Solving the Traveling Salesman Problem and establishing P = NP

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1) Introduction:

This paper will demonstrate a solution for the traveling salesman problem and as a consequence why P is equal to NP.

2.1)Brief description of the Traveling Salesman Problem (TSP):

You have n cities and have to find the shortest path to visit all and come back to the start city, passing only one time in every city. To do it you just know the distances between the cities.
This problem is classified as NP-hard.
But also can be NP complete. If the question is to check if there is a path that go in all cities once and came back to the start smaller than a given number.

2.2) Solving the TSP:

-----NP-hard version-----

*Get a matrix with the values;

*Sum the numbers of the matrix lines, generating one sum per line (The line has the distance from one city to others);

*Find the cities that have the 3 biggest sums and connect the cities with it other. (E.g. A-B-C-A);

*The previous step will generate rows (E.g. AB; AC; BC);

*For every next city (that will be the biggest remaining sum, coming from the sums made in the second step):

- -for all the already generated rows:
  - --sum the distance from the actual city to both of the row cities and subtract the row size (Generating a cost).
- -get the row with the smallest cost delete this row and add 2 new ones, from the actual city to the cities that were forming the deleted row.

-----NP-complete version-----

*Just do the steps from the NP-hard version and at the end sum the the path to see if it is smaller than the given number.

2.3) HTML page that solves the Traveling Salesman Problem:

```
<!DOCTYPE html>
<html>
<head>
</head>
<body>

<h1><b>Salesman</b></h1>
<h2>Make a better path.</h2>
<button onclick="findBestPath()">Find your way</button>
</br></br>
<p id="resultHere"></p>

<script>
//edit the "distancias" to inform the distances
/*8x8*/var distancias=[[888, 11, 15, 10, 22, 29, 38, 54],
                  [11, 888, 29, 11, 29, 24, 45, 48],
                  [15, 29, 888, 18, 15, 34, 28, 55],
                  [10, 11, 18, 888, 14, 15, 31, 41],
                  [22, 29, 15, 14, 888, 18, 14, 36],
                  [29, 24, 34, 15, 18, 888, 27, 22],
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                  [54, 48, 55, 41, 36, 22, 34, 888]];
////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////
/*10x10*//*var distancias = [[888, 11, 15, 10, 22, 29, 38, 54, 25, 32],
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                   [15, 29, 888, 18, 15, 34, 28, 55, 11, 23],
                   [10, 11, 18, 888, 14, 15, 31, 41, 20, 43],
                   [22, 29, 15, 14, 888, 18, 14, 36, 6, 41],
                   [29, 24, 34, 15, 18, 888, 27, 22, 29, 59],
                   [38, 45, 28, 31, 14, 27, 888, 34, 13, 50],
                   [54, 48, 55, 41, 36, 22, 34, 888, 44, 80],
                   [25, 36, 11, 20, 6, 29, 13, 44, 888, 35],
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////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////
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         5930.14     ,     3690.71     ,     3763.14     ,     5043.17     ,
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        1315.59     ,     3132.56     ,     88888888],];*/
/////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////
var g = 0;
var j = 0;
var t = 0;
var l = 0;
```

```
var i = 0;
var n = 0;
var p = 0;
var u = 0;
var s = 0;
var b = 0;

function findBestPath()
{
var sums = [];
var sums2 = [];
var finalRows = [];
var FB = [];
var time10 = [];

	for(p = 0; p<distancias.length; p++)
	{
		s = 0;
		for(u = 0; u<distancias.length; u++)
		{
			s += Number(distancias[p][u]);
		}
		sums.push(s);
	}
	sums2 = sums.slice();
	sums2.sort(function(a, b){return b-a});

	finalRows.push([sums.indexOf(sums2[0])]);
	finalRows[0].push(sums.indexOf(sums2[1]));
	finalRows.push([sums.indexOf(sums2[1])]);
	finalRows[1].push(sums.indexOf(sums2[2]));
	finalRows.push([sums.indexOf(sums2[2])]);
	finalRows[2].push(sums.indexOf(sums2[0]));

	for (n=3; n<distancias.length; n++)
	{
		FB = [];
		time10 = [];
		t = sums.indexOf(sums2[n]);
```

```
		for (g=0; g<n; g++)
		{
			j = finalRows[g][0];
			l = finalRows[g][1];
			FB.push(Number(distancias[t][j])+Number(distancias[t][l])-
Number(distancias[j][l]));
		}
		time10 = FB.slice();
		time10.sort(function(a, b){return a-b});

		finalRows.push([sums.indexOf(sums2[n])]);
		finalRows[n].push(finalRows[FB.indexOf(time10[0])][1]);
		finalRows[FB.indexOf(time10[0])][1] = t;
	}

	document.getElementById("resultHere").innerHTML=finalRows+
" ---- this has "+finalRows.length+" rows";
	window.alert(finalRows);
}
////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////
</script>
</body>
</html>
```

2.4)Explaining the HTML page:

First set the array with the distances (Note that the array starts with city 0 distance to other cities inside the first [ ], city 1 inside the second, city two inside the third and so on and when from n to n use a big number, this way when it sums will get a high result and not consider it.)

Sum the numbers of the array square brackets, generating one sum result for every square bracket;

Generates an array in descending order using these sums;

Connect the cities of the 3 first elements in this array with it other. (E.g. 0-1-2-0);

The previous step will generate paths (E.g. 0,1; 1,2 e 2,0);

For every next city from the array made with the square Brackets sums: get all the already generated paths, sum the distance from the actual city to both of the path cities and subtract the path size (Generating a cost for it path) and get the path with the smallest cost delete this path and add 2 new ones, from the actual city to the cities that were forming the deleted path.

The result will be pairs of numbers, those are the paths that form the best way. For exemple: 0,1,4,3,1,2,2,4,3,0, here the best way is from 0 to 1, 4 to 3, 1 to 2, 2 to 4 and 3 to 0.

Note that the code can run faster if used a better sorting algorithm.

3)P = NP

In short P stands for polynomial time and in this class are the problems that are easy to solve and check, NP stands for non polynomial time and this class contains the problems that are easy to check and hard to solve and the P vs NP question asks if all problems that can have the solution checked in polynomial time can be solved in polynomial time.

Thus is possible to say that P = NP because it has already been proved that all the problems in the NP class can be reduced to the ones in the NP-COMPLETE class. So if one NP-COMPLETE problem, the traveling salesman problem, is solved in polynomial time then all the NP class can be solved in polynomial time and if a problem can be solved and checked in polynomial time this problem is classified as P.

4) Conclusion:

It has already been established that any NP problem can be reduced to a NP-complete problem in polynomial time. Thus the solution above shows that all the NP class of problems is solvable in polynomial time, what means NP is equal to P.