateral. In this model, the financial assets are nominal, that is, asset j promises to deliver one unit non-contingent and demands the issuer a collateral $C_j \in \mathbb{R}_{++}$ to back it. All agents are subject to a collateral constraint, which means that they have to provide in state $s = 0$ the amount of durable to cover the collateral of the assets issued. The actual delivery of asset j in state $s = 1, \ldots, S$, will be $\min\{1, p_s C_j\}$. When the agents deliver $p_{s3}C_j$ in state s it is said that they default.[5]. Therefore, the assets differ from each other only in the amount of collateral used to back them. In this model, as well in Araujo et al. (2015), it is used the framework of Araujo et al. (2012) in which the collateral requirement is determined by the market. This is called *endogenous collateral* in the literature. It can be proved[6] that the agents trade at most S assets, with collateral requirement given by:

$$C_j = 1/p_{j3} \text{ with } j = 1, \ldots, S$$

Suppose that the states are ranked by the durable price so that $C_j = 1/p_{j,3} < 1/p_{j+1,3} = C_{j+1}$.[7] Note that, with this ranking, the asset 1 has the lowest collateral requirement, given by C_1, which gives default in all states, and the asset S has the highest collateral requirement, given by C_S, which never defaults, that is, always pays the promise in every state. Hence, throughout this thesis the term *subprime asset* may be used to designate asset 1 and the expression *prime asset* may be used to refer to asset S.

The two exogenous parameters, δ_j and ξ_{sj}, define the policy proposed in this chapter. A positive delta means that less collateral is being demanded to back the asset, that is, that the collateral requirement is being relaxed. As a consequence, the collateral constraint is modified to allow the agents to put less colateral $(1 - \delta_j)C_j$ for asset j and the delivery of the assets is adjusted to $\min\{1, p_{s3}(1-\delta_j)C_j\}$ since the commitment of the agents is limited to the collateral. Note that when $\delta_j > 0$ the structure of endogenous collateral is being abandoned, that is, the government leaves the original assets with endogenously chosen collateral requirement and offer to the agents a new set of assets with more favorable collateral requirement levels, but not chosen endogenously.

Besides the relaxation of the collateral requirement, embodied by the regulatory parameter δ, the central authority also intervenes in the second period with the parameter $\xi_{sj} \geq 0$ in order to compensate the less collateralized assets. The government's compensation works in a very simple way: it raises fund from the issuer of the assets and transfer the same value to the buyers of the assets. Thus, in practice, the issuers of assets have to pay more than the delivery of the relaxed assets in the states of period 2 and the buyers receive a delivery closer to the original

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[4]The positive price of the money in the first period is implicitly given by the interest rate i defined by the Central Bank.

[5]Note that given an asset j with its collateral C_j, the default is *not* an agent's decision. The condition to default is purely determined by market conditions, that is, the determination of p_{s3}. This is why either all agents default or no agent default.

[6]For more details on this result see Araujo et al. (2012) and Araujo et al. (2015).

[7]In the next section conditions will be given to assure this property.

one. To be economically meaningfull, the compensation is defined such that it is positive only if there is a difference between the delivery of the original asset and the delivery of the relaxed asset. Formally, $\xi_{sj} \equiv \alpha\left(\min\{1, p_{s3}C_j\} - \min\{1, p_{s3}(1-\delta_j)C_j\}\right)$, where $\alpha \in [0,1]$. The range of α says that the maximum compensation possible, which occurs with $\alpha = 1$, restores the delivery of the asset. And when $\alpha = 0$ there is no compensation in the second period.

Each household face the following maximization problem.

$$\max_{x^h, \psi^h, \varphi^h, \mu^h, x_3^h \geq 0} \quad u^h(x^h)$$

s.t.

$$\begin{aligned}
& p_0 \cdot (x_0^h - e_0^h) - p_{02}x_{03}^h + q \cdot (\psi^h - \varphi^h) + \mu^h - m^h \;\leq\; 0 \\
& \sum_{l=1}^{2} p_{sl}(x_{sl}^h - e_{sl}^h) + p_{s3}(x_{s3}^h - e_{s3}^h - x_{03}^h) - p_{s2}x_{s3}^h - \sum_j (\psi_j^h - \varphi_j^h)[\min\{1, p_{s3}(1-\delta_j)C_j\} + \xi_{sj}] + \\
& + \theta^h(1+i)m - (1+i)\mu^h \leq 0 \\
& x_{03}^h \;\geq\; \sum_j \varphi_j^h (1-\delta_j)C_j
\end{aligned}$$

where $m = \sum_h m^h$, $\delta_j \in [0,1]$ and $\xi_{sj} > 0$.[8]

Roughly speaking, the compensation of the new assets, guaranteed by the government, reveals that the government *shares* the original commitment of the asset with each issuer. Indeed, the commitment of the original assets is $p_{s3}C_j$ and relies completely on the private agents. In the new assets offered by the government, in contrast, the private agents are committed only to $p_{s3}(1-\delta_j)C_j$ and the government becomes committed to ξ_{sj}.

Note that in this setting the government intervention has *no* fiscal cost in equilibrium due to market clearing. Indeed, $\sum_h \sum_j \xi_{sj}\left(\varphi_j^{h^*} - \psi_j^{h^*}\right) = \sum_j \xi_{sj}\left[\sum_h \left(\varphi_j^{h^*} - \psi_j^{h^*}\right)\right] = 0$.[9]

The definition of equilibrium of this economy is the following one:

Definition 1. *Let $(u^h(\cdot), e^h)$ be the economy defined previously with monetary specification $(i, \{p_{s1} = 1\}_{s \in S})$. The equilibrium for this economy is a vector $((x^*, \{x_{s3}^*\}_{s\in S^*}, \psi^*, \varphi^*), \mu^*, p^*, q^*)$ consistent with the monetary policy specification such that:*

(i) $(x^{h^*}, \{x_{s3}^{h^*}\}_{s\in S^*}, \psi^{h^*}, \varphi^{h^*})$ *solves the optimization problem above given prices* (p^*, q^*) *for all* h;

(ii) $\sum_{h=1}^H x_{01}^{h^*} = \sum_{h=1}^H e_{01}^h$;

(iii) $\sum_{h=1}^H x_{02}^{h^*} = \sum_{h=1}^H e_{03}^h$;

(iv) $\sum_{h=1}^H x_{03}^{h^*} = \sum_{h=1}^H e_{03}^h$;

(v) $\sum_{h=1}^H x_{s1}^{h^*} = \sum_{h=1}^H e_{s1}^h$ *for* $s \in S$;

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[8] Note about the notation: the symbol $\cdot$ represents the inner product and the subindex of the variables refers to states and goods (x_{sl}^h, p_{sl}).

[9] In order to produce a fiscal cost or surplus it would be necessary to define a compensation for the lender different from the compensation/taxation for the borrower, that is $\xi_{sj}^{\psi} \neq \xi_{sj}^{\varphi}$. However, this would lead to a different modeling because this structure is not compatible with one price q_j for asset j in the initial state. Indeed, since the buyer and the seller face a different payoff structure for the same asset j, they would diverge on the price q_j due to a different perception on the "fair" present value of this asset. Thus, it would be necessary to allow for two prices q_j^{ψ} and q_j^{φ} in $s = 0$.

(vi) $\sum_{h=1}^{H} x_{sl}^{h^*} = \sum_{h=1}^{H} e_{03}^{h}$ *for* $s \in S$ *and* $l = 2, 3$;

(vii) $\sum_{h=1}^{H} (\psi^{h^*} - \varphi^{h^*}) = 0$;

(viii) $\sum_{h=1}^{H} \mu^{h^*} = m$

Note that the aggregate endowment of the service is the aggregate endowment of the durable. In the following subsections theoretical properties will be presented, some of them similar to those found in Araujo et al. (2015). First it will be analyzed the case in which the collateral requirement is relaxed without compensation and after that a positive compensation will be introduced.

2.2.2 Theoretical properties: without compensation ($\xi_{sj} = 0$)

The relaxation of the collateral requirement has consequences to the financial structure of the economy. As δ_j increases, it is more likely that asset j will default because its collateral tends to zero. When δ_j is sufficiently close to 1 for every j, then the rank of the matrix of returns collapses to 1. The matrix of returns of the assets is defined by:

$$V_\delta = \begin{bmatrix} \min\{1, p_{13}(1-\delta_1)C_1\} & \dots & \min\{1, p_{13}(1-\delta_J)C_J\} \\ \vdots & & \vdots \\ \min\{1, p_{S3}(1-\delta_1)C_1\} & \dots & \min\{1, p_{S3}(1-\delta_J)C_J\} \end{bmatrix}_{S \times J}$$

Lemma 1. *Suppose* $\xi_{sj} = 0$ *for all* s *and* j*. Then the economy satisfies the following properties:*

(a) If $\delta_j \geq 1 - \frac{1}{p_{\tilde{s}3} C_j}$*, then asset* j *will default in every state* $s \geq \tilde{s}$

(b) Let $\delta_j \in \left[1 - \frac{1}{p_{s3}C_j}, 1 - \frac{1}{p_{(s-1)3}C_j}\right]$ *for all* $j \geq s$*. Then rank(V)=s*

Proof. (a) Recall that $J = S$ in this economy due to the endogenous collateral constraint. If $\delta_j \geq 1 - \frac{1}{p_{\tilde{s}3}C_j}$, then $p_{s3}(1-\delta_j)C_j \leq \frac{p_{s3}}{p_{\tilde{s}3}}$. By the ordering $C_j = 1/p_{j3} < 1/p_{j+1,3} = C_{j+1}$ mentioned in the previous section, it is immediate that $\frac{p_{s3}}{p_{\tilde{s}3}} \leq 1$ for all $s \geq \tilde{s}$. Note that the condition $\delta_j \geq 1 - \frac{1}{p_{\tilde{s}3}C_j}$ is trivially satisfyed for every $\tilde{s} \geq j$ since for those cases $1 - \frac{1}{p_{\tilde{s}3}C_j} \leq 0$ and $\delta_j \in [0, 1]$ for every j. Therefore, asset j defaults in every state $s \geq \tilde{s}$.

(b) When $\delta_j = 0$ for all j, the matrix V becomes:

$$V = \begin{bmatrix} \min\{1, p_{13}C_1\} & \dots & \min\{1, p_{13}C_J\} \\ \vdots & & \vdots \\ \min\{1, p_{S3}C_1\} & \dots & \min\{1, p_{S3}C_J\} \end{bmatrix}_{S \times J}$$

$$\implies V = \begin{bmatrix} 1 = p_{13}C_1 & 1 & \dots & 1 & 1 \\ p_{23}C_1 & 1 = p_{23}C_2 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ p_{(S-1)3}C_1 & p_{(S-1)3}C_2 & \dots & 1 = p_{(S-1)3}C_{J-1} & 1 \\ p_{S3}C_1 & p_{S3}C_2 & \dots & p_{S3}C_{J-1} & 1 = p_{S3}C_J \end{bmatrix}_{S \times J}$$

The ordering $C_j = 1/p_{j3} < 1/p_{j+1,3} = C_{j+1}$ will assure $rank(V) = S$. Indeed, with the notation col_j to the columns and row_s to the rows of the matrix V, first replace col_j with $col_j - \frac{C_j}{C_J} col_J$ for every $j \leq J - 1$ to obtain

$$V \sim \begin{bmatrix} (p_{13}-p_{S3})C_1 & (p_{23}-p_{S3})C_2 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ (p_{23}-p_{S3})C_1 & (p_{23}-p_{S3})C_2 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ (p_{(S-1)3}-p_{S3})C_1 & (p_{(S-1)3}-p_{S3})C_2 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}_{S\times J}$$

Now replacing col_j with $col_j - \frac{C_j}{C_{J-1}} col_{J-1}$ for every $j \leq J-2$ and the matrix becomes

$$V \sim \begin{bmatrix} (p_{13}-p_{(S-1)3})C_1 & (p_{23}-p_{(S-1)3})C_2 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ (p_{23}-p_{(S-1)3})C_1 & (p_{23}-p_{(S-1)3})C_2 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ (p_{(S-2)3}-p_{(S-1)3})C_1 & (p_{(S-2)3}-p_{(S-1)3})C_2 & \dots & (p_{(S-2)3}-p_{(S-1)3})C_{J-1} & 1 \\ 0 & 0 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}_{S\times J}$$

Finally, repeating the procedure more $S-3$ times one gets

$$V \sim \begin{bmatrix} (p_{13}-p_{23})C_1 & (p_{23}-p_{33})C_2 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ 0 & (p_{23}-p_{33})C_2 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & (p_{(S-1)3}-p_{S3})C_{J-1} & 1 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}_{S\times J}$$

with the elements of the main diagonal given by $v_{ii} = (p_{i3} - p_{(i+1)3})C_i$ for all $i \leq S-1$ and $v_{SS} = 1$. Since $p_{s3} > p_{(s+1)3}$ for all $s = 1, \dots, S-1$, $rank(V) = S$ and the economy has complete markets when $\delta_j = 0$ for all j.

In order to simplify the notation, define $\eta_{sj} = p_{s3}(1-\delta_j)C_j$. Using the previous item and the hipothesis $1 - \frac{1}{p_{(s+1)3}C_j} \leq \delta_j \leq 1 - \frac{1}{p_{s3}C_j}$ for all $j \geq s$, the matrix of returns becomes:

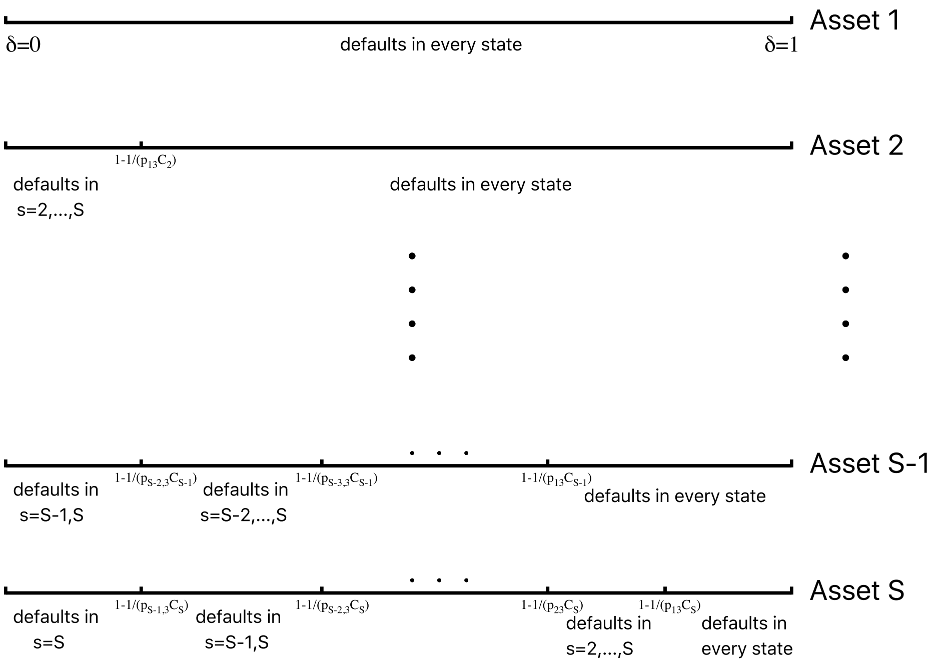
$$V_\delta = \begin{bmatrix} \eta_{11} & 1 & \dots & 1 & 1 & \dots & 1 & 1 \\ \eta_{21} & \eta_{22} & \dots & 1 & 1 & \dots & 1 & 1 \\ \vdots & \eta_{32} & \ddots & \ddots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & \dots & \eta_{(s-1)(s-1)} & 1 & \dots & 1 & 1 \\ \eta_{s1} & \eta_{s2} & \dots & \eta_{s(s-1)} & \eta_{ss} & \dots & \eta_{s(J-1)} & \eta_{sJ} \\ \vdots & \vdots & & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta_{S1} & \eta_{S2} & \dots & \eta_{S(s-1)} & \eta_{Ss} & \dots & \eta_{S(J-1)} & \eta_{SJ} \end{bmatrix}_{S\times J}$$

Since $row_{\tilde{s}} = \frac{p_{s3}}{p_{S3}} row_S$ for all $\tilde{s} \geq s$, the last $S-s+1$ rows become linear dependent and therefore $rank(V) = s$.

■

The above lemma says that if δ_j is sufficiently small, the financial market is complete, despite the financial friction of the collateral constraint. And if the δ_j is sufficiently high, completeness colapses into incompleteness. The proof of the first property shows that the interesting cases are those in which $\tilde{s} < j$ because in these cases it is possible that asset j defaults in states $s < j$. For example, asset 1 is the subprime, always has $\tilde{s} \geq 1$, and gives defaults in every states s. On the other hand asset $j = S$, which is the prime and never gives default, has $1 - \frac{1}{p_{\tilde{s}3}C_j} \geq 0$ for all $\tilde{s} \in S$. Thus, previous lemma says that making $\delta_j \geq 1 - \frac{1}{p_{\tilde{s}3}C_j}$ for $\tilde{s} < S$ implies that asset $j = S$

Figure 2.1: Default Structure of the Assets Depending on δ_j



starts giving default on states $\tilde{s} \leq s < S$ in which it had no default before. The next diagram summarizes these information.

An asset j is called *inessential* when the same allocation and prices of equilibrium can be obtained with another portfolio such that $\psi_j^h = \varphi_j^h = 0$ for all h, that is, as if the market of asset j were closed. The main role of the financial assets is to give the agents the opportunity to transfer wealth between states. The concept of inessentiability translates the idea that the economy has non-collateralized objects that mimics the role of a financial asset. The durable in this model, for example, since does not give utility to the agents, in some sense also has this property. Holding one unit of durable in $s = 0$ means receiving p_{s3} in state s of period 1. The money also has similar property.

When $\xi_{sj} = 0$, the result on inessentiability is similiar to the one presented in Araujo et al. (2015).

Lemma 2. *Let* $((x^*, \{x_{s3}^*\}_{s\in S^*}, \psi^*, \varphi^*), \mu^*, p^*, q^*)$ *be an equilibrium of the economy previously defined and* $\xi_{s1} = 0$ *for all* s*. Then:*

(a) If asset 1 is transacted, then $q_1^* = (p_{03}^* - p_{02}^*)(1 - \delta_1)C_1$.

(b) Asset 1 is inessential if, and only if, $x_{03}^{*h} \geq \varphi_1^{*h}(1 - \delta_1)C_1$ *for all agents.*

Proof. (a) First note that, due to market clearing conditions, asset 1 being transacted implies that exists h_1 such that $\psi_1^{*h_1} > 0$. Also note that one unit of asset 1 and $(1-\delta_1)C_1$ units of durable give the same payoff in period 1, $p_{s3}^*(1-\delta_1)C_1$, and neither of them gives utility for the agents. Thus, the agents treat them as equivalent objects in terms of payoff and utility. Hence, if $q_1^* > (p_{03}^* - p_{02}^*)(1 - \delta_1)C_1$, agent h_1 would prefer selling all his position of asset 1, paying $(p_{03}^* - p_{02}^*)$ for $\psi_1^{*h_1}$ units of $(1 - \delta_1)C_1$, receive the same payoff in period 1 and earn a profit of $[(q_1^* - (p_{03}^* - p_{02}^*)(1 - \delta_1)C_1]\psi^{*h_1} > 0$ in $s = 0$. Buying more durable also relaxes his collateral constraint. Thus, $\psi_1^{*h_1} > 0$ cannot be the optimizer choice of agent h_1. Contradiction.

Now suppose by contradiction that $q_1^* < (p_{03}^* - p_{02}^*)(1-\delta_1)C_1$. Also by market clearing, there exists h_1 such that $x_{03}^{*h_1} > 0$. If $\varphi_1^{*h_1} = 0$, he is able to sell $\varepsilon(1-\delta_1)C_1$ units of durable, for some $\varepsilon > 0$, receive $(p_{03}^* - p_{02}^*)(1-\delta_1)\varepsilon$ and buy ε units of asset 1, which would give him the same payoff in period 1, and profit $[(p_{03}^* - p_{02}^*)(1-\delta_1)C_1 - q_1^*]\varepsilon > 0$. This is a contradiction with $x_{03}^{*h_1}$ being maximizer for h_1. And if $\varphi_1^{*h_1} > 0$, he can diminish $\varepsilon(1-\delta_1)C_1$ units in durable and also decreases ε units of $\varphi_1^{*h_1}$ without violating his collateral constraint, even if it is binding, and profit the same value $[(p_{03}^* - p_{02}^*)(1-\delta_1)C_1 - q_1^*]\varepsilon > 0$. This also contradits the hypothesis that $x_{03}^{*h_1} > 0$ is maximizer for h_1. Therefore, $q_1^* = (p_{03}^* - p_{02}^*)(1-\delta_1)C_1$.

(b) For the "if" part of the item, the same reasoning of Araujo et al. (2015) applies, that is, since all the sellers of asset have more durable than the collateral required, they could simply sell the collateral directly to the buyer of the asset 1 and reduce the amount sold to zero. This procedure has the same effect in their payoffs in period 1 and also in state zero since, by previous item, $(p_{03}^* - p_{02}^*)(1-\delta_1)C_1 = q_1^*$. For the "only if" part, suppose not. Thus, there exists some agent h such that $x_{03}^{*h} < \varphi_1^{*h}(1-\delta_1)C_1$. But since he has less durable than the amount of collateral, even if he sells all the durable to the buyer of asset 1, and reduce the amount of asset 1 sold in the same proportion, it would remain a positive amount $\varphi_1^{*h} - \frac{x_{03}^{*h}}{(1-\delta_1)C_1} > 0$ of assets sold that is not backed by any durable. Thus, asset 1 would *not* be inessential in this case.

■

Note that, without compensation, the collateral constraint implies that asset 1 is always inessential. Next lemma show similar properties to the price of the prime asset.

Lemma 3. *Let* $((x^*, \{x_{s3}^*\}_{s\in S^*}, \psi^*, \varphi^*), \mu^*, p^*, q^*)$ *be an equilibrium and* $\xi_{sj} = 0$ *for* $j = S$ *and for all* s*.*

(a) Suppose that asset S *is transacted. Then*

i) $\delta_S = 0 \implies q_S^* = \frac{1}{1+i}$

ii) $\delta_S \in \left(0, 1 - \frac{1}{p_{13}^* C_S}\right) \implies q_S^* \in \left((p_{03}^* - p_{02}^*)(1-\delta_S)C_S, \frac{1}{1+i}\right)$

iii) $1 - \frac{1}{p_{13}^* C_S} \leq \delta_S \implies q_S^* = (p_{03}^* - p_{02}^*)(1-\delta_S)C_S$

(b) i) If $\delta_S = 0$ *and* $\mu^{*h} \geq \varphi_S^{*h} C_S$ *for all agents, then asset* S *is inessential*

ii) If $\delta_S > 1 - \frac{1}{p_{13}^* C_S}$*, then asset* S *is inessential*

Proof. (a) i) In this case the same argument of Araujo et al. (2015) applies.

ii) If $q_S^* > \frac{1}{1+i}$, no one would have the incentive to buy the asset S because of the money. And if $q_S^* < (p_{03}^* - p_{02}^*)(1-\delta_S)C_S$ there will be no incentive to sell asset S because of the durable $(1-\delta_S)C_S$.

iii) In this case the asset S turns out to be equivalent to the subprime. Thus, the same argument of the item (a) of the previous lemma applies to conclude that $q_S^* = (p_{03}^* - p_{02}^*)(1-\delta_S)C_S$.

(b) For the first case, the proof is analogous to the inessentiability case of the previous lemma. In this case, agent h could simply sell money μ^{*h} directly to the other agents buying asset S and reduce his position in this asset to zero. This procedure would not affect any payoff in any state or the equilibrium conditions, whence asset S is inessential. For the second case, since $\delta_S > 1 - \frac{1}{p_{13} C_S}$, the prime asset is now giving default in all states, therefore it is now subprime. Thus, the inessentiability follows from the collateral constraint in the same way as in the inessentiability of lemma 1.

■

A comment on the above lemmas. The inessentiability for asset S when $\delta_S = 0$ requires the additional hypothesis that $\mu^{*^h} \geq \varphi_S^{*^h} \frac{1}{1+i}$ because the non-collateralized object that mimics asset S in this case is the money, over wh ich there is no additional constraint similar to the collateral constraint. The agents choose the amount of money only subject to the positivity constraint. For $\delta_S > 0$, the money is no longer equivalent to the asset S because it starts giving default in some states and therefore loses its property of being riskless.

Corollary 1. *Suppose $\xi_{s1} = 0$ for all $s \in S$. Then the collateral requirement regulation in the subprime does not change the equilibrium.*

Proof. Direct consequence of lemma 2. ■

In the numerical analysis the model will be restricted to two states in period 1 ($S = 2$), with the same probability of occurance, and the same utility function for all agents, given by:

$$U^h(x^h) = u(x_{01}^h, x_{02}^h) + \tfrac{1}{2}u(x_{11}^h, x_{12}^h) + \tfrac{1}{2}u(x_{21}^h, x_{22}^h)$$

where $x^h \equiv (x_{01}^h, x_{02}^h, x_{11}^h, x_{12}^h, x_{21}^h, x_{22}^h)$ and $u(\cdot,\cdot)$ is a function that does not depend on the states or on the agents.

This is done to simplify the analysis and make the agent's choice of the demand depend exclusively on the endowment distribution. Hence, different demand choices would be due differences in the initial endowment. Since there are only two states in period 2, there will be only two financial assets, the prime and the subprime.

The technical hypothesis over $u(\cdot,\cdot)$ are: *Inada condition*, *continuity*, *strictly increasing*, *strictly concave* and *homothetic*. These hypothesis and the first order conditions of the optimization problem of the agents imply that $\frac{x_{s1}^h}{x_{s2}^h}$ is uniquely determined by $\frac{p_{s2}}{p_{s1}}$ due to following equality[10]:

$$\frac{\partial_2 u\left(1, \frac{x_{s1}^h}{x_{s2}^h}\right)}{\partial_1 u\left(1, \frac{x_{s1}^h}{x_{s2}^h}\right)} = \frac{p_{s2}}{p_{s1}}$$

Once the fraction $\frac{x_{s1}^h}{x_{s2}^h}$ is uniquely determined as a function of $\frac{p_{s2}}{p_{s1}}$ and is equal to every agent h, because they are subject to the same utility function $u(\cdot,\cdot)$ in every state, then in equilibrium the relative prices $\frac{p_{s2}}{p_{s1}}$ must be determined from the aggregate endowment of the economy[11]

[10]Using the equations and notations from the appendix, first note that the Inada condition implies that $x_{sl}^h > 0$ for all $s = 0, 1, 2$, $l = 1, 2$ and all h. Thus, ${}_{col}\mu_l^h = {}_x\mu_{sl}^h = 0$ for all $s = 0, 1, 2$, $l = 1, 2$ and all h. The first order condition for $l = 1, 2$ in $s = 0$ then is $\partial_{0l} u^h(x^h) - \mu_0^h p_{0l} = 0$, because $Y_{s_{l,l'}} = 0$ for $l = 1, 2$. And the first order condition for $s = 1, \ldots, S$ becomes $\partial_{sl} u^h(x^h) - \mu_s^h p_{sl} = 0$. Therefore, the equality $\frac{\partial_2 u(x^h)}{\partial_1 u(x^h)} = \frac{p_{s2}}{p_{s1}}$ holds. The uniqueness can be formalized in the following way. If another $\frac{\overline{x}_{s1}^h}{\overline{x}_{s2}^h} \neq \frac{x_{s1}^h}{x_{s2}^h}$ exists, each point $(\overline{x}_{s1}^h, \overline{x}_{s2}^h)$ and (x_{s1}^h, x_{s2}^h) belongs to a different ray passing through the origin. If, without loss of generality, $u(\overline{x}_{s1}^h, \overline{x}_{s2}^h) > u(x_{s1}^h, x_{s2}^h)$, one can use a standard argument with continuity and strongly increasing to conclude that exists $\bar{t} \in \mathbb{R}_+$ such that $u(\overline{x}_{s1}^h, \overline{x}_{s2}^h) = u(\bar{t}x_{s1}^h, \bar{t}x_{s2}^h)$. The homothetic property implies that $\partial_i u(\cdot,\cdot)$ is homogeneous of zero degree, therefore $\frac{\partial_2 u\left(1, \frac{\overline{x}_{s1}^h}{\overline{x}_{s2}^h}\right)}{\partial_1 u\left(1, \frac{\overline{x}_{s1}^h}{\overline{x}_{s2}^h}\right)} = \frac{\partial_2 u\left(1, \frac{\bar{t}x_{s1}^h}{\bar{t}x_{s2}^h}\right)}{\partial_1 u\left(1, \frac{\bar{t}x_{s1}^h}{\bar{t}x_{s2}^h}\right)} = \frac{p_{s2}}{p_{s1}}$. The same marginal utility of substitution at different points over the same indifference curve is incompatible with the strict concavity.

[11]Indeed, if $\frac{x_{s1}^h}{x_{s2}^h} = c$ for all h, then $\sum_h x_{s1}^h = c\sum_h x_{s2}^h \implies \sum_h e_{s1}^h = c\sum_h e_{s2}^h \implies c = \frac{\sum_h e_{s1}^h}{\sum_h e_{s2}^h}$, where the first implication is true only in equilibrium due to the market clearing.

through the following equation;

$$\frac{\partial_2 u\left(1, \frac{e_{s1}}{e_{s2}}\right)}{\partial_1 u\left(1, \frac{e_{s1}}{e_{s2}}\right)} = \frac{p_{s2}}{p_{s1}}$$

where $e_{sl} = \sum_h e_{sl}^h$ for $l = 1, 2$.

Considering that $p_{s2} = p_{s3}$ for all $s = 1, \cdots, S$ and that the Central Bank can guarantee the value of the money in the first period by defining the prices in state $s = 1, \cdots, S$ fixing $p_{s1} = 1$ for all $s = 1, \cdots, S$, then p_{s2} is also known and there is no price left to determine in the second period. In $s = 0$ the price of the money is not normalized and also the first order conditions does not imply anymore that $p_{02} = p_{03}$. Therefore, the only prices in the economy left to be determined is p_{01} and p_{03} because p_{02}/p_{01} is still fixed in the state 0.

In models with financial markets, the wealth available at state s is determined not only by the endowments at that state but also by the vector of transferences chosen by the agent. With the presence of financial assets and durable non-collateralized objects, agents have the possibility to carry resources interstate and interperiod. Thus, one can define the vector of net transferences of wealth to the states of the second period in the following way, in units of perishable:

$$y_s^h = \left(\frac{1+i}{p_{s1}}\right)\mu^h + \frac{1}{p_{s1}}\left(\psi_2^h - \varphi_2^h\right)\min\{1, p_{s3}(1-\delta_2)C_2\} + \left(\frac{p_{s3}}{p_{s1}}\right)\left[x_{03}^h + (\psi_1^h - \varphi_1^h)(1-\delta_1)C_1\right]$$

The third part of the sum is the *effective position in risky durable*, that is, the total position in subprime and in the durable, its "equivalent". The y_s^h represents the *purchase power* carried by agent h to state s and a negative vector, if possible, would mean that agent h is bringing resources from state s of the second period to $s = 0$.

Since the durable is one of the objects used to define the agent's transference vector $y^h = (y_1^h, y_2^h)$, it is reasonable to think that the collateral constraint would also impose some constraint over the set of y^h available to agent h. Although not formally proven, Araujo et al. (2015) suggested the following relation between the collateral constraint and the set of possible transferences for the case $\delta_j = 0$ for all j:

$$x_{03}^h \geq \varphi_1^h C_1 + \varphi_2^h C_2 \Longrightarrow \begin{cases} p_{21} y_2^h \leq p_{11} y_1^h \\ y_2^h \geq 0 \end{cases}$$

The inequality $p_{21}y_2^h \leq p_{11}y_1^h$ is called *short sale constraint* by them and $y_2^h \geq 0$ *leverage constraint*. Next lemma shows that a new short sale constraint must be defined in order to properly adapt the previous result to the model presented here.

Lemma 4. *Suppose that $\xi_{sj} = 0$ for all s, j. Consider the following three items:*

(a) $x_{03}^h \geq \varphi_1^h(1-\delta_1)C_1 + \varphi_2^h(1-\delta_2)C_2$

(b) $\begin{cases} p_{21}y_2^h \leq (1-\delta_2)p_{11}y_1^h + \delta_2(1+i)\mu^h + \delta_2 p_{13}[x_{03}^h + (1-\delta_1)(\psi_1^h - \varphi_1^h)C_1] \\ y_2^h \geq 0 \end{cases}$

(c) $\begin{cases} p_{21}y_2^h \leq \dfrac{p_{11}y_1^h}{p_{13}C_2} + \left(1 - \frac{1}{p_{13}C_2}\right)(1+i)\mu^h + \left(1 - \frac{1}{p_{13}C_2}\right)p_{13}[x_{03}^h + (1-\delta_1)(\psi_1^h - \varphi_1^h)C_1] \\ y_2^h \geq 0 \end{cases}$

Then,

- $\delta_2 \leq 1 - \frac{1}{p_{13}C_2}$ *implies that* (a) $\Longrightarrow$ (b)
- $\delta_2 \geq 1 - \frac{1}{p_{13}C_2}$ *implies that* (a) $\Longrightarrow$ (c)

Proof. $\boxed{(a) \implies (b)}$

First note that if $\delta_2 \leq 1 - \frac{1}{p_{13}C_2}$ then $\min\{1, p_{13}(1-\delta_2)C_2\} = 1$. Thus, the right hand side of the new short sale constraint in (b) can be rewritten in the following way:

$$\begin{aligned}
&(1-\delta_2)p_{11}y_1^h + \delta_2(1+i)\mu^h + \delta_2 p_{13}[x_{03}^h + (1-\delta_1)(\psi_1^h - \varphi_1^h)C_1] \\
=&(1+i)\mu^h + (1-\delta_2)(\psi_2^h - \varphi_2^h) + p_{13}[x_{03}^h + (\psi_1^h - \varphi_1^h)(1-\delta_1)C_1] \qquad (1)
\end{aligned}$$

Recalling that the prime asset ($j = 2$) defaults in the bad state of nature ($s = 2$), the left hand side of the constraint becomes:

$$\begin{aligned}
p_{21}y_2^h &= (1+i)\mu^h + (\psi_2^h - \varphi_2^h)p_{23}(1-\delta_2)C_2 + p_{23}[x_{03}^h + (\psi_1^h - \varphi_1^h)(1-\delta_1)C_1] \\
&= (1+i)\mu^h + (1-\delta_2)(\psi_2^h - \varphi_2^h) + p_{23}[x_{03}^h + (\psi_1^h - \varphi_1^h)(1-\delta_1)C_1] \qquad (2)
\end{aligned}$$

where the last equality comes from $p_{23}C_2 = p_{23}\frac{1}{p_{23}} = 1$. The hypothesis (a) implies that $x_{03}^h + (\psi_1^h - \varphi_1^h)(1-\delta_1)C_1 \geq 0$. Thus, since $p_{13} > p_{23}$, then a direct comparison between the addends of expressions (1) and (2) shows that the new short sale constraint must hold.

For the leverage constraint, just note that

$$\begin{aligned}
y_2^h &= \tfrac{(1+i)}{p_{21}}\mu^h + \tfrac{p_{23}}{p_{21}}[x_{03}^h + (\psi_1^h - \varphi_1^h)(1-\delta_1)C_1 + (\psi_2^h - \varphi_2^h)(1-\delta_2)C_2] \qquad (3) \\
&\geq 0
\end{aligned}$$

because $\mu^h \geq 0$ and $x_{03}^h + (\psi_1^h - \varphi_1^h)(1-\delta_1)C_1 + (\psi_2^h - \varphi_2^h)(1-\delta_2)C_2 \geq 0$ due to the collateral constraint.

$\boxed{(a) \implies (c)}$

Just follow the same reasoning of $(a) \implies (b)$, noting that $\delta_2 \geq 1 - \frac{1}{p_{13}C_2}$ implies that $\min\{1, p_{13}(1-\delta_2)C_2\} = p_{13}(1-\delta_2)C_2$ and the right hand side of the short sale constraint in (c) becomes exactly expression (1).

■

The last lemma has important properties and counterintuitives behavior with this policy. First, the interesting consequences for the feasible set of transferences of the agents happens in equilibrium, when μ^h is limited by $m = \sum_h m^h$. Without this limitation the transference cone of the agents is indeed the same of Araujo et al. (2015). When μ is limited, however, it is possible to see the counterintuitive fact that relaxing the collateral requirement actually leads to a *tightening* of the feasible transferences vectors in equilibrium.

Note that a quick and unadvised analysis of the model may induce one to conclude that relaxing the collateral requirement would favor a relaxation of the short sale constraint. Using the original short sale of Araujo et al. (2015), $p_{21}y_2^h \leq p_{11}y_1^h$, as a reference, if $\delta_2 > 0$ and $\mu^h \leq m$, then the prime asset starts giving default in the bad state. Therefore, the buyer of asset 2 will transfer $\psi_2^h \min\{1, p_{13}(1-\delta_2)C_2\}$ to state 1 and $\psi_2^h(1-\delta_2)$ for state 2. Since $\min\{1, p_{13}(1-\delta_2)C_2\} > (\psi_2^h - \varphi_2^h)(1-\delta_2)$ for every $\delta_2 \in (0, 1]$, because $p_{13} > p_{23}$, then $p_{11}y_1^h > p_{21}y_2^h$ for all $\delta_2 > 0$. This would "prove" that the collateral requirement relaxation leads to a short sale relaxation.

However, the above reasoning is misleading. The concept of being short sale constrained is not defined by the equality of the original short sale constraint of Araujo et al. (2015), that is, that agent h chooses transfers y_1^h and y_2^h such that $p_{11}y_1^h = p_{21}y_2^h$. Being short sale constrained means that the agent is using the financial assets and non-collateralized objects of the economy in order to bring as much resource as possible from the good state to the bad state. If the objects of the economy only allows for $0.5p_{11}y_1^h = p_{21}y_2^h > 0$, then if an agent h has $0.5p_{11}y_1^h = p_{21}y_2^h$ one should redefine the short sale constraint to consider that he is actually constrained, although it is true that $p_{11}y_1^h > p_{21}y_2^h$. The inequality of the new short sale constraint is less sharper than the inequality of the original short sale constraint, that is, the collateral requirement relaxation creates a new gap between the transferences of the two states wich tightens the constraint. Formally, the set of feasible transferences *in equilibrium* of (b) and (c), are proper subsets of the original set of feasible transferences in equilibrium obtained in Araujo et al. (2015).

Lemma 4 also wrongly induces one to think that a binding collateral constraint is equivalent to either a binding short sale constraint or a binding leverage constraint. Indeed, the next very simple result shows that a binding collateral constraint is not enough to conclude that one of the constraints over the transferences, the short sale constraint or the leverage constraint, is binding. More information on the portfolio of the agents is needed.

Lemma 5. *Suppose $\delta_j < 1$ for all j. Then, the following equivalence holds:*

(a) the new short sale constraint is binding $\iff \begin{cases} x_{03}^h = \varphi_1^h(1-\delta_1)C_1 \\ \psi_1^h = 0 \end{cases}$

(b) the leverage constraint is binding $\iff \begin{cases} x_{03}^h = \varphi_1^h(1-\delta_1)C_1 + \varphi_2^h(1-\delta_2)C_2 \\ \psi_1^h = \psi_2^h = 0 \\ \mu^h = 0 \end{cases}$

Proof. (a) Based on expression (2) of the previous lemma, one can write the left hand side of the short sale constraint for any $\delta_2 \in [0,1]$ as

$$p_{21}y_2^h = (1+i)\mu^h + \psi_2^h(1-\delta_2) + p_{23}\psi_1^h(1-\delta_1)C_1 + p_{23}[x_{03}^h - \varphi_1^h(1-\delta_1)C_1 - \varphi_2^h(1-\delta_2)C_2] \quad (1)$$

and from expression (1) of the previous lemma the right hand side can be written in the following way

$$\begin{aligned} &(1+i)\mu^h + (\psi_2^h - \varphi_2^h)(1-\delta_2) + p_{13}[x_{03}^h + (\psi_1^h - \varphi_1^h)(1-\delta_1)C_1] \\ =&(1+i)\mu^h + \psi_2^h(1-\delta_2) + p_{13}\psi_1^h(1-\delta_1)C_1 + (p_{13}C_2 - 1)\varphi_2^h(1-\delta_2) + \\ &p_{13}[x_{03}^h - \varphi_1^h(1-\delta_1)C_1 - \varphi_2^h(1-\delta_2)C_2] \end{aligned} \quad (2)$$

If the short sale constraint is binding, then

$$(p_{13} - p_{23})\psi_1^h + (p_{13} - p_{23})[x_{03}^h - \varphi_1^h(1-\delta_1)C_1 - \varphi_2^h(1-\delta_2)C_2] = -(p_{13}C_2 - 1)\varphi_2^h(1-\delta_2)$$

and finally

$$(p_{13} - p_{23})\psi_1^h + (p_{13} - p_{23})[x_{03}^h - \varphi_1^h(1-\delta_1)C_1] = 0$$

Due to the collateral constraint and the fact that $\psi_1^h \geq 0$ all terms of the left hand side of the last equation is not negative. Then each of them must be zero. Considering that $p_{13} > p_{23}$, then $x_{03}^h - \varphi_1^h(1-\delta_1)C_1 = 0$ and $\psi_1^h = 0$. On the other way, if $x_{03}^h - \varphi_1^h(1-\delta_1)C_1 = 0$ and $\psi_1^h = 0$, then a direct comparison between expressions (1) and (2) shows that the short sale constraint must also bind.

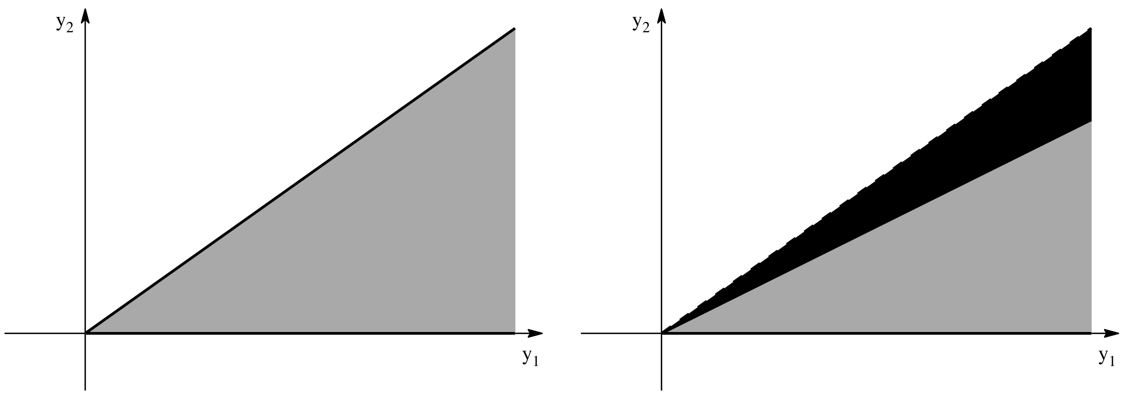
(b) Consider $y_2^h = (1+i)\mu^h + \psi_2^h(1-\delta_2) + p_{23}\psi_1^h(1-\delta_1)C_1 + p_{23}[x_{03}^h - \varphi_1^h(1-\delta_1)C_1 - \varphi_2^h(1-\delta_2)C_2]$. Since all the terms of y_2^h is positive, if the leverage constraint is binding, then $\mu^h = \psi_1^h = \psi_2^h = x_{03}^h - \varphi_1^h(1-\delta_1)C_1 - \varphi_2^h(1-\delta_2)C_2 = 0$. The other implication is trivial.

■

The next figure depicts the difference between the original consumer's set of feasible transferences in equilibrium and the same set, also in equilibrium, with a relaxed collateral requirement. This is also related to Araujo et al. (2012) because in that paper they study changes in the collateral requirement of the assets.

Figure 2.2: Feasible Transference Set

(a) Original Araujo et al. (2015) set (b) Set with $\delta_j > 0$



The black region of figure 2.2 (b) is the set of transferences that were feasible in the original cone with $\delta_j = 0$ and changes to unfeasible when $\delta_j > 0$ in equilibrium. This figure represents the case with $m = 0$, which is approximately the value defined in the numerical analysis.

2.2.3 Theoretical properties: with positive compensation

The following analysis will consider the case in which $\alpha > 0$ and therefore ξ_{sj} can be positive. It will be shown that this provides important changes in the results previously proven. Furthermore, the following analysis together with the numerical results of the next section suggest that the compensation ξ_{sj} is crucial to understand how this regulation enables for Pareto improvement effects.

The definition $\xi_{sj} \equiv \alpha\left(\min\{1, p_{s3}C_j\} - \min\{1, p_{s3}(1-\delta_j)C_j\}\right)$ given before allows for three possible cases. First, if $s \geq j$, then the original asset defaults for any $\delta_j \in [0,1]$ because $p_{s3}C_j \leq 1$. In this case, the relaxed asset also defaults and the compensation should be $\xi_{sj} = \alpha p_{s3}C_j\delta_j$. If $s > j$, then $p_{s3}C_j > 1$ and the original asset does not default in state s. In this case the definition of the compensation ξ_{sj} will depend on whether the relaxed asset will default or not. And the default of the relaxed asset will depend on the size δ_j of the relaxation. If $\delta_j \leq 1 - \frac{1}{p_{s3}}$ then it does not default and there should be no government intervention, that is, $\xi_{sj} = 0$. But if $\delta_j > 1 - \frac{1}{p_{s3}C_j}$ the compensation should be $\xi_{sj} = \alpha\left(\min\{1, p_{s3}C_j\} - \min\{1, p_{s3}(1-\delta_j)C_j\}\right) = \alpha(1 - p_{s3}(1-\delta_j)C_j) = \alpha(p_{s3}C_j\delta_j - p_{s3}C_j + 1)$. This is summed up in the following way:

$$\xi_{sj} = \begin{cases} 0 & \text{if } s < j \text{ and } \delta_j \leq 1 - \frac{1}{p_{s3}C_j} \\ \alpha(p_{s3}C_j\delta_j - p_{s3}C_j + 1) & \text{if } s < j \text{ and } \delta_j > 1 - \frac{1}{p_{s3}C_j} \\ \alpha p_{s3}C_j\delta_j & \text{if } s \geq j \end{cases}$$

When the government is in the economy to guarantee the proportion $\alpha \in [0,1]$ of the additional default caused by the relaxation δ of the collateral requirement, some of the previous results changes. For example, $\alpha > 0$ will sustain the rank of the original matrix of returns V even for high values of δ_j and the price of the subprime will be higher than when $\xi_{sj} = 0$.

Lemma 6. *Let* $((x^*, \{x^*_{s3}\}_{s\in S^*}, \psi^*, \varphi^*), \mu^*, p^*, q^*)$ *be an equilibrium with* $\alpha > 0$*. Then:*

(a) If asset 1 is transacted, then $q_1^* = (p^*_{03} - p^*_{02})(1 - (1-\alpha)\delta_1)C_1$.

(b) If $\alpha' > \alpha$*, then* $q_1^{*'} > q_1^*$ *for every* $\delta_1 > 0$.

(c) Asset 1 is inessential if, and only if, $x^h_{03} \geq \varphi^h_1(1 - (1-\alpha)\delta_1)C_1$ *for all agents.*

Proof. (a) The proof is analogous to the proof of lemma 2, the only difference is that in this case one unit of subprime is equivalent in payoff to $(1-(1-\alpha)\delta_1)C_1$ units of durable.

(b) Direct corollary of the previous item.

(c) This proof is also analogous to the proof of lemma 2.

■

The *effective* matrix of returns of the assets, denoted by V_δ^ξ, is defined as the matrix of returns of the assets with relaxed collateral requirement V_δ plus the matrix of compensation $\xi = (\xi_{sj})_{sj}$ given by the government:

$$\begin{aligned} V_\delta^\xi =& V_\delta + \xi \\ =& \begin{bmatrix} \eta_{11} & 1 & \dots & 1 & 1 \\ \eta_{21} & \eta_{22} & \dots & 1 & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ \eta_{(S-1)1} & \eta_{(S-1)2} & \cdots & \eta_{(S-1)(J-1)} & 1 \\ \eta_{S1} & \eta_{S2} & \cdots & \eta_{S(J-1)} & \eta_{SJ} \end{bmatrix}_{S\times J} + \begin{bmatrix} \xi_{11} & \dots & \xi_{1J} \\ \xi_{21} & \dots & \xi_{2J} \\ \vdots & & \vdots \\ \xi_{(S-1)1} & \cdots & \xi_{(S-1)J} \\ \xi_{S1} & \cdots & \xi_{SJ} \end{bmatrix}_{S\times J} \end{aligned}$$

Next lemma shows that even when δ_j is high enough for each j, so that all the assets of the economy defaults in every state, the compensation succeed in maintaining the financial markets complete, that is, $rank(V_\delta^\xi) = S$.

Lemma 7. *Suppose that $\alpha > 0$ and δ_j is such that every asset j defaults in every state s. Then $rank(V_\delta^\xi) = S$.*

Proof. With the notation $\eta_{sj}^\alpha = p_{s3}(1-(1-\alpha)\delta_j)C_j$ and $\tilde{\eta}_{sj}^\alpha = p_{s3}(1-(1-\alpha)\delta_j - \alpha)C_j + \alpha$, when all the assets defaults in all states V_δ^ξ becomes

$$V_\delta^\xi = \begin{bmatrix} \eta_{11}^\alpha & \tilde{\eta}_{12}^\alpha & \cdots & \tilde{\eta}_{1(J-1)}^\alpha & \tilde{\eta}_{1J}^\alpha \\ \eta_{21}^\alpha & \eta_{22}^\alpha & \cdots & \tilde{\eta}_{2(J-1)}^\alpha & \tilde{\eta}_{2J}^\alpha \\ \vdots & \vdots & & \vdots & \vdots \\ \eta_{(S-1)1}^\alpha & \eta_{(S-1)2}^\alpha & \cdots & \eta_{(S-1)(J-1)}^\alpha & \tilde{\eta}_{(S-1)J}^\alpha \\ \eta_{S1}^\alpha & \eta_{S2}^\alpha & \cdots & \eta_{S(J-1)}^\alpha & \eta_{SJ}^\alpha \end{bmatrix}_{S\times J}$$

Note that $\eta_{js}^\alpha, \tilde{\eta}_{js}^\alpha > 0$ for all $\alpha > 0$. Substituting row_s with $row_s - \frac{p_{s3}}{p_{S3}} row_S$ for all $s \leq S-1$

$$V_\delta^\xi \sim \begin{bmatrix} 0 & \alpha(1-p_{13}C_2) & \dots & \alpha(1-p_{13}C_{J-1}) & \alpha(1-p_{13}C_J) \\ 0 & 0 & \dots & \alpha(1-p_{23}C_{J-1}) & \alpha(1-p_{23}C_J) \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \alpha(1-p_{S3}C_J) \\ \eta_{S1}^\alpha & \eta_{S2}^\alpha & \cdots & \eta_{(J-1)S}^\alpha & \eta_{SJ}^\alpha \end{bmatrix}_{S\times J}$$

and $\alpha(1-p_{s3}C_j) < 0$ for all $s < j$. Finally, substituting col_j with $col_j - \frac{\eta_{jS}^\alpha}{\eta_{JS}^\alpha} col_S$ for all $j \geq 2$,

$$V_\delta^\xi \sim \begin{bmatrix} 0 & \alpha(1-p_{13}C_2) & \dots & \alpha(1-p_{13}C_{J-1}) & \alpha(1-p_{13}C_J) \\ 0 & 0 & \dots & \alpha(1-p_{23}C_{J-1}) & \alpha(1-p_{23}C_J) \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \alpha(1-p_{S3}C_J) \\ \eta_{S1}^\alpha & 0 & \cdots & 0 & 0 \end{bmatrix}_{S\times J}$$

which shows that $rank(V_\delta^\xi) = S$. ■

The previous lemma has a simple but relevant corollary. If there is any government compensation in the second period, it is possible to implement an Arrow Debreu equilibrium with a sufficiently high collateral relaxation.

Corollary 2. *Consider that* $(x^*_\delta, x^*_{s3_\delta}, \psi^*_\delta, \varphi^*_\delta, \mu^*_\delta, p^*_\delta, q^*_\delta)$ *is an equilibrium for a given regulation* $\delta = (\delta_1, \ldots, \delta_J)$. *If* $\alpha > 0$ *and* $min\{\delta_1, \ldots, \delta_J\} \longrightarrow 1$, *then the equilibrium* $(x^*_\delta, x^*_{s3_\delta}, \psi^*_\delta, \varphi^*_\delta, \mu^*_\delta, p^*_\delta, q^*_\delta)$ *converges to an Arrow Debreu equilibrium.*

Proof. First note that by lemma 7 the financial markets are complete. Thus, for any level of δ, if the agents are not subject to the collateral constraint, an Arrow Debreu equilibrium can be achieved with this financial market structure.

The main consequence of the condition $min\{\delta_1, \ldots, \delta_J\} \longrightarrow 1$ is that the collateral constraint of all agents vanishes. Thus, the required level of financial transactions necessary to achieve the Arrow Debreu equilibrium with the complete matrix V^ξ_δ will be eventually not constrained by the collateral constraint. Thus, the Arrow Debreu equilibrium will be achieved for some $min\{\delta_1, \ldots, \delta_J\} = \overline{\delta} < 1$ and will be constant for $\overline{\delta} \leq min\{\delta_1, \ldots, \delta_J\} \leq 1$.

Note also that the elements of the matrix V^ξ_δ converges to elements of matrix V^ξ_1. ■

One important feature is that, with $\alpha > 0$, asset 1 may loses its property of being inessential. It may happen that $\varphi^h_1(1-\delta_1)C_1 \leq x^h_{03} < \varphi^h_1(1-(1-\alpha)\delta_1)C_1$, that is, although the collateral constraint is accomplished, the condition given by item (c) of lemma 6 may fail. In this case, lemma 1 is no longer true because the compensation creates the possibility of new transferences between the states that did not exist without it.

Indeed, with a new definition of the vector of transferences this important consequence can be seen clearly. It is necessary to consider the effect of the taxation/compensation, made by the government in period 2, in the agent's originally chosen vector of transferences y^h.

$$\tilde{y}^h_s = y^h_s + \tfrac{1}{p_{s1}} \sum_j (\psi^h_j - \varphi^h_j)\xi_{sj}$$

The vector $\tilde{y}^h = (\tilde{y}^h_1, \tilde{y}^h_2)$ is the *real* or *effective* transference made by the agents. Substituting the variables in the right hand side of the equivalence of lemma 4, if $\delta_j \leq 1 - \frac{1}{p_{13}C_2}$ one gets:

$$\begin{cases} p_{21}\tilde{y}^h_2 \leq (1-\delta_2)p_{11}\tilde{y}^h_1 + \sum_j(\psi^h_j - \varphi^h_j)(\xi_{2j} - (1-\delta_2)\xi_{1j}) + \delta_2(1+i)\mu^h + \\ \qquad +\delta_2 p_{13}[x^h_{03} + (1-\delta_1)(\psi^h_1 - \varphi^h_1)C_1] \\ \tilde{y}^h_2 \geq \frac{1}{p_{21}} \sum_j(\psi^h_j - \varphi^h_j)\xi_{2j} \end{cases}$$

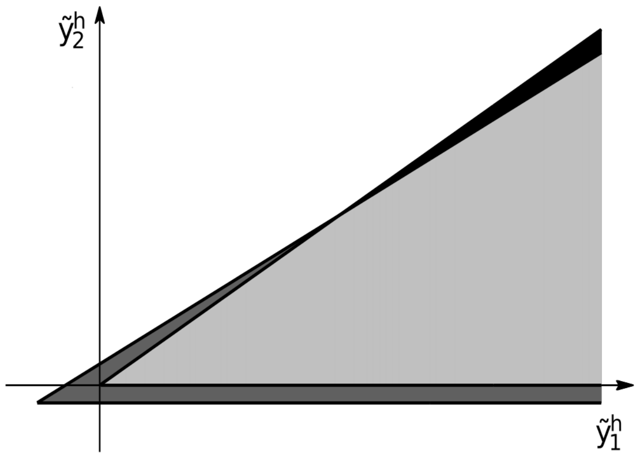
And if $\delta_j \geq 1 - \frac{1}{p_{13}C_2}$ the system becomes:

$$\begin{cases} p_{21}\tilde{y}^h_2 \leq \dfrac{p_{11}\tilde{y}^h_1}{p_{13}C_2} + \displaystyle\sum_j(\psi^h_j - \varphi^h_j)\left(\xi_{2j} - \dfrac{\xi_{1j}}{p_{13}C_2}\right) + \\ \qquad +(1 - \frac{1}{p_{13}C_2})(1+i)\mu^h + (1 - \frac{1}{p_{13}C_2})p_{13}[x^h_{03} + (1-\delta_1)(\psi^h_1 - \varphi^h_1)C_1] \\ \tilde{y}^h_2 \geq \frac{1}{p_{21}} \sum_j(\psi^h_j - \varphi^h_j)\xi_{2j} \end{cases}$$

Analyzing the above system the first thing to point out is that it can relax both the leverage and the short sale constraint, depending if the portfolio is chosen carefully by the agent. If an agent wants to relax both constraints, he typically needs to sell one asset in order to relax the leverage constraint and buy the other so he can also relax the short sale. This is a major difference from the unconventional monetary policy, whereby there is no possibility to relax the short sale constraint. Another important difference from the unconventional monetary policy is that in this model the level of relaxement is ultimately defined by the portfolio chosen by the agents, that is, depends on choice variables. The unconventional monetary policy affects equally the set of feasible effective transferences for all agents, because the effect depends only on the amount of durable bought by the central bank, which is given exogenously, and on the prices, which is not a choice variable. Note that, as mentioned before, the subprime *is* effective now since it will necessarily changes the leverage constraint of the agents in an important way: the seller will now have the possibility to transfer negative values to the bad state ($s = 2$). This is not possible without government compensation.

The figure below describes a typical change in the feasible set of the effective transferences:

Figure 2.3: Feasible Transference Set After Compensation



The dark grey area is an expansion of the set of transferences, in comparison with the original transference cone, which is an area of relaxement of the leverage constraint or the short sale constraint, and the black area is a contraction of the set of transferences caused by the collateral requirement relaxation.

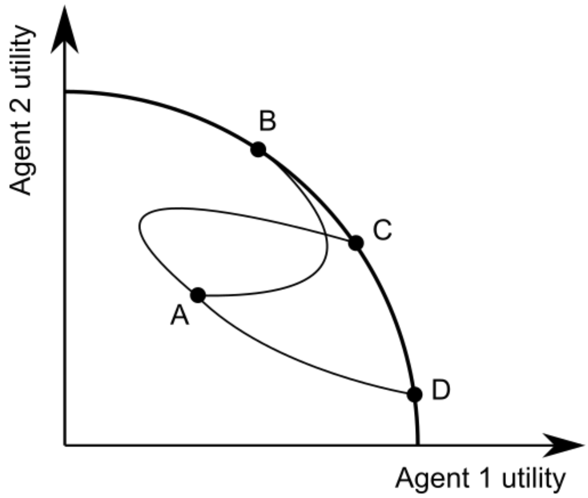
The above discussion about the properties of the compensation, holding the rank of the matrix of returns complete and providing a relaxation of the leverage or short sale constraints, allows to interpret the compensation ξ_{sj} as if the government were introducing a new unsecured asset in the economy.[12] This new asset would be unsecured by the private agents but totally secured by the government. In this intepretaion, the issuer of an asset j put less collateral than its effective delivery. Indeed, with positive compensation the private collateral only protects $\min\{1, p_{s3}(1-\delta_j)C_j\}$ and the ξ_{sj} is guaranteed by the goverment. This is the precise meaning of the government *sharing* the commitment with the agents. From the private point of view, this process ultimately provides a relaxation in the collateral constraint (should not be confused with collateral requirement), which is the direct motive for the translation of the transference cone that produces the dark grey area.

In view of corollary 2, the policy will reach the Arrow-Debreu equilibrium if the parameters of the policy are high enough because it has the property of relaxing the collateral constraint while holding the complete markets. However, it is important to emphasize that there is no *a priori* information on what path is taken by the economy to reach the efficient Arrow Debreu Pareto frontier. The next figure shows different paths the economy A can take to reach the Arrow Debreu Pareto frontier. For example, path $\overline{AD}$ reaches the frontier without any Pareto improvement.

The numerical results will show that the economies typically reaches the AD equilibrium

[12] Villalba (2016) studies an economy with two assets being one secured and the other unsecured. The unsecured asset produces effects allowing new transferences in a similar way with those analyzed here.

Figure 2.4: Possible Paths to the Arrow Debreu Pareto Frontier



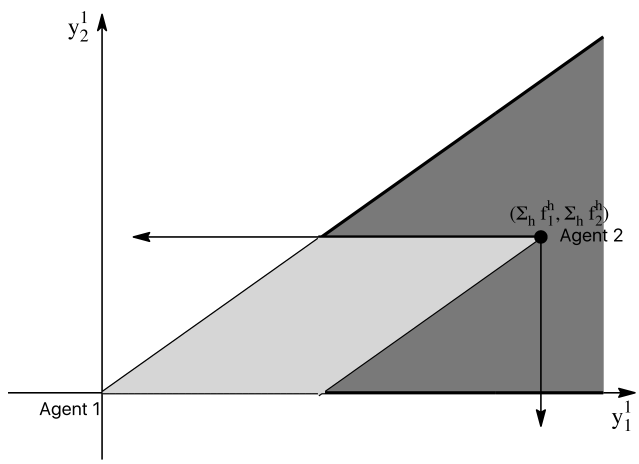
through a path with Pareto improvement.

To end this section, following Araujo et al. (2015), note that all effective transferences of the economy are limited to the aggregate amount of non-collateralized objects of the economy that can transfer wealth through time, which are money and durable. The financial assets are also limited to the durable because of the collateral constraint. Thus, defining $f_s^h = \frac{p_{s3}}{p_{s1}} e_{03}^h + \frac{1+i}{p_{s1}} m^h$, in equilibrium it should be true that:

$$\sum_h^H \tilde{y}_s^h = \sum_h^H f_s^h \text{ for } s = 1, 2$$

This is similar to a market clearing when it is considered that the agent is solving his optimization problem choosing the best effective transferences $\tilde{y}^h = (\tilde{y}_1^h, \tilde{y}_2^h)$. The next figure describes the set of feasible transferences in equilibrium in the case without compensation:

Figure 2.5: Set of Feasible Transferences in Equilibrium with $\delta_j = 0$ and $\xi_{sj} = 0$



2.3 Numerical Results

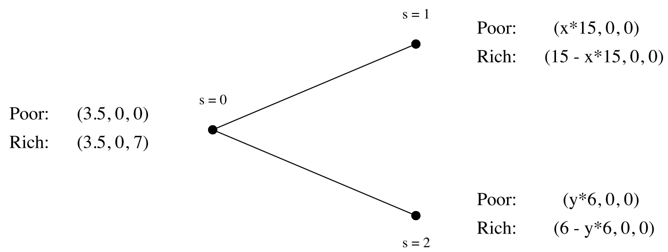
In this section a particular framework of the economy previously described will be defined for the numerical analysis. There will be no calibration to represent any specific society, the objective is to offer a conceptual and pictorial description of some characteristics of the economy related to the regulation of the collateral requirement proposed. In this sense, the endowment distribution does not refer to any concrete distribution of the real world. The analysis involves computing the equilibria of several economies, for different values of the regulatory parameter, and see the impact of this instrument on the endogenous variables and on the welfare of the agents. The computation of the equilibria follows Schommer (2013), which uses KKT conditions to characterizes the equilibrium as a system of equations and then implements it in the software ALGENCAN[13]. Such characterization is possible because each agent's optimization problem is convex. This program uses a Lagrangian Augmented method as described in Andreani et al. (2008).[14]

All the agents have the same usual cobb-douglas utility function:

$$u^h(x) = \sum_l \log(x^h_{0l}) + \sum_s \frac{1}{S} \sum_l \log(x^h_{sl})$$

where each state s has the same subjective probability $\frac{1}{S}$.

The endowment distribution analyzed is the following one:



This endowment distribution is the asymetric case.[15] The aggregate endowment in $s = 0$ is $(7, 0, 7)$, in $s = 1$ is $(15, 0, 0)$ and $(6, 0, 0)$ in $s = 2$. The difference between the aggregate endowment at $s = 1$ and $s = 2$ characterizes the good and bad state of nature, being $s = 1$ the good state and $s = 2$ the bad state. Since the agents are equal except for the endowments, the endowment distribution will typically induce the roles of the agents in the financial markets. One could interpret as if the poor agent in $s = 0$ was, in fact, the rich agent in the last period before $s = 0$ but became poor because of the occurance of the crisis. A bank would fit this description, for example. In this interpretation, the government relaxes the collateral requirement of the bank in $s = 0$, period of crisis, and makes the compensation in the following period. The variables x and y in the endowments of the agents indicates the methodology to create the figures. The x-axis will represent the *proportion* of perishable owned by the poor (borrower) in state 1 (i.e., $x = e^1_{11}/\sum_h e^h_{11} \in [0, 1]$) and the y-axis will represent the proportion of perishable owned by the

[13]See TANGO project (2013).

[14]It was used a Macbook Pro with an Intel Core i7, 2.5GHz and 16 GB 1600 MHz DDR3 for the computations. The tolerance in all cases was 10^{-08}.

[15]One may interpret the state 0 as the occurance of a bad state of an economy that started in a previous period, with less aggregate of perishable and higher inequality of durable. It can also be interpreted in this context that the poor agent in state 0 was, in fact, richer in the previous period but became poorer because of the effects of the occurrance of the bad state.

poor at state 2 (i.e., $y = e^1_{21}/\sum_h e^h_{21} \in [0,1]$).

The endogenous collateral of the subprime and the prime are previously known because of the properties of the model. Indeed, as mentioned in the previous section, all equilibrium relative prices for goods 1 and 2 of the states $s = 1, 2$ depend only on the proportion of aggregate endowments of the corresponding goods. Adding the fact that the utility function is logarithm[16], one gets: $\dfrac{\sum_h e^h_{sl'}}{\sum_h e^h_{sl}} = \dfrac{p^*_{sl}}{p^*_{sl'}} = \dfrac{x^{*h}_{sl'}}{x^{*h}_{sl}}$ $\forall h$. Therefore, in this model:

$$\frac{p^*_{11}}{p^*_{12}} = \frac{\sum_h e^h_{12}}{\sum_h e^h_{11}} = \frac{7}{15} = 0.46666666 \quad \text{and} \quad \frac{p^*_{21}}{p^*_{22}} = \frac{\sum_h e^h_{22}}{\sum_h e^h_{21}} = \frac{7}{6} = 1.16666666$$

Since the central bank set $p_{s1} = 1$ and the model has $p_{s2} = p_{s3}$ for $s = 1, 2$, then $C_1 = \frac{1}{p_{13}} = 0.4666666$ and $C_2 = \frac{1}{p_{23}} = 1.1666666$.

The interest rate i is set to 0.1, the taxation is $\theta^1 = 0.9$ and $\theta^2 = 0.1$ and the endowment of money is $m^1 = 0.0009$ and $m^2 = 0.0001$. These parameters follows Araujo et al. (2015). The relaxation of the assets is made simultaneously making $\delta_1 = \delta_2 = \delta$. Finally, the compensation of the subprime with this setting becomes $\xi_{11} = \alpha\delta_1$ and $\xi_{21} = \alpha 0.4\delta_1$ and the compensation of the prime will be

$$\begin{cases} \xi_{12} = 0 & \text{if } \delta_2 \leq 0.6 \\ \xi_{12} = \alpha(2.5\delta_2 - 1.5) & \text{if } \delta_2 > 0.6 \\ \xi_{22} = \alpha\delta_2 & \text{for all } \delta_2 \in [0,1] \end{cases}$$

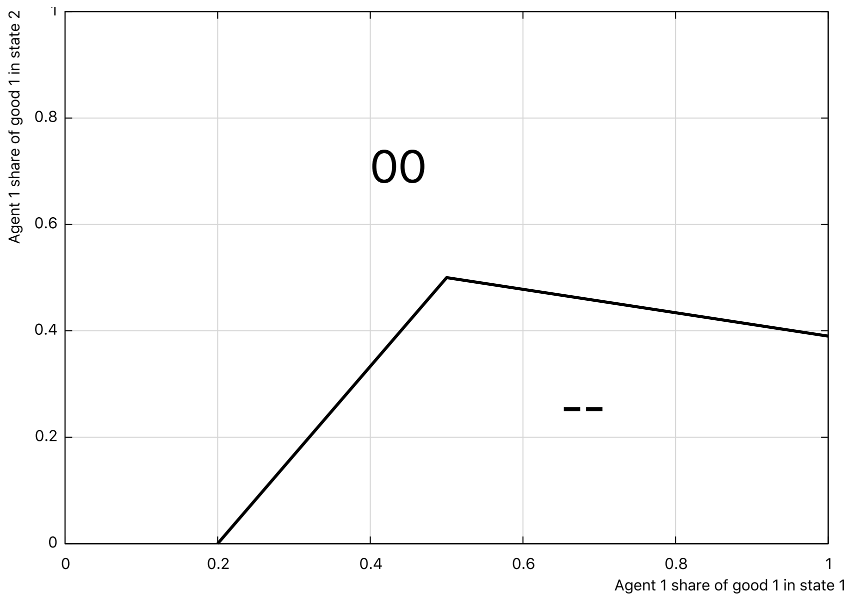
It will be analyzed in the next subsections the following cases: no compensation ($\alpha = 0$), ideal compensation ($\alpha = 1$) and a case of partial compensation ($\alpha = 0.5$).

2.3.1 Case $\alpha = 0$:

The results in this subsection confirm the result in Araujo et al. (2012) because there is no Pareto improvement for any level of relaxation. Each dot of the next graphics is an economy with different distribution of perishable in the second period. The point $(0,0)$ is the economy where agent 1 (poor) has no perishable in period 2, for example. The equilibrium for each economy was calculated first for $\delta = 0$ and second for $\delta = 0.001$ and the utilities of the agents compared. The symbol $--$, for example, indicates that the utility of both agents decreases when the δ changes from 0 to 0.001. The symbols $-+$, $++$ or $+-$ are analogous. And the 00 stands for the case in which the variation of δ has no effect in the utility of the agents. This happens typically when their constraints on the transferences are not binding.

[16]Indeed, from the previous section it is known that $\dfrac{\partial_{s2} u^h(x^{*^h})}{\partial_{s1} u^h(x^{*^h})} = \dfrac{p^*_{s2}}{p^*_{s1}}$ $\forall s$. Using $\partial_{sl} u^h(x^{*^h}) = \dfrac{1}{2x^{*h}_{sl}}$ it follows $\dfrac{x^{*h}_{s1}}{x^{*h}_{s2}} = \dfrac{p^*_{s2}}{p^*_{s1}}$ $\forall h$. Again from the last section this implies $\dfrac{\sum_h e^h_{s1}}{\sum_h e^h_{s2}} = \dfrac{p^*_{s2}}{p^*_{s1}} = \dfrac{x^{*h}_{s1}}{x^{*h}_{s2}}$ $\forall h$.

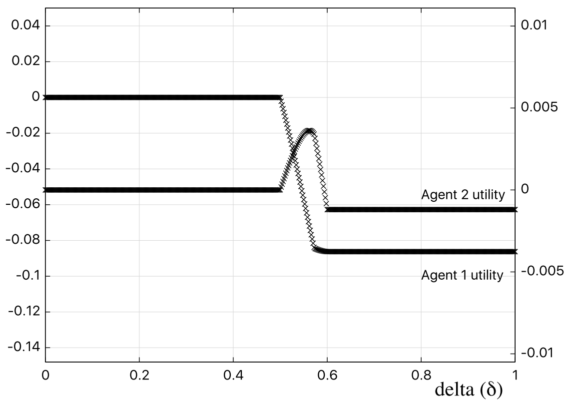
Figure 2.6: Case δ=0.0 and 0.001



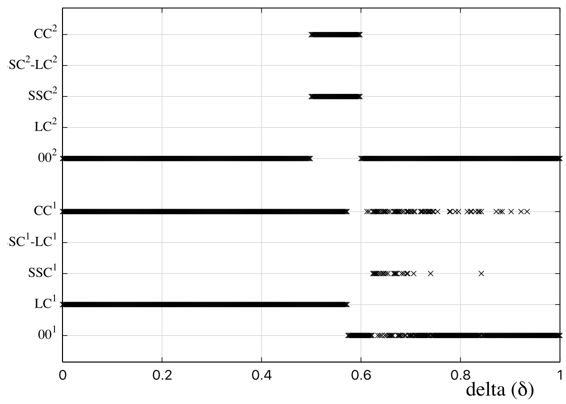
In the next graphic an endowment distribution is fixed and the behavior of the utility of the agents is analyzed as a funcion of δ_j ranging in $[0, 1]$. On the right hand side there is a graphic hightlighting wether the collateral constraint, short sale constraint or the leverage constraint of the agents is binding for each value of δ_j. The symbols CC^h, SSC^h and LC^h are used when the collateral constraint of agent h is binding, the short sale constraint of h is binding and leverage constraint of h is binding, respectively. The symbol 00 is used when the agent is not binding in either in the short sale or the leverage constraint. The endowment distribution, called "Economy A", is: $e^1 = ((3.5, 0, 0), (3, 0, 0), (4.8, 0, 0))$ and $e^2 = ((3.5, 0, 7), (12, 0, 0), (1.2, 0, 0))$. This economy is inside the 00 region.

Figure 2.7: Economy A: utility and constraints over the transferences

(a) Utility



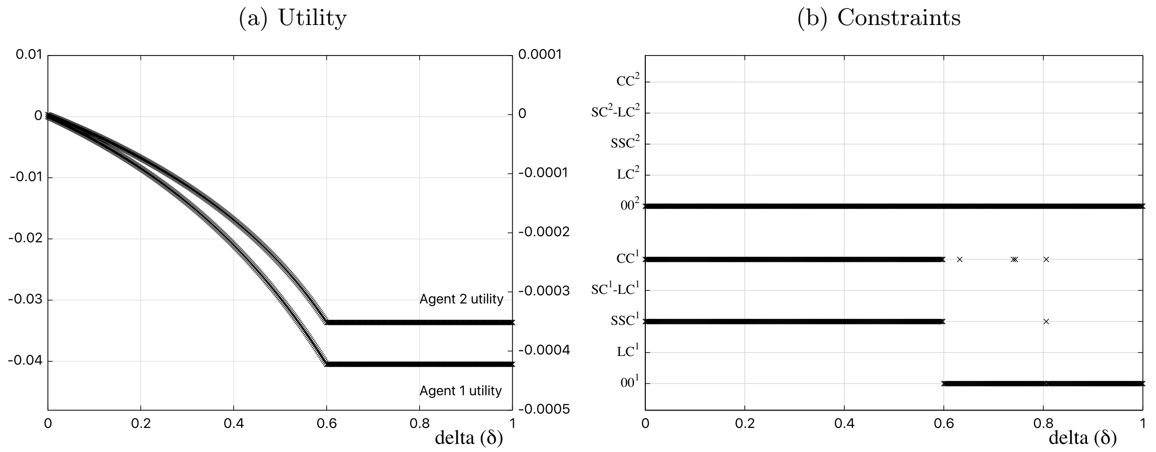
(b) Constraints



The above graphic shows that, although the poor is leverage constrained, the collateral requirement relaxation has no effect in the economy because without compensation there is no effect in the leverage constraint. As explained in the theoretical properties, the collateral requirement relaxation without compensation can only tighten the short sale constraint of the agents. This is exactly what happens when δ is between 0.5 and 0.6, where panel (b) shows that the rich agent is short sale constrained. Before 0.5 the rich agent ($h = 2$) is not constrained in the leverage or in the short sale but between 0.5 and 0.6 he starts being short sale constrained. Thus an increasing in δ will hurt him more and there will be effect in the equilibrium. After $\delta = 0.6$ there is no effect in the economy anymore due to lemma 1 because above this level of δ the prime asset becomes subprime, defaulting in all states.

Below there is another graphic of the utility of the agents, for another endowment distribution in the second period, called "economy B": $e^1 = ((3.5, 0, 0), (9, 0, 0), (1.2, 0, 0))$ and $e^2 = ((3.5, 0, 7), (6, 0, 0), (4.8, 0, 0))$. This economy is within the $--$ region of the figure 2.6.

Figure 2.8: Economy B: utility and constraints



Again, there is effect in the equilibrium only when some agent is short sale constrained. And after $\delta = 0.6$ it no longes produces effect in the equilibrium for the same reason of the economy A.

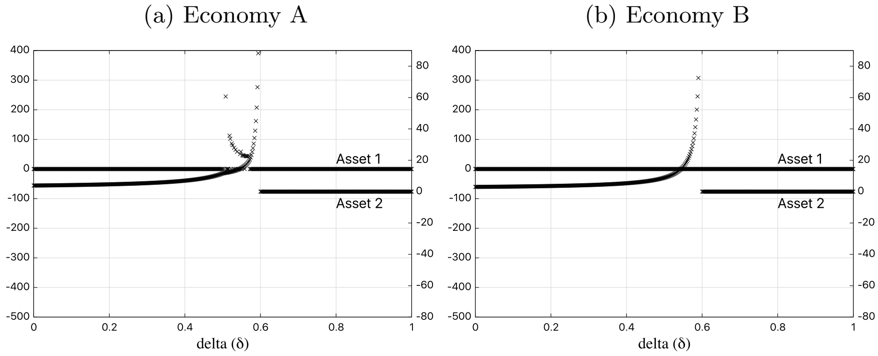
In Geanankoplos (2009) the term "leverage" is used with a different meaning, it is a real number associated to each asset. In this model his definition can be adapted, for each asset j, to the following expression:

$$lev_j = \frac{(p_{03} - p_{02})(1 - \delta_j)C_j}{(p_{03} - p_{02})(1 - \delta_j)C_j - q_j}$$

This definition of leverage for the asset has the common intuition that the value is higher when the amount borrowed q_j when issuing asset j is close to the value of collateral needed to buy to back this promise. The more the value q_j is close to the value $(p_{03} - p_{02})(1 - \delta_j)C_j$ of its collateral, the greater the leverage is. This means that the lender is less secured. For example, an asset that finances 90 and asks for a collateral costing 100 is less leveraged than another one that finances 99 asking for the same collateral.

The next graphic confirm Geanakoplos' intuition that relaxing the collateral requirement increases the leverage of the asset. Relaxing the collateral requirement of the prime leads to an increase in the leverage up to $\delta = 0.6$. After that the prime becomes subprime and therefore $q_2 = (p_{03} - p_{02})(1 - \delta_2)C_2$. The subprime behaves in the same way for all δ, thus it does not changes its leverage. The leverage is not defined for the cases $q_j = (p_{03} - p_{02})(1 - \delta_j)C_j$, therefore when it happens in the graphic this value is adjusted to zero.

Figure 2.9: Leverage of the assets

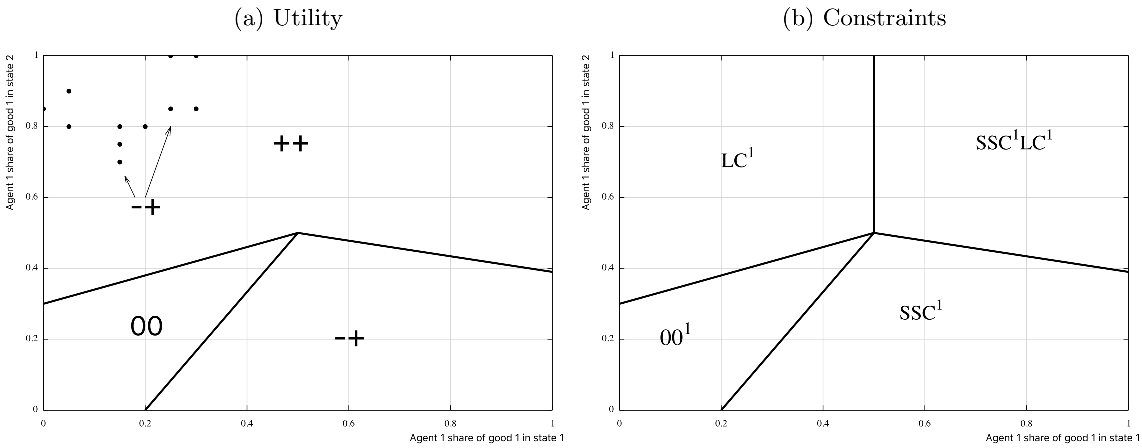


2.3.2 Case $\alpha = 1$:

In this subsection it will be analyzed the case in which the government gives the ideal compensation in order o restore the delivery of the original asset.

This first figure shows that there is a big region of Pareto improvement when the government enters in the economy in period 2 to guarantee the delivery of the new assets with relaxed collateral requirement.

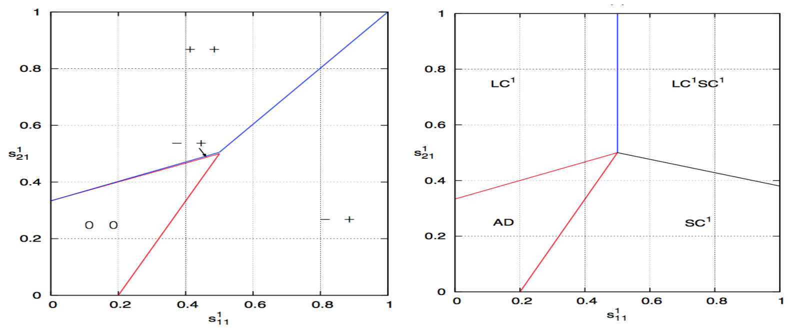
Figure 2.10: Case δ=0.0 and 0.001



The graphic in panel (b) shows the constraints over the transferences of agent 1, short sale or leverage, in each economy of the box. As in the previous subsection, the symbol LC^h is used when the leverage constraint of agent h is binding and SSC^h is similar to the short sale constraint. The constraints of agent 2 are not shown because he is not binding in neither of the constraints, thus being 00^2 in all the economies of the box. The result above is similar to the one found in Araujo et al. (2015) for an initial value of the unconventional monetary policy, as next figure shows:

Figure 2.11: Case ω=0.0 and 0.001

(a) Utility (b) Constraints



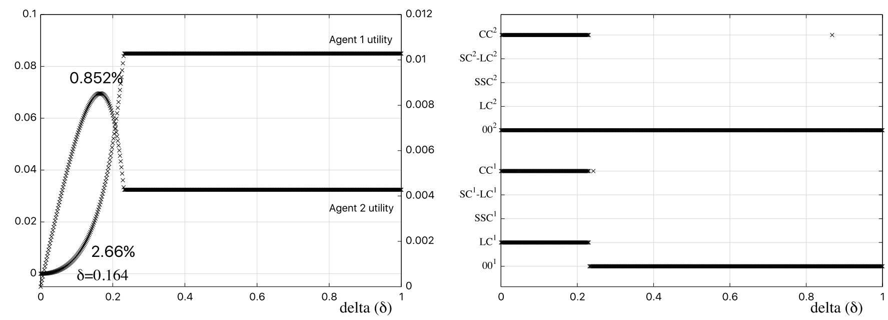
Source: Araujo et al. (2015)

where $\omega \in [0, 1]$ is the proportion of the aggregate of durable bought at $s = 0$ by the Central Bank, which represents the unconventional monetary policy.

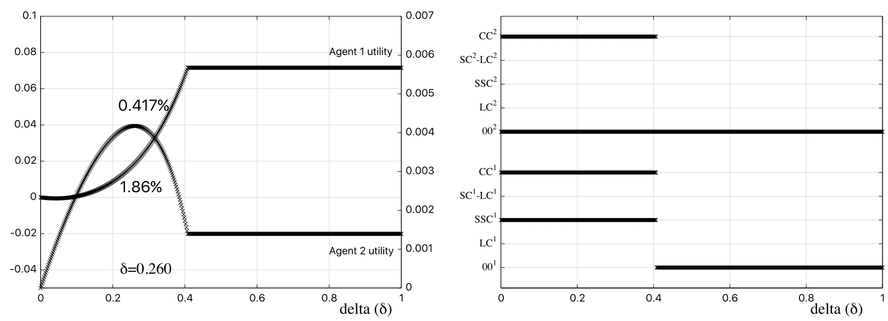
Two differences must be emphasized, however. First, the region of Pareto improvement in policy proposed in this thesis reaches economies in the top left side of the box that is not reached by the unconventional monetary policy of Araujo et al. (2015). It almost covers all the region where agent 1 (poor) is leverage constrained. Second, the slight imprecision in the top left of the box, where some black $-+$ points appears, is due to the fact that in these economies the agent's 1 marginal utility with $\delta = 0$ is almost zero. However, for values slightly greater than 0.001, such as 0.002, the region of Pareto improvement covers all the leverage constrained economies. This is shown by the next graphics of the utility of the agents of the economy A, which lays in the region LC^1, and economy B, which is in the region SSC^1:

Figure 2.12: Utility and Constraints of the Agents

(a) Economy A: Utility (b) Economy A: Constraints



(c) Economy B: Utility (d) Economy B: Constraints

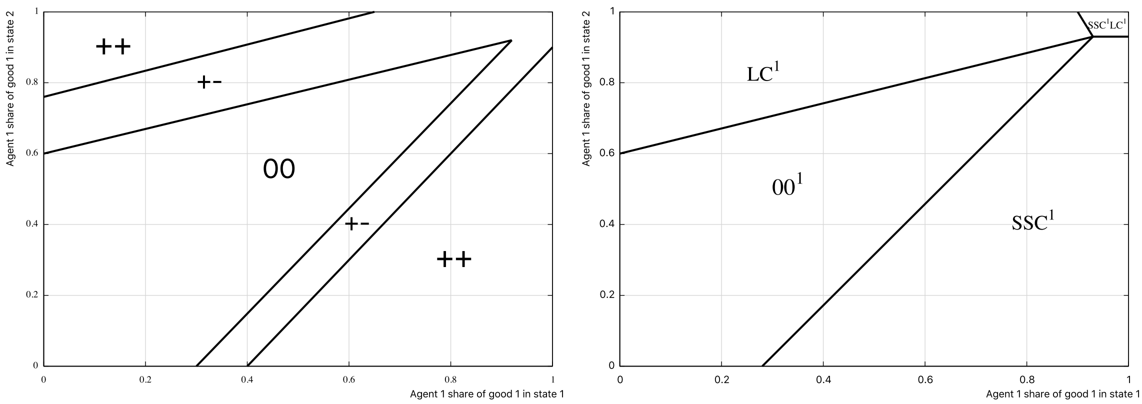


In the case of economy A one can easily see that the marginal utility of agent 1 is close to zero when δ is zero. In the economy B it is clear that the utility of the agent decreases for the initial values of δ. However, even for low values of δ, such as 0.15 and 0.2, his utility starts to increase together with the utility of agent 2, which shows a Pareto improvement in an economy exclusively short sale constrained. There is no analogous result with the unconventional monetary policy of Araujo et al. (2015).

Hence, in order to see the size of Pareto improvement in the region SSC^1 a new graphic of the utilities of the agents, such as figure 2.10, is shown below. But now the initial delta is $\delta = 0.175$ and it is compared with $\delta = 0.176$. It unveals a big region of Pareto improvement in the short sale region:

Figure 2.13: Case δ=0.175 and 0.176

(a) Utility (b) Constraints



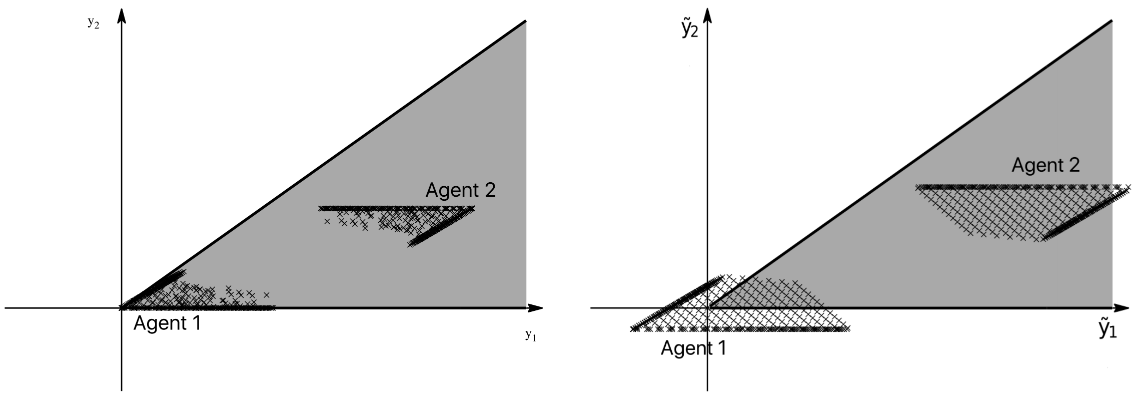
It can be seen that the region $-+$ disappears and a new region $+-$ appears surrounding the 00 region. This $+-$ in some sense is a thick border separating the regions 00 and $++$.

It is important to emphasize that a Pareto improvement is obtained in the short sale constrained economies *in spite of* the fact that the collateral requirement relaxation provides a tightening of this type of constraint, as shown in the case $\alpha = 0$ and in the previous section. The government compensation is crucial to achieve this Pareto improvement.

The next graphic shows all the chosen transferences y^h for all the economies of the figure 2.13. It also shows in panel (b) the corresponding transferences after the government compensation, $\tilde{y}^h$. The black dots in the top right are the transferences chosen by the lender (rich) and the dots around the vertix of the cone are the choices of the borrower (poor). The grey area behind these dots is the original transference set with $\delta = 0$ and highlights how the set of transferences changes when δ increases, with and without compensation.

Figure 2.14: Agent's Transferences at $\delta = 0.175$

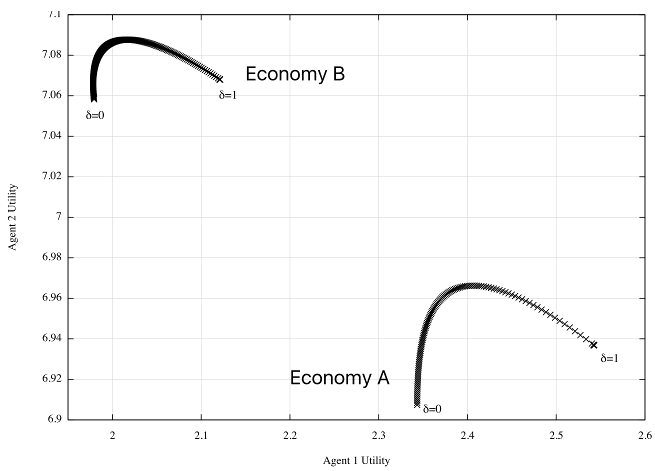
(a) Before compensation: y^1 and y^2 (b) After compensation: $\tilde{y}^1$ and $\tilde{y}^2$



The above graphics shows how the government intervention solves the problem of the tightening of the short sale constraint due to the increase of δ. The short sale constraint, with $\delta = 0$, is binding when the dots agent 1 are over identity straight line. Note that in panel (a), with $\delta = 0.175$, the dots of agent 1 are no longer over the identity, but in the interior of the original cone. This happens because at $\delta = 0.175$ the straight line that defines the boundary of the transference cone (and the short sale constraint) suffers a rotation, decreasing its angle. Thus, although the dots are inside the original transference cone, they are still short sale constrained. The graphic of $\tilde{y}$ shows that the government intervention translates the shortened transference cone in a direction such that *both* the leverage and short sale constraint are relaxed.

Note that figures 2.12 and 2.13 show that the Arrow Debreu equilibrium is indeed achieved. In the flat region of figure 2.12 the equilibrium does not change and the agents are no longer constrained. The 00 region of 2.13 increases first in the diagonal connecting the points $(0, 0)$ and $(1, 1)$ and after that increases towards the top-left and bottom-right corners of the box, making all the economies of the box unconstrained. Next figure shows the path taken by economies A and B to reach the Arrow Debreu frontier.

Figure 2.15: Economies A and B Reaching the Pareto Frontier

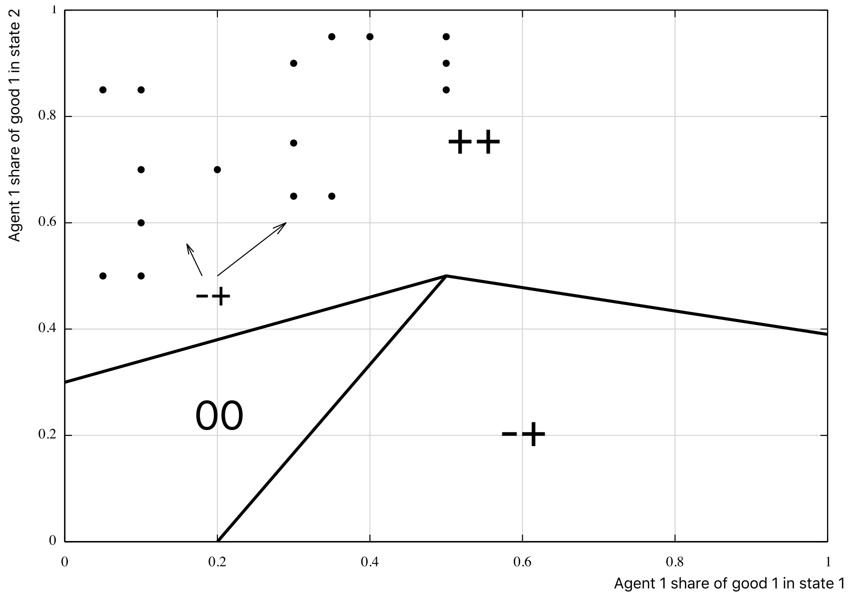


The next case shows that an ideal compensation is not necessary to a Pareto improvement in any of those regions, leverage or short sale constrained.

2.3.3 Case $\alpha = 0.5$:

In this section it will be presented the same graphics of the case $\alpha = 1$ to compare the results. Although the effects are typically the same, they have less intense results.

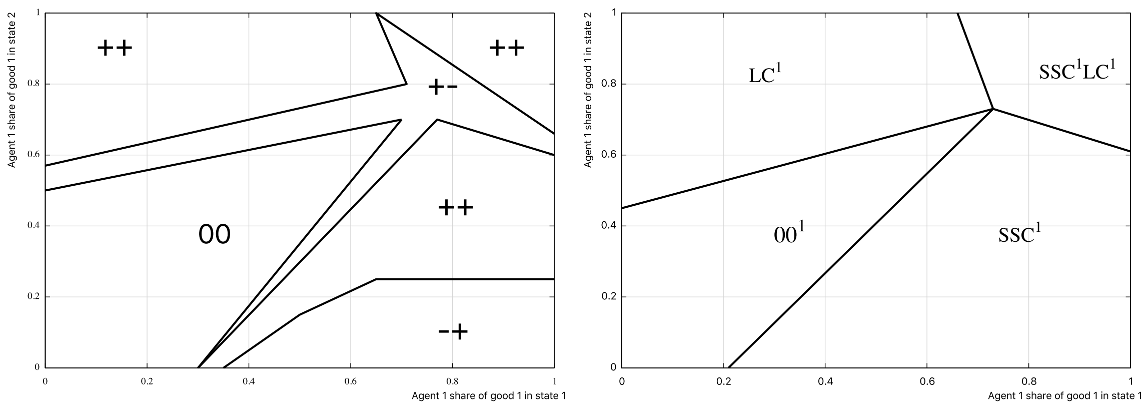
Figure 2.16: Case delta=0 and 0.001



The above graphic shows that the top-left region, where the borrower is only LC^1, is more ambiguous and has more subtle Pareto improving. The graphic with $\delta = 0.175$ below shows the process of reaching the Arrow Debreu equilibrium is slower with less compensation. There is still a $-+$ region in the bottom-right corner that in the case $\alpha = 1$ had disappeared. It is important to emphasize that even with $\alpha = 0.5$ there is a large region of Pareto improvement in the SSC^1 region. Thus, it is not necessary that the government guarantees all the additional default with $\alpha = 1$ in order to produce the Pareto improvement in this region.

Figure 2.17: Case δ=0.175 and 0.176

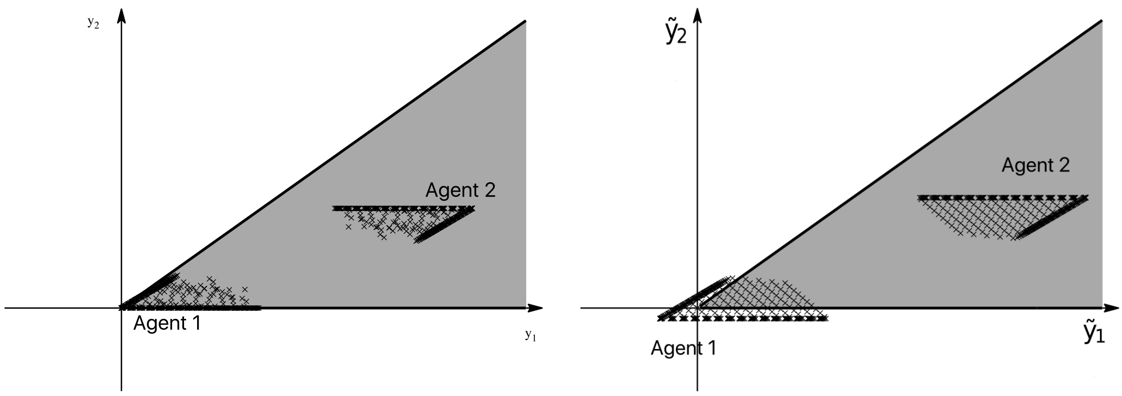
(a) Utility (b) Constraints



The transferences of the agents behave in the same way as before.

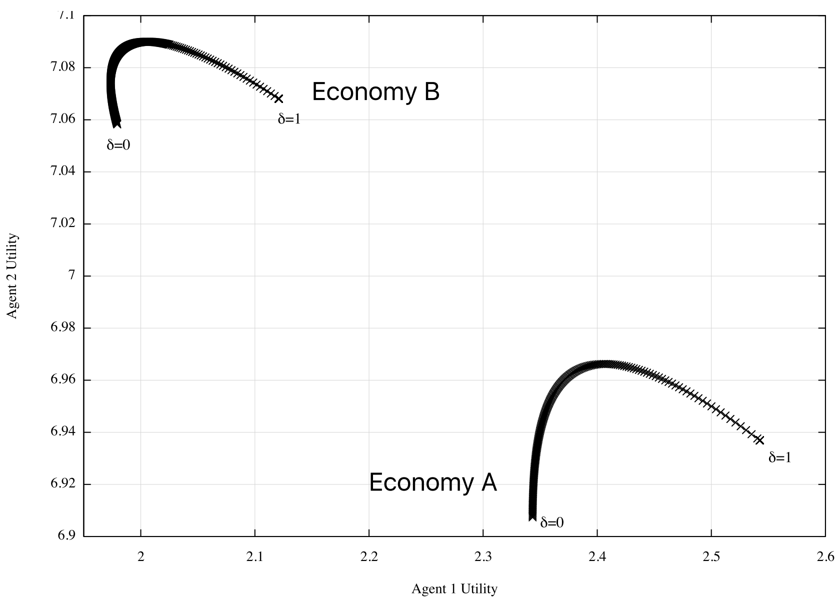
Figure 2.18: Agent's Transferences at $\delta = 0.175$

(a) Before compensation: y^1 and y^2 (b) After compensation: $\tilde{y}^1$ and $\tilde{y}^2$



And the path taken by the economies A and B to the Arrow Debreu Pareto frontier is almost equal. The only remark is that the economy B has a more profound initial decrease in agent's 1 utility before the Pareto improvement.

Figure 2.19: Economies A and B Reaching the Pareto Frontier



2.4 Conclusions

In this chapter it was studied a new kind of policy, involving the relaxation of the collateral requirement and some government compensation, that can be used in crisis moments. It has a relationship with the intuition of Geanankoplos (2009) about changing the assets of the economy in order to give the agents new assets with more favourable collateral requirement to increase the leverage and benefit the economy.

The numerical findings showed that the government compensation is crucial to allow for a Pareto improvement in the economy. Without government compensation there is no Pareto improvement and the agents surprisingly becomes more short sale constrained, even though the leverage of the asset increases. This tightening of the short sale constraint provides a new perspective to the Araujo et al. (2012) claim that there is no Pareto improvement with other assets different from the endogenous ones in the case of heterogenous utilities. The decreasing of the set of feasible transferences when the collateral requirement is relaxed does not depend on the utility of the agents.

The Pareto improvement appears only when the collateral requirement relaxation is combined with a government compensation in the second period. This combination ultimately provides a relaxation in the collateral constraint, which in turn allows for relaxation in *both* constraints over the transferences, the leverage and the short sale constraints. The possibility of Pareto improvement in short sale constrained economies is a major step up in comparison with the unconventional monetary policy of Araujo et al. (2015), whose Pareto improving effects are limited to leverage constrained economies.

In the direct comparison of the asymetric economy, the region of Pareto improvement with the collateral requirement relaxation and government compensation reaches leverage economies not reached by the unconventional monetary policy. However, the unconventional monetary policy remains more effective than the policy proposed for initial values of the policies in leverage constrained economies in which the borrower is poor in the good state of nature and rich in the bad one. In these economies the marginal utility of the borrower is approximately zero for initial values of the collateral requirement relaxation even for the highest possible compensation.

The policy proposed models the government structure to implement the compensation in a very simple way. It does not capture all the frictions, costs or trade off effects of this structure. In this sense, the model is more likely to better represent reality for lower values of collateral requirement relaxation. Thus, reaching the Arrow Debreu equilibrium for high values of collateral requirement relaxation and/or high values of compensation must be seen much more as a theoretical or extreme possibility. Indeed, a larger government structure would be needed to manage the exogenous mechanism of transference involving very high values of wealth created by high values of relaxation or high values of compensation. Furthermore, high values of relaxation means that the collateral structure of the economy would be completely destroyed, which is an extreme possibility and unlike to be implemented.

Finally, the analysis of the regulation proposed allows to conclude that it stands as a true alternative policy to be used in moments of crisis, especially as an alternative to the unconventional monetary policy, which is the one effectively compared here. Furthermore, if the economy is only short sale constrained, the regulation of the collateral requirement relaxation is in a better position than the unconventional monetary policy since the first one can get Pareto improvement and the second *never* leads to a Pareto improvement in these economies.

2.5 Appendix: Details of the Implementation

The complete system of equalities and inequalities implemented in ALGENCAN is show below. Recall that $m = \sum_h m^h$.

$\boxed{\text{Inequality consumption } t = 0 \text{ (collateral constraint):}}$

$$-x_{0l}^h + \sum_j (1-\delta_j) C_{jl} \varphi_j^h \leq 0 \quad \forall h, \, \forall l$$

$\boxed{\text{Inequality to } r_{sj}\text{:}}$

$$2r_{sj} - 1 - (1-\delta_j) \sum_l p_{sl} \left(\sum_{l'} Y_{s_{l,l'}} C_{jl'} \right) \leq 0 \quad \forall s \in S, \, \forall j$$

Remark: $r_{sj} = \min\{1, (1-\delta_j) p_{s3} C_j\}$

$\boxed{\text{First-order condition of } x \text{ in } t=0\text{:}}$

$$\partial_{0l}u^h(x^h) - \mu_0^h p_{0l} + \sum_{s=1} \mu_s^h \sum_{l'} Y_{s_{l'},l} p_{sl'} + {}_{col}\mu_l^h = 0 \quad \forall h, \text{ for } l = 1,2$$

$$\implies \frac{1}{x_{01}} - \mu_0^h p_{0l} + {}_{col}\mu_l^h = 0 \quad \forall h, \text{ for } l = 1,2$$

and

$$\partial_{03}u^h(x^h) - \mu_0^h(p_{03} - p_{02}) + \sum_{s=1} \mu_s^h \sum_{l'} Y_{s_{l'},3} p_{sl'} + {}_{col}\mu_3^h = 0 \quad \forall h, \text{ for } l = 3$$

$$\implies -\mu_0^h(p_{03} - p_{02}) + {}_{col}\mu_3^h + \sum_{s=1} \mu_s^h p_{s3} = 0 \quad \forall h, \text{ for } l = 3$$

$\boxed{\text{First-order condition of } x \text{ in } t=1\text{:}}$

$$\partial_{sl}u^h(x^h) - \mu_s^h p_{sl} + {}_x\mu_{sl}^h = 0 \quad \forall h, \forall s \in S, \forall l = 1,2$$

$$\implies \frac{0.5}{x_{sl}^h} - \mu_s^h p_{sl} = 0 \ \ \forall h, \forall s \in S, \forall l = 1,2$$

and

$$\partial_{s3}u^h(x^h) - \mu_s^h(p_{s3} - p_{s2}) + {}_x\mu_{s3}^h = 0 \quad \forall h, \forall s \in S, l = 3$$

$$\implies -\mu_s^h(p_{s3} - p_{s2}) + {}_x\mu_{s3}^h = 0 \ \ \forall h, \forall s \in S, l = 3$$

Remark: Due to market clearing of the durable, for each state s there is some agent h that accomplishes the following equality $-\mu_s^h(p_{s3} - p_{s2}) = 0 \implies p_{s3} = p_{s2}$.

$\boxed{\text{Budget constraint at } t=0\text{:}}$

$$\sum_l p_{0l}(x_{0l}^h - e_{0l}^h) - p_{02}x_{03}^h + \sum_j q_j(\psi_j^h - \varphi_j^h) + \mu^h - m^h = 0 \quad \forall h$$

$\boxed{\text{Budget constraint at } t=1\text{:}}$

$$\sum_l p_{sl}(x_{sl}^h - e_{sl}^h - \sum_{l'} Y_{s_{l,l'}} x_{0l'}^h) - p_{s2}x_{s3}^h - \sum_j (\psi_j^h - \varphi_j^h)(r_{sj} + \xi_{sj}) +$$
$$+\theta^h(1+i)m - (1+i)\mu^h = 0 \quad \forall h, \forall s \in S$$

$\boxed{\text{First-order conditions of } \psi\text{:}}$

$$_{\psi}\mu_j^h + \sum_s \mu_s^h(r_{sj} + \xi_{sj}) - \mu_0^h q_j = 0 \quad \forall h, \forall j$$

$\boxed{\text{First-order conditions of } \varphi\text{:}}$

$$_{\varphi}\mu_j^h - \sum_s \mu_s^h(r_{sj} + \xi_{sj}) + \mu_0^h q_j - (1 - \delta_j) \sum_l {}_{col}\mu_l^h C_{jl} = 0 \quad \forall h, \forall j$$

$\boxed{\text{First-order conditions of } \mu\text{:}}$

$$-\mu_0^h + \sum_{s=1}^{S} \mu_s^h(1+i) + {}_{\mu}\mu^h = 0 \quad \forall h$$

$\boxed{\text{Market clearing for } x \text{ at } t=0\text{:}}$

$$\sum_h (x^h_{01} - e^h_{01}) = 0 \quad (l = 1)$$
$$\sum_h (x^h_{0l} - e^h_{03}) = 0 \quad (l = 2, 3)$$

$\boxed{\text{Market clearing for } x \text{ at } t=1\text{:}}$

$$\sum_h (x^h_{s1} - e^h_{s1}) = 0 \quad \forall s \in S,\ (l = 1)$$
$$\sum_h (x^h_{sl} - e^h_{s3} - Y_{s3,3} e^h_{03}) = 0 \quad \forall s \in S,\ (l = 2, 3)$$

$\boxed{\text{Market clearing for } \psi \text{ and } \varphi\text{:}}$

$$\sum_h (\psi^h_j - \varphi^h_j) = 0 \quad \forall j$$

$\boxed{\text{Market clearing for the money } \mu^h\text{:}}$

$$\sum_h \mu^h - m = 0$$

$\boxed{\text{Boundary conditions:}}$

$$
\begin{aligned}
{}_x\mu^h_{sl} x^h_{sl} &= 0 \quad \forall h,\ \forall s,\ \forall l \\
{}_\psi\mu^h_j \psi^h_j &= 0 \quad \forall h,\ \forall j \\
{}_\varphi\mu^h_j \varphi^h_j &= 0 \quad \forall h,\ \forall j \\
{}_\mu\mu^h \mu^h &= 0 \quad \forall h \\
{}_{col}\mu^h_l (-x^h_{0l} + \sum_j (1 - \delta_j) \varphi^h_j C_{jl}) &= 0 \quad \forall h,\ \forall l
\end{aligned}
$$

$\boxed{\text{Portfolio condition or ortogonality of } \psi \text{ and } \varphi\text{:}}$

$$\varphi^h_j \psi^h_j = 0 \quad \forall h,\ \forall j$$

$\boxed{\text{Price normalization at } t=1\text{:}}$

$$p_{s1} - 1 = 0 \quad \forall s \in S$$

$\boxed{\text{Equality for } r_{sj}\text{:}}$

$$(r_{sj} - 1)(r_{sj} - (1 - \delta_j) \sum_l p_{sl} \sum_{l'} Y_{s_{l,l'}} C_{jl'}) = 0 \quad \forall s \in S,\ \forall j$$

Chapter 3

Financial Crisis Interventions with Heterogeneous Expectations

3.1 Introduction

The analysis of the unconventional monetary policy, presented in Araujo et al. (2015), and also the new policy of the previous chapter are developed assuming that the agents have homogeneous beliefs of the states of nature of the second period. The optimism/pessimism of the agents can play an important role in a crisis cycle. For example, the behavior of the optimistic agents can contribute to produce the conditions for the crisis, such as mortgage bubbles, by "betting" too much in the good state. As observed by Geanakoplos in his model with optimistic/pessimistic agents in Geanankoplos (2009), once the crisis happened the optimistic agents may bankrupt, lose all their wealth and hence get out of the market. This may amplify the fall of the asset prices, worsening the crisis because the optimistic agents constitutes an important raising force for the asset's prices, thus helping to stop the spiral fall of the prices in the crisis. In this context, Geanakoplo's suggests that "...bailing out crucial players or injecting optimistic capital into the financial system..." (Geanakoplos, 2009,p. 4) would be an important intervention to bring optimistic agents back to the market and therefore hold the prices. More recently, the unpublished work of Tsomocos and Yan (2016) studies optimism in a context of crisis using a variant of Geanakoplo's model with three period and bayesian update.

In this chapter the concept of optimism of Geanakoplos is combined with two crisis interventions: the unconventional monetary policy of Araujo et al. (2015) (from now on called ω-model) and the collateral requirement regulation with compensation presented in the previous chapter (from now on called δ-model). The main contribution is an overview of the effect of these new policies in economies with heterogeneous beliefs. There is no similar analysis in the literature and this chapter intends to fill this gap. The analysis is developed using the following methodology: introduce the heterogeneous beliefs in each model, produce numerical examples for several cases and analyze the results. The aim is to assess the impact, effects and relations between the these two policies and the expectations of the agents.

Generally speaking, the numerical results suggest that the relative optimism is important to determine which constraint will be binding for the agents. The effects of the policies in economies with heterogeneous expectations shows that the unconventional monetary policy and the collateral requirement regulation are much more complementary than substitutes. The unconventional monetary policy should be prefered in economies where the poor is relatively optimistic. In this case the unconventional monetary policy effects are potentialized and the collateral requirement regulation has less effect. In economies where the poor is relatively pessimistic the collateral requirement regulation should be prefered because it potentializes its effects and the region of Pareto improvement is greater than the region of the unconventional monetary policy.

The remainder of this chapter is organized as follows: section 3.2 presents the ω-model with its main theoretical properties; section 3.3.1 presents the numerical results for the ω model and section 3.3.2 presents the results for the δ-model. Section 3.4 concludes. The appendix shows the equations and inequations used to compute the equilibrium of the ω-model.

3.2 Unconventional Monetary Policy

3.2.1 ω-Model

Since the ω-model is used as a reference to the δ-model of the previous chapter, they share most of the basic properties. Thus, the presentation will be short. The main theoretical reference of the ω-model is the model of Araujo et al. (2015), with endogenous collateral requirement, in which they introduce money and an exogenous parameter ω to represent the unconventional monetary policy.

Formally, the model is a general equilibrium with collateral and money. It has two periods with one state in the first period and $S \in \mathbb{N}$ states in the second period. The symbol $S^* = S+1$ will denote all states in the economy. There are $H \in \mathbb{N}$ agents, $L \in \mathbb{N}$ goods and $J \in \mathbb{N}$ assets in the economy. The preferences of the agents are incorporated in their utility functions $u^h(\cdot)$.

There are three goods, being good 1 perishable, good 3 durable and good 2 the service of the durable. The durable is used as collateral or to enjoy its service. Each household face the following maximization problem.

$$\max_{x^h,\psi^h,\varphi^h,\mu^h,x_3^h\geq 0} u^h(x^h)$$

s.t.

$$\begin{aligned}
&p_{01}(x^h_{01}-e^h_{01})+p_{02}(x^h_{02}-x^h_{03})+p_{03}(x^h_{03}-e^h_{03})+q\cdot(\psi^h-\varphi^h)+\mu^h-m^h \ \leq\ 0\\
&p_{s1}(x^h_{s1}-e^h_{s1})+p_{s2}(x^h_{s2}-x^h_{03})-\sum_j(\psi^h_j-\varphi^h_j)\min\{1,p_{s3}C_j\}+\\
&+\theta^h(1+i)[(p_{03}-p_{02})\omega e_{03}+m]-\theta^h p_{s3}\omega e_{03}-(1+i)\mu^h\leq 0\\
&x^h_0 \ \geq\ \sum_j \varphi^h_j C_j
\end{aligned}$$

where $e_{03}=\sum_h e^h_{03}$ and $m=\sum_h m^h$.[1]

Once the agent chooses the amount of durable x^h_{s3} in state s, he is able rent it and therefore receive the value $-p_{s2}x^h_{s3}$. The agents have no endowment of the service, $e^h_{s2}=0$, for all h and s and no durable in the second period, $e^h_{s3}=0$ for $s=1,2$ and for all h. In this model it is also true that $p_{s2}=p_{s3}$ for $s=1,2$ in equilibrium.

The money is represented by m^h and is a risk-free bond paying $1+i$ non-contingent in the second period. The agent's choice of money if μ^h. The Central Bank is present in the model not as a maximizer agent, but as an accounting equality that must hold. When he does the unconventional monetary policy he buys a fraction $\omega \in [0,1]$ of the aggregate of durable in the economy at price $p_{03}-p_{02}$ and increases the aggregate of money in $(p_{03}-p_{02})\omega e_{03}$. Since the Central Bank (CB) cannot buy more than the aggregate amount of durable, thus ω should be between 0 and 1. Hence, the total amount of money in this economy is $M=m+(p_{03}-p_{02})\omega e_{03}$, composed by the total endowment of money m plus the money issued by the CB. Note that $p_{03}-p_{02}$ is the price of the durable out of the rental income that will be received by the CB while he owns this good. All this money should be redeemed by the CB at period 1 to clean the economy. This operation may result in loss or profit to the CB, but it will be anyway distributed

[1] Note about the notation: the symbol $\cdot$ represents the inner product and the subindex of the variables refers to states and goods (x^h_{sl}, p_{sl}).

across the households as lump-sum transfers with proportions θ^h. In period 1 the amount M of money will value $(i+1)M$ and the amount ωe_{03} of durable bought by CB must be returned to the market for the value $p_{s2}\omega e_{03}$ (recall that $p_{s2} = p_{s3}$). Thus, the net lump-sum tax obligation of agent h in period 1 should be $\theta^h[(1+i)M - p_{s2}\omega e_{03}]$ in state s. The uncconventional monetary policy can therefore be summed up in setting/changing the parameter ω in view of some desired objective, such as a Pareto improvement. The CB also can guarantee the value of the money in the second period by normalizing the prices of the perishable in states $s = 1, \cdots, S$. He sets $p_{s1} = 1$ for all $s = 1, \cdots, S$.

Every financial asset j is nominal promising one unit non-contingent and demanding the issuer a collateral $C_j > 0$ to back it. The actual delivery of asset j in state $s = 1, \dots, S$, will be $\min\{1, p_s C_j\}$. Therefore, the assets differ from each other only in the amount of collateral used to back them. Since the collateral is endogenous, they are defined by:

$$C_j = 1/p_{j3} \text{ with } j = 1, \dots, S$$

It is possible to rank the states by the durable price so that $C_j = 1/p_{j3} < 1/p_{j+1,3} = C_{j+1}$. Asset 1 will be called subprime and asset S will be called prime.

The definition of equilibrium is given below:

Definition 2. *Let $(u^h(\cdot), e^h)$ be the economy defined previously with monetary specification $(i, \{p_{s1}\}_{s\in S})$. The equilibrium for this economy is a vector $((x^*, \{x^*_{s3}\}_{s\in S^*}, \psi^*, \varphi^*), \mu^*, p^*, q^*)$ consistent with the monetary policy specification such that:*

(i) $(x^{h^*}, \{x^{h^*}_{s3}\}_{s\in S^*}, \psi^{h^*}, \varphi^{h^*})$ *solves the optimization problem above given prices (p^*, q^*) for all h;*

(ii) $\sum_{h=1}^H x^{h^*}_{01} = \sum_{h=1}^H e^h_{01}$;

(iii) $\sum_{h=1}^H x^{h^*}_{02} = \sum_{h=1}^H e^h_{03}$;

(iv) $\sum_{h=1}^H x^{h^*}_{03} = (1-\omega)\sum_{h=1}^H e^h_{03}$;

(v) $\sum_{h=1}^H x^{h^*}_{s1} = \sum_{h=1}^H e^h_{s1}$ *for $s \in S$;*

(vi) $\sum_{h=1}^H x^{h^*}_{sl} = \sum_{h=1}^H e^h_{03}$ *for $s \in S$ and $l = 2, 3$;*

(vii) $\sum_{h=1}^H (\psi^{h^*} - \varphi^{h^*}) = 0$;

(viii) $\sum_{h=1}^H \mu^{h^*} = m + (p_{03} - p_{02})\omega e_{03}$

The item (iv) is justified by the fact that when CB buys durable in $s = 0$ he holds his position until the next period, thus the agents have less aggregate durable to the market clearing. However, the service of the durable is still tradeable because the Central Bank uses the durable in his possession to trade its service. Analogously, the item $(viii)$ of the definition of the equilibrium requires that the aggregate amount of money chosen by the agents in $s = 0$ equal the total amount of money available.

In the following subsection the main properties of this model is presented.

3.2.2 Theoretical properties of the ω-model

The most important theoretical properties of the model is presented below. The proofs can be found in Araujo et al. (2015). The next lemma shows the conditions in which the subprime and the prime are inessential in this model.

Lemma 8. *Suppose $((x^*, \{x^*_{s,3}\}_{s\in S^*}, \psi^*, \varphi^*), \mu^*, p^*, q^*)$ is an equilibrium to the economy previously defined. Then:*

(a) If asset 1 is transacted, then $q_1^* = (p_{0,3}^* - p_{0,2}^*)C_1$

(b) If asset S is transacted, then $q_S^* = \frac{1}{1+i}$

(c) Asset 1 is inessential if, and only if, ${x^*}_{0,3}^h \geq {\varphi^*}_1^h C_1$ *for all agents*

(d) If one of the following items is satisfied:

- ${\mu^*}^h \geq {\varphi^*}_S^h C_S$ *for all agents*
- $q_S^* > \frac{1}{1+i}$

then asset S is inessential

Note that the subprime is *always* inessential in this model due to the collateral constraint.

In this model, the definition of transference to state s of period 2, in units of perishable, is given by:

$$y_s^h = \left(\frac{1+i}{p_{s1}}\right)\left[\mu^h + \tfrac{1}{1+i}(\psi_2^h - \varphi_2^h)\right] + \left(\frac{p_{s3}}{p_{s1}}\right)\left[x_{03}^h + (\psi_1^h - \varphi_1^h)C_1\right]$$

The first part of the sum is the *effective position in cash*, that is, the total amount of the prime asset and its "equivalent", the money. Analogously, the second part of the sum is the *effective position in risky durable*, that is, the total position in subprime and in its "equivalent", the durable. Considering only two states in the second period the following characterization of the collateral constraint holds:

Lemma 9. *In this economy,*

$$x_{03}^h \geq \varphi_1^h C_1 + \varphi_2^h C_2 \iff \begin{cases} p_{21} y_2^h \leq p_{11} y_1^h \\ y_2^h \geq 0 \end{cases}$$

The first constraint, $p_{21}y_2^h \leq p_{11}y_1^h$ is called *short sale constraint* and the second one $y_2^h \geq 0$ is called *leverage constraint*. With only two states in period 2, there are only two financial assets in the economy, the prime and the subprime.

The utility of the agents, with heterogeneous expectation, will be:

$$U^h(x^h) = u(x_{01}^h, x_{02}^h) + p_1^h u(x_{11}^h, x_{12}^h) + p_2^h u(x_{21}^h, x_{22}^h)$$

where $x^h = (x_{01}^h, x_{02}^h, x_{11}^h, x_{12}^h, x_{21}^h, x_{22}^h)$, and $u(\cdot,\cdot)$ is a function that does not depend on the states or on the agents and $p^h = (p_1^h, p_2^h)$ is the subjective probability of agent h.[2] In this setting, differences on the demand choices would be due differences in the endowment position *and* in the subjective probability of the agents.

Since $u^h(\cdot)$ is separable in each state, it is possible to substitute each component $u(x_s^h, x_s^h)$ with the corresponding indirect utility function $\tilde{u}(c_s^h)$, where c_s^h is the total wealth available at that state. And the total wealth available at each state depends on the choice of transferences y^h by the agents, that can increase or decrease this value at the states. Using the definition of transference of wealth y_s^h, the previous lemma, the state prices a_1, a_2 from the non-arbitrage conditions and some basic algebra, the original agent's optimization problem can be rewritten in the following way, in terms only of the vector of transferences and the state prices.:

$$\max_{y_1^h, y_2^h \in \mathbb{R}} \tilde{u}(c_0^h(y^h)) + p_1^h \tilde{u}(c_1^h(y^h)) + p_2^h \tilde{u}(c_2^h(y^h))$$

[2]Remark on the notation: the symbol p_s^h should not be confused with p_{sl}. The first one, with superscript depending on the agents h, is the subjective probability and the second one is the price of good l in state l, which does not depend on the agents.

s.t.

$$p_{21}y_2^h \leq p_{11}y_1^h$$
$$y_2^h \geq 0$$

where $c_0^h(y^h) = e^h + a_1 f_1^h + a_2 f_2^h - a_1 y_1^h - a_2 y_2^h$; $c_s^h(y^h) = g_s^h + y_s^h - \frac{\theta^h}{p_{s1}}[(1+i)(p_{03}-p_{02})\omega e_{03} - p_{12}\omega e_{03}]$; $e_{03} = \sum_h e_{03}^h$; $e^h = e_{01}^h + \frac{p_{02}}{p_{01}} e_{03}^h$; $f_s^h = (\frac{p_{s2}}{p_{s1}}) e_{03}^h + (\frac{1+i}{p_{s1}}) m^h$; $g_s^h = e_{s1}^h - \theta^h (1+i) \frac{m}{p_{s,1}}$; and $\tilde{u}^h(\cdot)$ is the indirect utility function of $u^h(x_{s1}^h, x_{s2}^h)$ subject to the constraint $x_{s1}^h + \frac{p_{s2}}{p_{s1}} x_{s2}^h \leq c_s^h$.

Note that the two constraints that remains are due to the collateral constraint because the budget constraint at each state are already incorporated in the indirect utility function through c_s^h.

The transferences y^h chosen by the agents are affected in the second period by the unconventional monterary policy of the CB through the distribution of the net balance of this intervention. Thus, the effective transferences of each agent can be defined in the following way:

$$\tilde{y}_s^h = y_s^h - \frac{\theta^h}{p_{s1}}[(1+i)(p_{03}-p_{02})\omega e_{03} - p_{s2}\omega e_{03}]$$

Finally, substituting the variables, the agent's optimization problem becomes:

$$\max_{\tilde{y}_1^h, \tilde{y}_2^h \in \mathbb{R}} \tilde{u}(e^h + a_1 f_1^h + a_2 f_2^h - a_1 \tilde{y}_1^h - a_2 \tilde{y}_2^h) + p_1^h \tilde{u}(g_1^h + \tilde{y}_1^h) + p_2^h \tilde{u}(g_2^h + \tilde{y}_2^h)$$

s.t.

$$p_{21}\tilde{y}_2^h \leq p_{11}\tilde{y}_1^h - (p_{12} - p_{22})\theta^h \omega e_{03}$$
$$\tilde{y}_2^h \geq -\theta^h \phi(a_1, a_2)\omega e_{03}$$

where $\phi(a_1, a_2) = \frac{a_1(p_{12}-p_{22})}{a_1 p_{2,1} + a_2 p_{11}} > 0$.

The problem now depends only on the transference choices $\tilde{y}^h$, already considering the Central Bank unconventional monetary policy, so making explicit its relationship with the set of possible transferences. Note that when the CB does an unconventional monetary policy with $\omega > 0$, it tightens the short sale constraint and relaxes the leverage constraint. Intuitively, the net balance of the CB intervention, given by $[(1+i)(p_{03}-p_{02})\omega e_{03} - p_{s2}\omega e_{03}]$, is positive in the good state and negative in the bad state. Thus, it allows for negative transferences y_2^h for the bad state and increases the gap between y_1^h and y_2^h, thus making the agent more short sale.

Finally, given state prices (a_1, a_2), the problem is well defined and a maximizer $(\tilde{y}_1^h, \tilde{y}_2^h)$ can be found. Thus the equilibrium definition can be reformulated in the following way:

Definition 3. *Given the policy $(p_{11}, p_{21}, i, \omega)$, an equilibrium is a vector of state prices and transferences $(a_1, a_2, \tilde{y}_1^h, \tilde{y}_2^h)$ such that*

(i) for each h, $\tilde{y}^h$ maximizes the last optimization problem

(ii) for each $s = 1, 2$

$$\sum_h^H \tilde{y}_s^h = \sum_h^H f_s^h$$

The terms f_s^h can be interpreted as the initial transferences of each agent for each asset, given by their initial endowments endowments.

3.3 Numerical Results

In this section a particular framework of the economy will be defined for the numerical analysis. There will be no calibration to represent any specific society. As in the previous chapter, the computation of the equilibria follows Schommer (2013) with the software ALGENCAN[3][4]

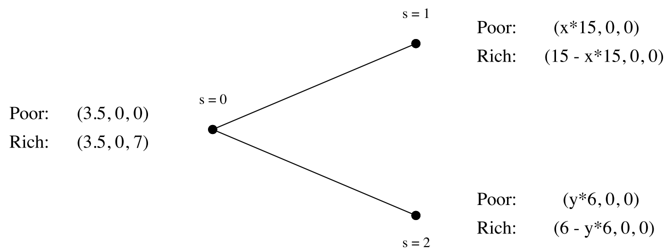
All the agents of both models, ω-model and δ-model, will have a cobb-douglas utility function. Each utility function differ from each other only in the subjective probability:

$$u^h(x) = \sum_l \log(x^h_{0l}) + \sum_s p^h_s \sum_l \log(x^h_{sl})$$

In both models the only prices left to be determined is p_{01} and p_{03} because p_{s2}/p_{s1} if fixed for all s, $p_{s2} = p_{s23}$ for $s = 1, 2$ and the central bank sets $p_{s1} = 1$ for $s = 1, 2$. Also in both models, due to the logarithm function, it is true that the following relationship in equilibrium:

$$\frac{\sum_h e^h_{sl'}}{\sum_h e^h_{sl}} = \frac{p^*_{sl}}{p^*_{sl'}} = \frac{{x^*}^h_{sl'}}{{x^*}^h_{sl}} \quad \forall h$$

The endowment distribution analyzed is the same of the previous chapter:



The asymmetry in the durable in $s = 0$ represents a moment of crisis with high inequality. This can be interpreted as the case in which the agent 1 become poor right after the crash due to frustrations from the crisis. The aggregate endowment will remain fixed along the entire analysis: $e_0 = \sum_h e^h_0 = (7, 0, 7)$, $e_0 = \sum_h e^h_1 = (15, 0, 0)$, $e_0 = \sum_h e^h_2 = (6, 0, 0)$. Note that, with aggregate risk in the economy, the state $s = 1$ can be interpreted as the good state and $s = 2$ can be interpreted as the bad state.

Therefore, in this model:

$$\frac{p^*_{11}}{p^*_{12}} = \frac{\sum_h e^h_{12}}{\sum_h e^h_{11}} = \frac{7}{15} = 0.46666666 \quad \text{and} \quad \frac{p^*_{21}}{p^*_{22}} = \frac{\sum_h e^h_{22}}{\sum_h e^h_{21}} = \frac{7}{6} = 1.16666666$$

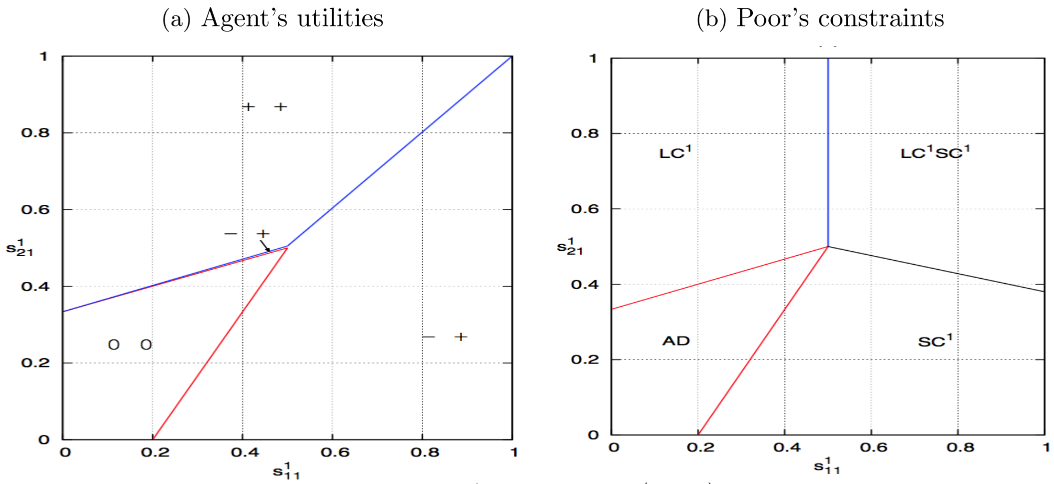
Since the central bank set $p_{s1} = 1$ and the model has $p_{s2} = p_{s3}$ for $s = 1, 2$, then $C_1 = \frac{1}{p_{13}} = 0.4666666$ and $C_2 = \frac{1}{p_{23}} = 1.1666666$. The interest rate i is 0.1, the taxation is $\theta^1 = 0.9$ and $\theta^2 = 0.1$ and the endowment of money is $m^1 = 0.0009$ and $m^2 = 0.0001$. These parameters follows Araujo et al. (2015).

The next graphics, from Araujo et al. (2015), will be used as a reference point in the analysis of the numerical results.

[3]See TANGO project (2013).

[4]It was used a Macbook Pro with an Intel Core i7, 2.5GHz and 16 GB 1600 MHz DDR3 for the computations. The tolerance in all cases was 10^{-08}.

Figure 3.1: Agent's utilities and constraints with homogeneous expectations ($p_s^h = 0.5$ for all h and s)



Source: Araujo et al. (2015)

where $s_{11}^1 = e_{11}^1 / \sum_h e_{11}^h \in [0, 1]$ and $s_{21}^2 = e_{21}^1 / \sum_h e_{21}^h \in [0, 1]$. The methodology and notation of the graphics is the same of the previous chapter. In the left figure, for each economy of the box, the equilibrium was calculated first with $\omega = 0$ and second with $\omega = 0.001$. The symbol ++ indicates that the utilities of both agents increase when ω increases. In the right figure the symbols LC^1, SC^1 indicate which constraint of the agent is binding with $\omega = 0$. And AD means that neither the leverage or the short sale constraint is binding for each agent. Note that there is Pareto improvement in the economy only when agent 1 is leverage constrained. The rich's constraints are not binding in the entire box.

3.3.1 Unconventional Monetary Policy with Heterogeneous Expectation

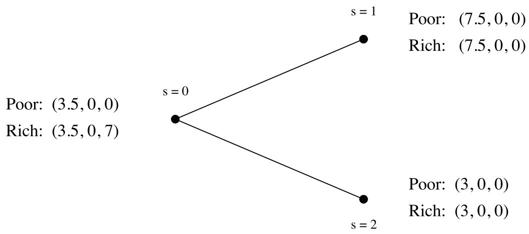
First it is important to make an observation about the organization of the results presented. They will classified in three different groups. In the first group the endowments will be fixed and the subjective probabilities p_s^h will vary. In this case the graphic with different probabilities will be shown in a box where the x-axis is p_1^1, which is the probability of agent 1 to the good state $s = 1$, and the y-axis is p_1^2, which is the probability of agent 2 to the good state. The second group is the opposite, the subjective probabilities will be fixed and the endowments of the agents in the second period will be allowed to vary. The graphic of this group is similar to 3.1 (a) and (b). Finally, in the third group both the endowments and the probabilities will be fixed so that it can be analyzed the case in which the parameters of the policies ranges from 0 to 1.

In all cases, for each economy of the boxes of group 1 and 2, it is computed the equilibrium first with $\omega = 0$ and second with $\omega = 0.001$.

Fixed endowments and various probabilities distributions

In this case the endowment distribution is fixed for the analysis of various probabilities distribution:

Figure 3.2: Economy **middle**

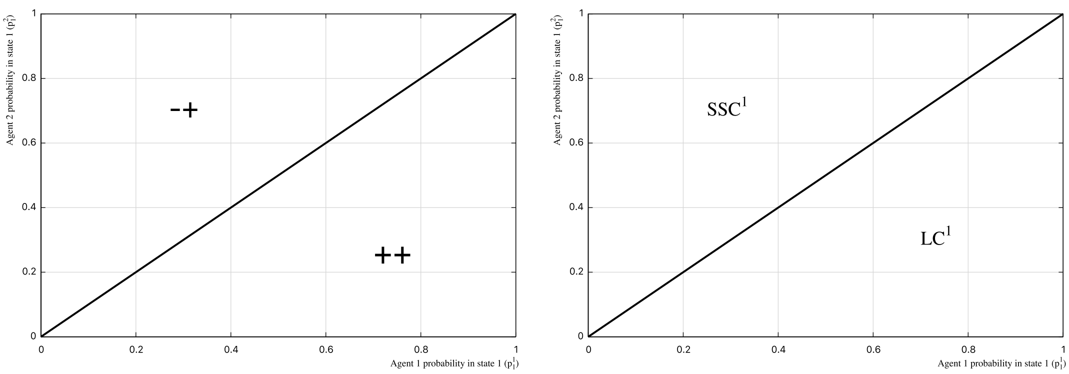


This endowment distribution is exactly in the middle of the box in figure 3.1 (a), which is approximately in the frontier of all regions $++$, 00 and $-+$. Note also that since the endowment distribution is symmetric in this economy for the second period, the result is more likely to be caused by differences in the subjective probability of the agents.

Figure 3.3 shows that the *relative* optimism is important. Panel (b) shows the relevance of the relative optimism to determine which constraint of the poor agent will be binding. When he is relatively more optimistic than the rich he will be leverage constrained and when he is relatively more pessimistic he will be short sale. Panel (a) shows that there will be Pareto improvement whenever the poor agent is more optimistic than the rich. This is consistent with the fact that the unconventional monetary policy only relaxes the leverage constraint (it tightens the short sale constraint) and also consistent with the numerical results of Araujo et al. (2015), in which the Pareto improvement is found only in leveraged constrained economies.

Figure 3.3: Agent's utilities and constraints at economy **middle**

(a) Agent's utilities (b) Poor's constraints

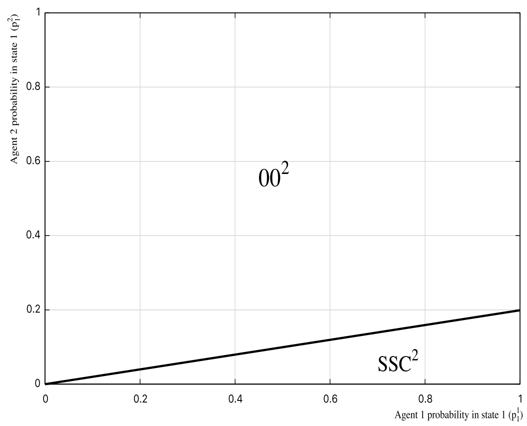


This result reinforces the interpretation that more optimism tends to lead to leverage constrained agents and more pessimism tends to lead to short sale constrained agents.

The above numerical fact supports the commmom intuition that optimistic agents tend to be more leveraged. Indeed, the optimistic agent thinks that the good state $s = 1$ is more likely to happen and, symmetrically, that the bad state is not going to occur. Therefore, he wants to bring the maximum wealth possible from the poor state $s = 2$ to the good state $s = 1$. The need to accomplish this kind of transference makes him more prone to be leverage constrained, making $y_2^1 = 0$ and *not* short sale constrained because he will probably choose $y_1^1 > y_2^1$. The constraints for the pessimistic rich agent has a symmetric analysis. Since he is pessimistic, he wants to bring wealth for the poor state and therefore will probably have $y_2^2 > 0$, thus not leverage constrained, and also $y_2^2 = y_1^2$, thus short sale constrained, because he chooses to decrease the wealth transfered to the good state y_1^h and increase the transference y_2^2 to the bad state.

Figure 3.4 (b) shows that even the rich may be short sale constrained in the cases he is extremely pessimistic.

Figure 3.4: Rich's constraints at economy **middle**



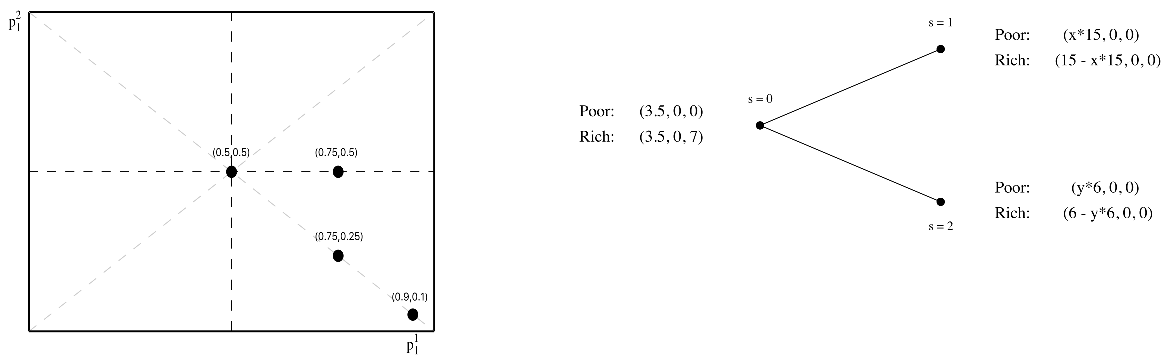
The numerical result of figure 3.3 can be related to figure 3.1 (a) because it suggests that the region of Pareto improvement may increase if the poor is more optimistic than the rich. This is confirmed in the results of the following case.

Fixed probabilities and various endowments distributions

The panel (a) of the figure 3.5 shows the chosen fixed probabilities inside "box of probabilities". In this box, the x-axis is p_1^1, which is the probability of agent 1 to the good state $s = 1$, and the y-axis is p_1^2, which is the probability of agent 2 to the good state. Thus, dot $(0.9, 0.1)$ is the case where the poor (agent 1) is optimistic and the rich is pessimistic. Three cases were considered here: $(0.75, 0.5)$, $(0.75, 0.25)$ and $(0.9, 0.1)$. They are representative to show the more important effects.

Figure 3.5: Fixed probabilities and endowments varying

(a) Fixed probabilities (b) Endowments varyies

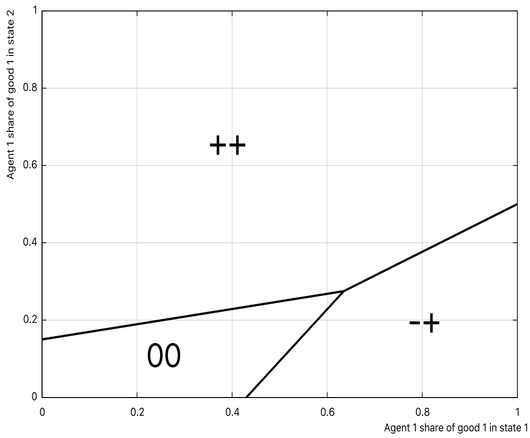


The variables x and y in the endowments of the agents indicates the methodology to create the figures. The x-axis will represent the proportion of perishable owned by the poor in state 1 (i.e., $x = e^1_{11}/\sum_h e^h_{11} \in [0,1]$) and the y-axis will represent the proportion of perishable owned by the poor at state 2 (i.e., $y = e^1_{21}/\sum_h e^h_{21} \in [0,1]$). The probability is fixed in $p^1_1 = 0.9$ and $p^2_1 = 0.1$, which means that the poor is optimistic and the rich is pessimistic. The dot $(0.5, 0.5)$ highlights the case with homogeneous expectation which was analyzed in Araujo et al. (2015).

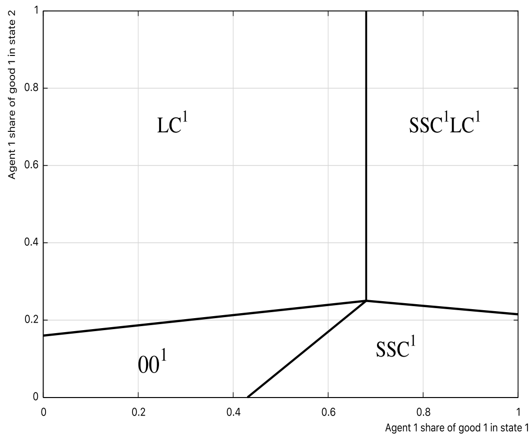
The cases $(p^1_1, p^2_1) = (0.75, 0.5)$ and $(p^1_1, p^2_1) = (0.75, 0.25)$ are shown together in the next graphic. The figure shows that increasing the relative optimism of the poor agent will increase the area of economies that are only leverage constrained and therefore amplify the region of Pareto improvement.

Figure 3.6: Agent's utilities and constraints

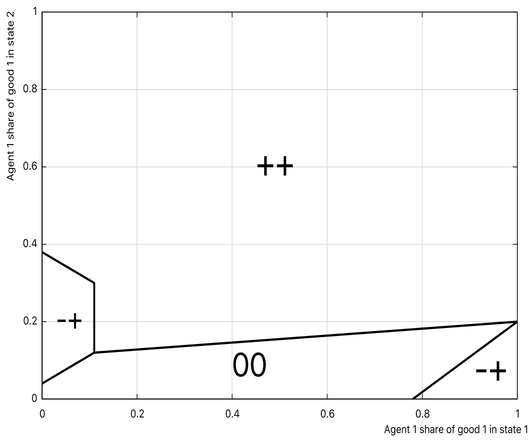
(a) Agent's utilities at $(p^1_1, p^2_1) = (0.75, 0.5)$



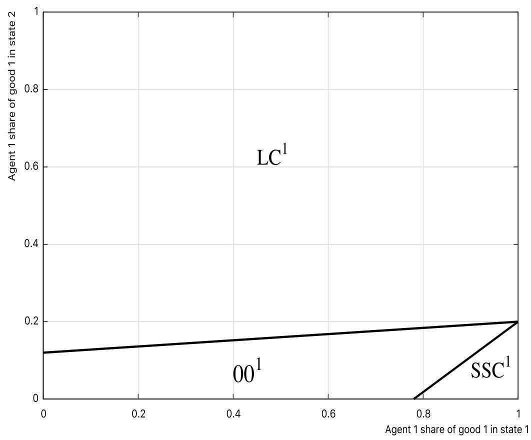
(b) Poor's constraints at $(p^1_1, p^2_1) = (0.75, 0.5)$



(c) Agent's utilities at $(p^1_1, p^2_1) = (0.75, 0.25)$

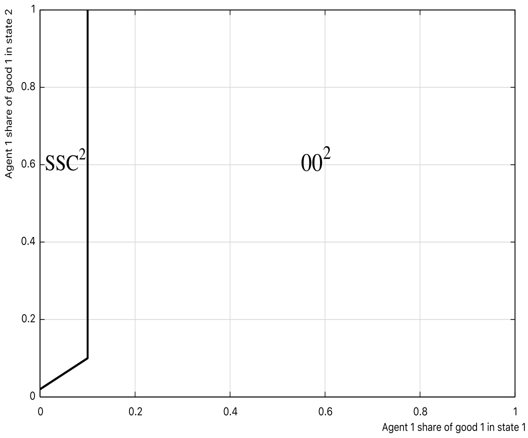


(d) Poor's constraints at $(p^1_1, p^2_1) = (0.75, 0.25)$



The bottom left region of panel (c), however, shows an important feature: when the relative optimism of the poor is higher, the rich agent becomes short sale constrained and this may cancel the Pareto improvement effect. The rich's constraints for $(p^1_1, p^2_1) = (0.75, 0.25)$ is shown below:

Figure 3.7: Rich's constraints with $(p_1^1, p_1^2) = (0.75, 0.25)$



In the case $(0.75, 0.5)$ the rich is not constrained. The Pareto improvement region remains more effective with more relative optimism until around the corner $(1, 0)$. In this case extreme case, the rich becomes strongly short sale constrained making the $-+$ region prevail in almost all economies of the box and a new $--$ region appears in the extreme bottom of the box, especially in the bottom left region. When p_1^1 is close to 1, the Pareto improvement region will be greater if p_1^2 is higher, more close to 0.5, so it can soften the short sale constraint of the rich. The next figure is the case $(0.9, 0.1)$ to illustrate this feature.

Figure 3.8: Agent's utilities and constraints with $(p_1^1, p_1^2) = (0.9, 0.1)$

(a) Agent's utilities (b) Poor's constraints

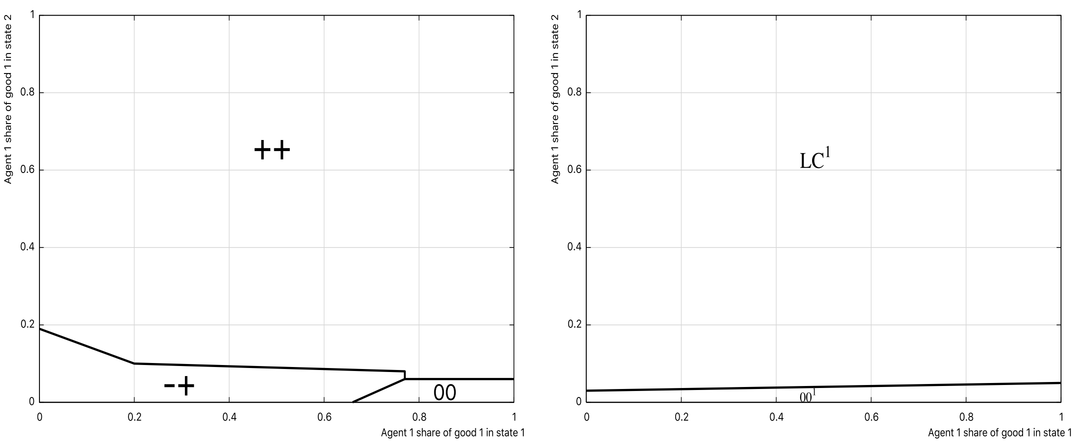
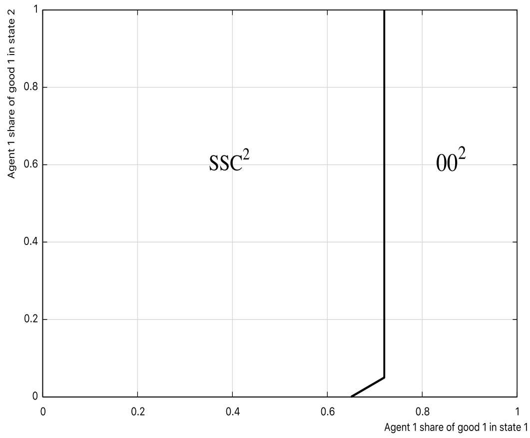


Figure 3.9: Rich's constraints with $(p_1^1, p_1^2) = (0.9, 0.1)$



Note also that the cases $(0.75, 0.25)$ and $(0.9, 0.1)$ show a pattern of "orthoganility" of the agent's constraints in the endowment distribution of the second period when the poor is too much relatively optimistic. If the poor agent has very low amount of perishable in the bad state $s = 2$, then he will bring some wealth to $s = 2$ thus making $y_2^1 > 0$ and being not leverage constrained anymore. In this case the amount of perishable in the good state does not change his structure of constraints since this perishable in $s = 1$ does not help him in being leverage. Similar analysis can be made to interpret the richs' constraints. When he has a few endowment in the good state, he will try to transfer a positive wealth to this state relaxing the short sale constraint. This is the 00^2 region. The numerical result shows that only when he has almost all perishable in the bad state he stops transfering wealth to this state and also relaxes the short sale. This happens between 0.65 and 0.7 in the x-axis of the figure 3.9 and from 0 to 0.1 in figure 3.7.

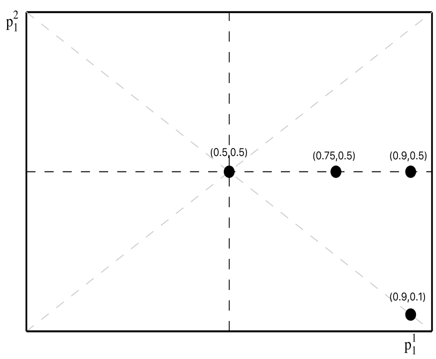
The case in which the poor is relatively pessimistic is symmetric.

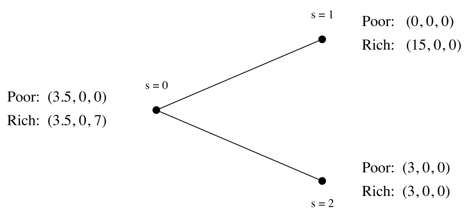
Fixed endowment and fixed probabilities

Finally, an endowment distribution is fixed and three probability distributions were chosen as described in the picture below, in order to analyze the utility of the agents for $\omega \in [0, 1]$. The case with homogeneous expectation were also computed for comparison purposes.

Figure 3.10: Fixed endowment and fixed probabilities

(a) Probabilities analyzed (b) Fixed endowment





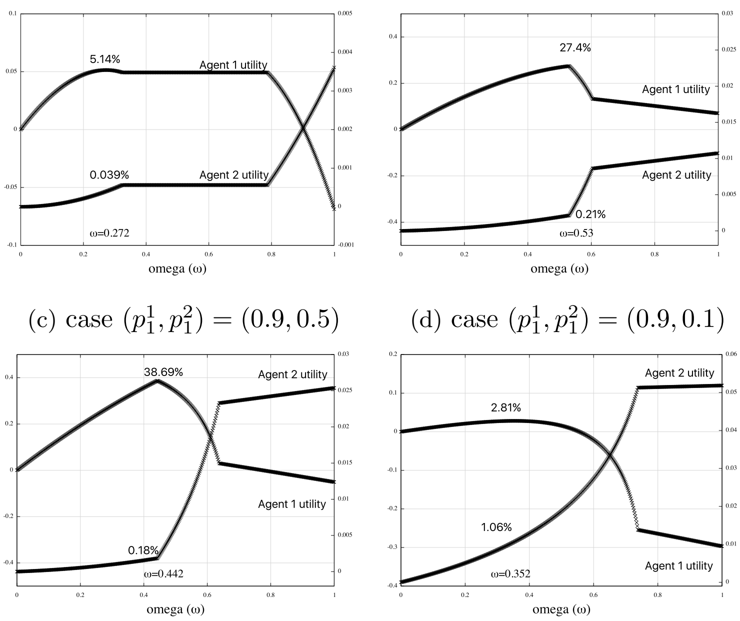
Inside each chart of figure 3.11 below there is the indication of the value of ω where the maximum Pareto improvement gain occurs. The percentages shows the relative gain of each agent in comparison with the initial $\omega = 0$ case. Generally speaking, this figure shows that the poor agent gains more when he is optimistic and the rich is neutral. When approaching $(p_1^1, p_1^2) = (1, 0.5)$, the improvement in agent's 1 utility increases, being 38,69% in the case (0.9,0.5) and only 5.14% in the homogeneous case. The rich's gain also increases in this line, from 0.039% in the case (0.5,0.5) to 0.18% in the case (0.9,0.5). However, in the case (0.9,0.1), where the poor is optimistic and the rich pessimistic, although everyone still gains, the poor gains less than in the homogeneous case and the rich gains more than if he is neutral. Thus, the interaction between the rich pessimistic and the poor optimistic favors the rich in terms of utility gains.

Note also that the range of Pareto improvement in ω changes depending on the beliefs of the agents. It is higher in the case $(0.75, 0.5)$, with the interval $[0, 0.53]$.

Figure 3.11: Fixed endowment and fixed probabilities

(a) case $(p_1^1, p_1^2) = (0.5, 0.5)$ (b) case $(p_1^1, p_1^2) = (0.75, 0.5)$

(c) case $(p_1^1, p_1^2) = (0.9, 0.5)$ (d) case $(p_1^1, p_1^2) = (0.9, 0.1)$



3.3.2 Collateral Requirement Regulation with Heterogeneous Expectations

In the analysis of the δ-model the same methodology of the previous analysis was applied, that is, the graphics were organized in three groups: fixed probabilities and endowments varying, varying probabilities and fixed endowments and both probabilities and endowments fixed. The only difference is that the parameter of reference is δ, not ω. The results on the ω-model will be used for comparison to highlight how differently the policies behave in a context of heterogeneous expectation. The government compensation in the second period is the maximum possible, that is, $\alpha = 1$ in every case where $\delta > 0$.

Fixed endowments and various probabilities distributions

Two endowment distributions were chosen to analyze different probabilities distribution. Each of them shows important differences between the ω-model and δ-model. They are called economy top-left and economy top-right because of their position in the box of different endowment distribution in the second period.

Figure 3.12: Economy **top-left**

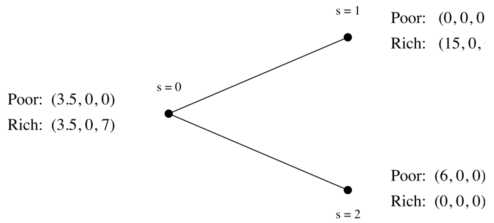
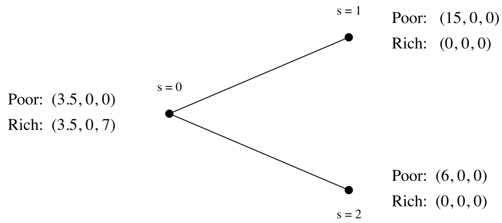


Figure 3.13: Economy **top-right**

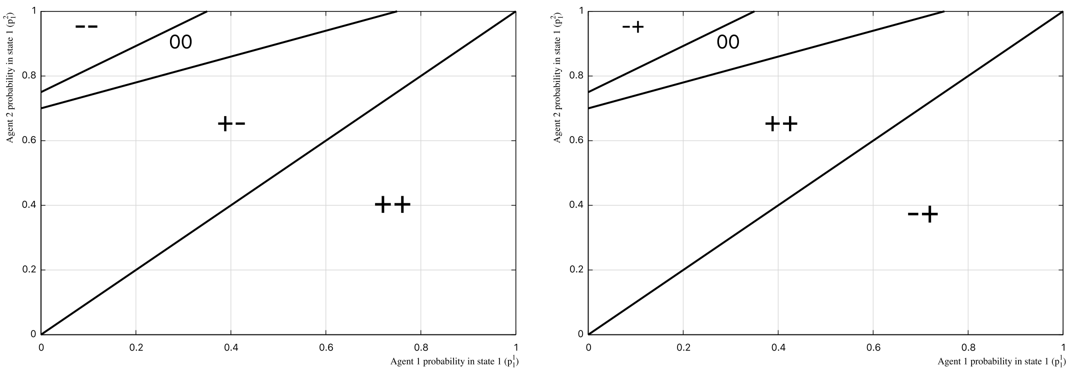


The next graphic shows the behavior of the utility of the agents in the top-left economy in both models:

Figure 3.14: Agent's utilities at economy **top-left**

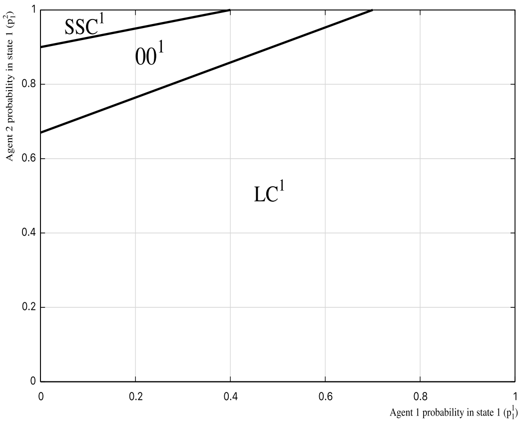
(a) Agent's utilities at ω-model

(b) Agent's utilities at δ-model



First note that in ω-model it behaves in a similar way of the previous section. Although the poor agent is leverage constrained in almost all economies of the box, as depicted in figure 3.15 below, there will be Pareto improvement only when he is relatively more optimistic than the rich. In contrast, in the δ-model the region of Pareto improvement appears clearly in the region where the poor is relatively pessimistic than the rich. Note that an ambiguous region, where the marginal utility of the agent is close to zero, appears in the top right corner. In the next group of graphics it will be shown that, differently from the ω-model, making the poor relatively pessimistic does not increase the Pareto improvement region.

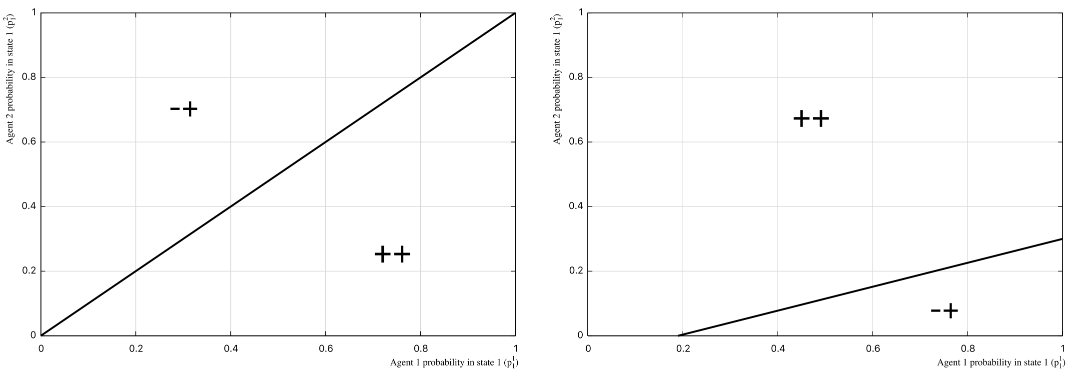
Figure 3.15: Poor's constraints



The following graphic shows that the Pareto improvement of the δ-model with heterogeneous expectation is limited to the cases with relatively pessimistic agent 1. In figure 3.16 there is a region of Pareto improvement in economies with the poor relatively optimistic.

Figure 3.16: Agent's utilities at economy **top-right**

(a) Agent's utilities at ω-model (b) Agent's utilities at δ-model

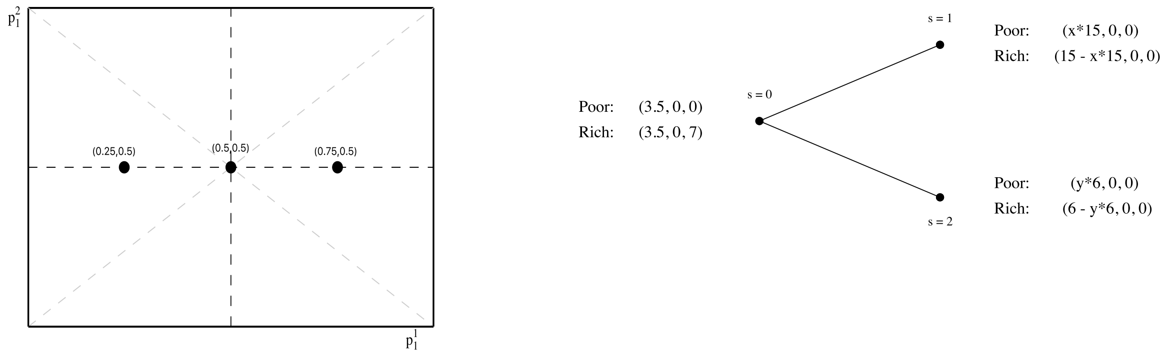


Fixed probabilities and various endowments distributions

The probabilities chosen to the analysis in this case is shown in panel (a) of the next figure.

Figure 3.17: Fixed probabilities and endowments varying

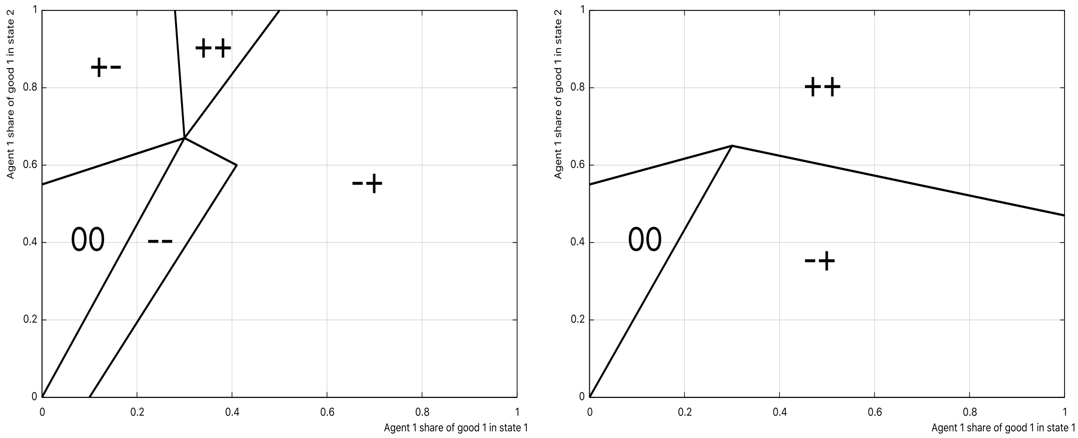
(a) Fixed probabilities (b) Endowments varyies



In the following case the poor is pessimistic and the rich is neutral. In this case, consistent with the previous analysis, the Pareto improvement region diminishes since the leverage constrained economies also decreases as seen in figure 3.20 (a). In the opposite direction, the region of Pareto improvement in the δ-model covers all the leverage constrained economies in a more solid and effectively way. Indeed, the ambiguous region in the top left of the box disappears because the marginal utility of the poor is not close to zero anymore, but positive. However, the size of the Pareto improvement region still decreases in comparison with the homogeneous case.

Figure 3.18: Agent's utilities at $(p_1^1, p_1^2) = (0.25, 0.5)$

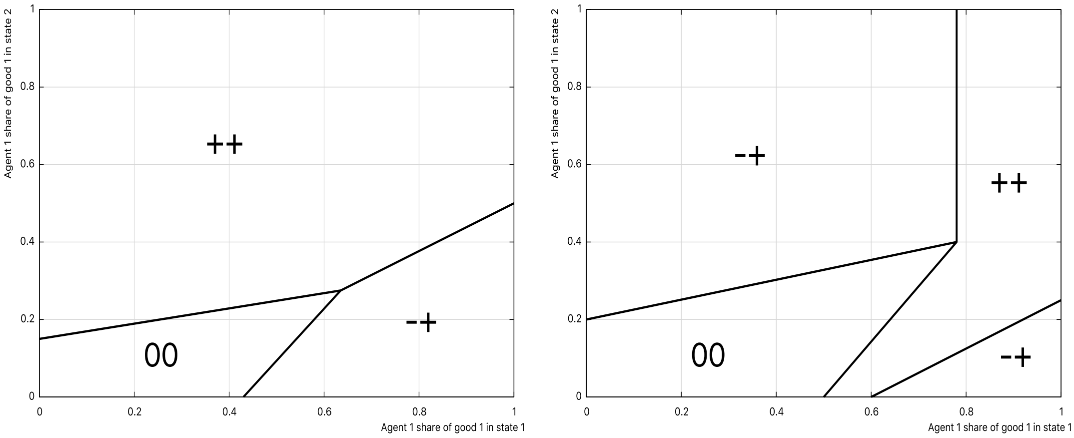
(a) Agent's utilities in ω-model (b) Agent's utilities in δ-model



When the poor is relatively optimistic, the numerical results shows the reverse of the previous paragraph. This is depicted in figure 3.19. The Pareto improvement region of the ω-model covers all the leverage constrained economies and the Pareto improvement of the δ-model decreases because it loses the region where the poor is only leverage constrained. However, an interesting small region of Pareto improvement appears inside the short sale constrained region in the bottom right of the box, even for initial values of δ, 0 and 0.001. Recall that in the previous chapter, with homogeneous expectations, the Pareto improvement in short sale economies was found only for positive values of δ.

Figure 3.19: Agent's utilities at $(p_1^1, p_1^2) = (0.75, 0.5)$

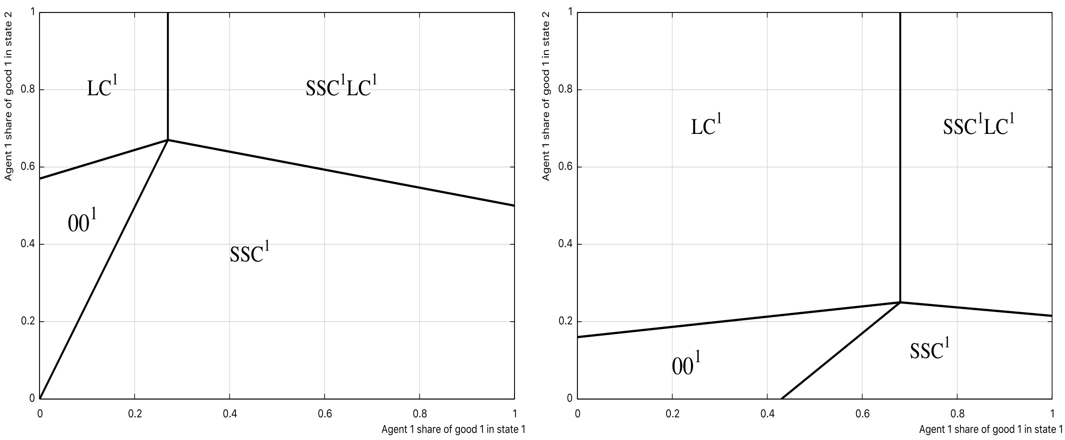
(a) Agent's utilities in ω-model (b) Agent's utilities in δ-model



Finally, the constraints of the poor behave, as expected, in the same way for both models because it is determined by the relative optimism (utility function) and the endowment. More relative optimism favors the leverage constrained region and the relative pessimism favors the short sale constrained region.

Figure 3.20: Poor's constraints

(a) At $(0.25, 0.5)$ (b) At $(0.75, 0.5)$

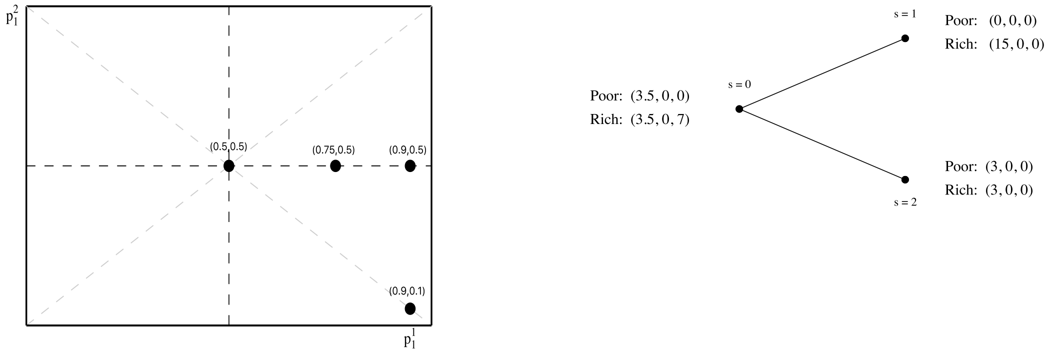


Fixed endowment and fixed probabilities

In this last analysis the same subjective probabilities and endowments of the ω-model were chosen:

Figure 3.21: Fixed endowment and fixed probabilities

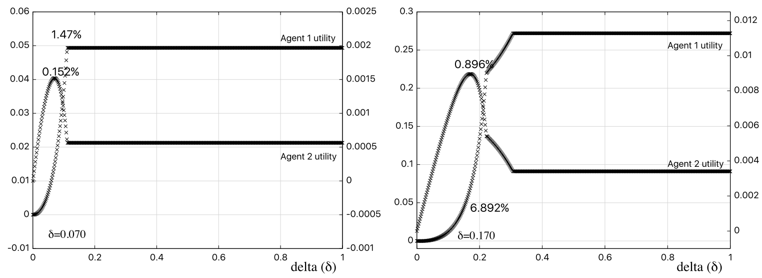
(a) Probabilities analyzed (b) Fixed endowment



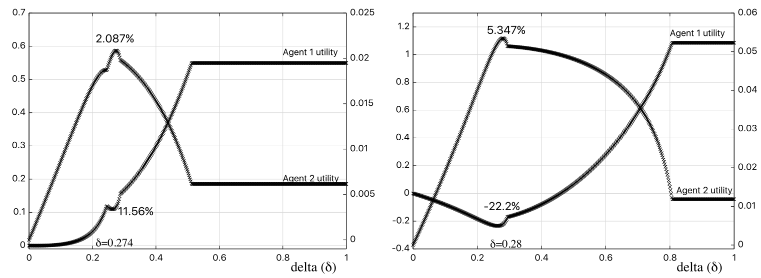
The sequence of figures (a)-(d) shows that when the relative optimism is higher, the slope of the marginal utility function of the poor in $\delta = 0$ becomes negative. Also note, in chart (d), that when the poor is highly relatively optimistic the region of Pareto improvement almost vanishes. This is an interesting feature because the δ-model has the property of reaching the Arrow Debreu Pareto frontier, which in this case happens after $\delta = 0.8$.

Figure 3.22: Fixed endowment and fixed probabilities

(a) case $(p_1^1, p_1^2) = (0.5, 0.5)$ (b) case $(p_1^1, p_1^2) = (0.75, 0.5)$



(c) case $(p_1^1, p_1^2) = (0.9, 0.5)$ (d) case $(p_1^1, p_1^2) = (0.9, 0.1)$



Finally, the sudden changes in curves of chart (c) and (d), around 0.3, matches with the moment in which the agents change the structure of the assets traded. For example, in figure (c), the poor is initially selling the subprime, then they stop trading this asset for a small interval and after that the rich starts selling it.

3.4 Conclusions

In this chapter it was studied the relation between the heterogeneous beliefs of the agents and two crisis policies: the unconventional monetary policy and the collateral requirement relaxation with compensation. This analysis in ω-model is missing in Araujo et al. (2015). The contribution is not only in filling this gap but also advancing in the study of the effects of different crisis policies in economies with heterogeneous beliefs. As the numerical results suggests, the presence of heterogeneous beliefs can potentiate or sterilize the policies in different ways. Furthermore, the role played by optimistic agents in the economy or in moments of crisis is still being understood and investigated by the researchers, such as in Geanankoplos (2009) and Tsomocos and Yan (2016).

The numerical findings suggests that the relative optimism has huge influence in determining the constraints of the agents. The one relatively more optimistic tends to be leverage constrained and the one relatively more pessimistic tends to be short sale constrained. This is in tune with Geanakoplos' intuition. The effects of the policies also seem to depend on the relative optimism, not only through the constraints of the agents but through intrinsic mechanisms of each policy.

On the unconventional monetary policy, it was found that when the borrower is relatively optimistic the policy is *more* effective in the sense that the set of economies with Pareto improvement increases. Thus, optimism *potentiates* the effect of this policy and, in contrast, pessimism *softens* the effect. The theoretical reason behind this is that the unconventional monetary policy can only produce a Pareto improvement in economies with leverage constrained agents, and the optimism tends to make the agents leverage constrained. Generally, the greater the relatively optimism the greater the region of Pareto improvement. This effect, however, has limitations. When approaching the extreme case of maximum relative optimism, the Pareto improvement region quickly vanishes.

The numerical results also showed that the optimism/pessimism affects the room for the unconventional monetary policy in the sense that it allows for Pareto improvement even for higher levels of purchase of durable by the Central Bank. And it was found that the maximum gains in the utilities happens in the region where the poor is more optimistic and the rich is neutral.

On the collateral requirement relaxation with compensation it was found that when the poor is relatively pessimistic, there is a *better*, but not greater, Pareto improvement region. The better Pareto improvement is in the sense that the poor gains more utility with a marginal relaxation of the collateral requirement than the case of homogeneous beliefs, thus the Pareto improving is more clear and effective. The size of the Pareto improvement region is indeed smaller than the homogeneous case, but still greater than the unconventional monetary policy with the same heterogeneous expectations. When the poor is relatively optimistic, although the Pareto improvement region of the collateral requirement relaxation is smaller than the unconventional monetary policy, it can still achieve Pareto improvement in economies that are short sale constrained, not leverage. And this happens even for initial values of the relaxation, which is a new result in comparison with the homogeneous case. Also an interesting result is that when the relative optimism of poor is sufficiently high, the Pareto improvement region almost disappear even though the relaxation of the collateral requirement with compensation keeps the financial markets complete and leads to the Arrow Debreu Pareto frontier.

Finally, when the analysis of the two models with heterogeneous beliefs are put together, they seem to be much more complementary than substitutes. Loosely speaking, the unconventional monetary policy should be prefered when the poor is relatively optimistic because its effects will be potentialized. However, if the central authority is looking for a short sale constrained economy, the collateral requirement regulation can be useful even in this case. And when the poor is relatively pessimistic the collateral requirement regulation should be prefered because, although diminishing, its Pareto improvement region remains greater and more effective than the unconventional monetary policy.

3.5 Appendix: details of the implementation

The complete system of equalities and inequalities implemented in ALGENCAN is depicted below. Recall the notation: $m = \sum_h m^h$ and $e_{03} = \sum_h e^h_{03}$.

Inequality consumption $t = 0$ (collateral constraint):

$$-x^h_{0l} + \sum_j \varphi^h_j C_{lj} \leq 0 \quad \forall h,\, \forall l$$

Inequality to r:

$$2r_{sj} - 1 - \sum_l p_{sl} \sum_{l'} Y_{s_{ll'}} C_{l'j} \leq 0 \quad \forall s \in S,\, \forall j$$

First-order condition of x in $t = 0$:

$$\partial_{0l} u^h(x^h) - \mu^h_0 p_{0l} + \sum_{s=1} \mu^h_s \sum_{l'} Y_{s_{l'l}} p_{sl'} + {}_{col}\mu^h_l = 0 \quad \forall h,\ \text{ for } l = 1,2$$

$$\Longrightarrow \frac{\alpha_{01}}{x_{01}} - \mu^h_0 p_{0l} + {}_{col}\mu^h_l = 0 \quad \forall h,\ \text{ for } l = 1,2$$

and

$$\partial_{03} u^h(x^h) - \mu^h_0 (p_{03} - p_{02}) + \sum_{s=1} \mu^h_s \sum_{l'} Y_{s_{l'3}} p_{sl'} + {}_{col}\mu^h_3 = 0 \quad \forall h,\ \text{ for } l = 3$$

$$\Longrightarrow -\mu^h_0 (p_{03} - p_{02}) + {}_{col}\mu^h_3 + \mu^h_s p_{s3} = 0 \quad \forall h,\ \text{ for } l = 3$$

First-order condition of x in $t = 1$:

$$\partial_{sl} u^h(x^h) - \mu^h_s p_{sl} + {}_x\mu^h_{sl} = 0 \quad \forall h,\, \forall s \in S, \forall l = 1,2$$

$$\Longrightarrow \frac{prob(s)\alpha^h_{sl}}{x^h_{sl}} - \mu^h_s p_{sl} = 0 \ \ \forall h, \forall s \in S, \forall l = 1,2$$

and

$$\partial_{s3} u^h(x^h) - \mu^h_s (p_{s3} - p_{s2}) + {}_x\mu^h_{s3} = 0 \quad \forall h,\, \forall s \in S, l = 3$$

$$\Longrightarrow -\mu^h_s (p_{s3} - p_{s2}) = 0 \ \ \forall h, \forall s \in S, l = 3$$

Budget constraint at $t = 0$:

$$\sum_l p_{0l}(x^h_{0l} - e^h_{0l}) - p_{02} x^h_{03} + \sum_j q_j(\psi^h_j - \varphi^h_j) + \mu^h - m^h = 0 \quad \forall h$$

Budget constraint at $t = 1$:

$$\sum_l p_{sl}(x^h_{sl} - e^h_{sl} - \sum_{l'} Y_{s_{ll'}} x^h_{0l'}) - p_{s2} x^h_{s3} - \sum_j (\psi^h_j - \varphi^h_j) r_{sj} +$$
$$+\theta^h (1+i)[(p_{03} - p_{02})\omega e_{03} + m] - \theta^h p_{s3} \omega e_{03} - (1+i)\mu^h = 0 \quad \forall h,\, \forall s \in S$$

First-order conditions of ψ:

$$_\psi\mu^h_j + \sum_s \mu^h_s r_{sj} - \mu^h_0 q_j = 0 \quad \forall h,\, \forall j$$

First-order conditions of φ:

$$ {}_{\varphi}\mu_j^h - \sum_s \mu_s^h r_{sj} + \mu_0^h q_j - \sum_l {}_{col}\mu_l^h C_{lj} = 0 \quad \forall h,\ \forall j $$

First-order conditions of μ:

$$ -\mu_0^h + \sum_{s=1}^{S} \mu_s^h (1+i) + {}_{\mu}\mu^h = 0 \quad \forall h $$

Market clearing for x at $t = 0$:

$$ \begin{aligned} \sum_h (x_{01}^h - e_{01}^h) &= 0 \quad (l = 1) \\ \sum_h (x_{02}^h - e_{03}^h) &= 0 \quad (l = 2) \\ \sum_h (x_{03}^h - (1-\omega) e_{03}^h) &= 0 \quad (l = 3) \end{aligned} $$

Market clearing for x at $t = 1$:

$$ \begin{aligned} \sum_h (x_{s1}^h - e_{s1}^h) &= 0 \quad \forall s \in S,\ (l = 1) \\ \sum_h (x_{sl}^h - e_{s3}^h - Y_{s33} e_{03}^h) &= 0 \quad \forall s \in S,\ (l = 2, 3) \end{aligned} $$

Market clearing for ψ and φ:

$$ \sum_h (\psi_j^h - \varphi_j^h) = 0 \quad \forall j $$

Market clearing for the money μ^h:

$$ \sum_h \mu^h = m + (p_{03} - p_{02}) \omega e_{03} $$

Boundary conditions:

$$ \begin{aligned} {}_{x}\mu_{sl}^h x_{sl}^h &= 0 \quad \forall h,\ \forall s,\ \forall l \\ {}_{\psi}\mu_j^h \psi_j^h &= 0 \quad \forall h,\ \forall j \\ {}_{\varphi}\mu_j^h \varphi_j^h &= 0 \quad \forall h,\ \forall j \\ {}_{\mu}\mu^h \mu^h &= 0 \quad \forall h \\ {}_{col}\mu_l^h (-x_{0l}^h + \sum_j \varphi_j^h C_{lj}) &= 0 \quad \forall h,\ \forall l \end{aligned} $$

Portfolio condition or ortogonality of ψ and φ:

$$ \varphi_j^h \psi_j^h = 0 \quad \forall h,\ \forall j $$

$\boxed{\text{Price normalization at } t = 1\text{:}}$

$$p_{s1} = 1 \quad \forall s \in S$$

$\boxed{\text{Equality for } r_{sj}\text{:}}$

$$(r_{sj} - 1)(r_{sj} - \sum_{l} p_{sl} \sum_{l'} Y_{s_{ll'}} C_{l'j}) = 0 \quad \forall s \in S,\, \forall j$$

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