Automatic reconstruction of ancient Portuguese tile panels

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Portuguese tile panels, or *azulejos*, are one of Portugal's cultural icons, and a representative cultural heritage of this country. Portugal's museums currently have a large collection of loose tiles that are often reassembled manually, which represents a laborious and challenging work. In this article, we explore the problem of automatically reconstructing ancient tile panels, mapping this problem to the reconstruction of an image from an unordered collection of rectangular non-overlapping tiles, an interest and important formulation of the jigsaw puzzle problem. Here we analyze, in a preliminary study, the application of image puzzle solvers in the assembling of ancient tile panels provided by the National Tile Museum. We compare the obtained results in different formulations of the problem, depending on the prior knowledge – known or unknown panel dimension and tile orientation –, and with missing tiles.

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Fig. 1. Tile Panel at the “Igreja da Misericórdia de Arraiolos”, named “Obras de Misericórdia – Enterrar os mortos”, 1753.

1. INTRODUCTION

Tile panels (or *azulejos* in Portuguese) is a form of Portuguese (and also Spanish) painted, tin-glazed, ceramic tilework that has become one of the most important forms of artistic representation in Portugal for the last five centuries (Figure 1).

The artists of azulejos were often inspired by or used to copy famous paintings or prints of those paintings [da Silva et al. 2011]. As a result, thousands upon thousands of tiles were produced. Not only in Portugal, but also in other Portuguese colonies (e.g., Brazil), *azulejos* are still commonly found in the interior and exterior of churches, palaces, castles, houses, restaurants, and railway stations. These tile panels usually cover large sections of walls, floors, or even ceilings for decorative purposes. When one of these buildings needs to be renovated or demolished, these *azulejos* can also be destroyed unless they are carefully removed from the building structure.

The National Tile Museum (*Museu National do Azulejo*, MNAz), in Lisbon, Portugal, currently stores a large collection of tiles that have been removed from several buildings in Portugal. In order to study these tiles, several works have been devoted to their characterization, treatment, and conservation [Carvalho et al. 2006; Silva et al. 2013; Pereira et al. 1992; Pessoa et al. 1996]. In a work of heritage preservation, the MNAz tries to reassemble the panels back together, in a program called *Devolver ao Olhar* (Giving Back to the View) [Temática 2011].

The challenges involved in reassembling tile panels are huge because some of them have been removed in such a way that mounting instructions are non-existing, there can be missing tiles, and information about the shape and size of the panels is not always available. Furthermore, a single box of loose tiles can actually contain several (incomplete) panels.

Fig. 2. A person assembling a panel made of Portuguese tiles at The National Tile Museum (*Museu National do Azulejo*, MNAz), in Lisbon, Portugal. Image provided by [Fonseca 2012].

Figure 2 shows an art historian of MNAz working on the reassembling of a tile panel. The tasks involved in reassembling a tile panel comprise placing the tiles from a single origin on the floor and cataloging the position, orientation, and panel identity of each tile. These tasks usually involve the investigation of hundreds of tiles and can become quite time consuming, even for an expert in *azulejos*.

A tile panel assembling process can be seen as the assembly of a jigsaw puzzle, where each tile corresponds to one piece in the puzzle. However, the level of difficulty can be higher than an usual puzzle because sometimes no information about the final appearance is available, which means that it may be necessary to determine the position and orientation of each tile. Another difficulty is that all the pieces of this puzzle are equal and roughly square, thus failing to provide any information about its orientation and neighboring tiles, except from color continuity. In addition, there is still the problem that many of the tiles are in an advanced state of degradation, making the pairing of them based on appearance a challenging task.

Similar cultural heritage problems have been studied previously, with two important examples being the Thera Frescoes project [Brown et al. 2008; Shin et al. 2012; Toler-Franklin et al. 2010] and the Digital Forma Urbis Romae project [Koller et al. 2006; Koller and Levoy 2006]. The Thera Frescoes project aims at the automated digitization and matching of free-form fragments of wall paintings (frescoes) recovered from the archaeological site of Akrotiri on the island of Thera. It has three main components: acquisition, matching algorithms that compute candidate matches between fragments, and a user interface that allows users to evaluate the proposed matches. The Digital Forma Urbis Romae project aims at reconstructing the Severan Marble Plan of Rome, an enormous map, carved between 203–211 CE, that covered an entire wall inside the Templum Pacis in Rome. It employs digital technologies to try to reconstruct the map, creating digital photographs and 3D models of all $1,186$ fragments, and building a fully searchable database.

In this work we analyze the application of image puzzle solvers to the automatic reconstruction of Portuguese tile panels provided by the MNAz. These solvers address the problem of reconstructing images from rectangular non-overlapping puzzle pieces of identical shape and size. This type of application has been explored before, but with panels very limited in size. We extend it to other scenarios, yet in a preliminary study, by considering larger and mixed panels, and missing tiles.

The application of image puzzle solvers to the problem of reconstructing tile panels has the advantage of not requiring complicated pipelines and equipment. With a standard camera, one has to simply digitize the tiles. Each tile is automatically corrected by the adjustment of its shape and size and then, using all the prior knowledge available, a solver can reconstruct the entire panel or pieces of it. The tiles used in this work have not received any treatment due to their deterioration, yet it is possible to reconstruct the panels entirely or several parts of them.

In Section 2 we cover the literature on image puzzle solvers and in Section 3 we present the formulation of relevant solvers and their comparison with other methods. Section 4 shows the application of such solvers to the task of reassembling Portuguese tile panels, and finally Section 5 concludes our work.

2. BACKGROUND ON IMAGE PUZZLE SOLVERS

The problem of automatically reconstructing an image from a collection of unordered non-overlapping pieces, or tiles, is computationally complex in a sense that no efficient algorithm is known capable of solving it in a deterministic manner when the compatibility between the tiles is uncertain, i.e., when it is not possible to determine the adjacent tiles without ambiguities [Demaine and Demaine 2007].

The problem has an inherent difficulty revealed from its *global nature*. It is hard to construct an entire image puzzle when dealing only with local matches, because no exact measure of similarity between tiles is known to date. Moreover automatic puzzle solvers have to overcome the *combinatorial nature* of the problem, in which the number of possible solutions increases super-exponentially with the number of available tiles, because every possible permutation of the tiles can be a valid solution.

Generally image puzzle solvers are developed for two kinds of puzzle: *pictorial*, in which the correctly assembled pieces form an image, and *apictorial*, where there is no chromatic difference between the pieces, and their distinct shapes, when assembled correctly, form an unique plane.

The first solver was proposed by Freeman and Garder [1964] to solve 9-piece apictorial puzzles, and it is considered the basis to many subsequent works. Thirty years later, the method by Kosiba et al. [1994] was the first to consider chromatic information, successfully assembling small pictorial puzzles with traditional pieces.

In this work, we consider pictorial puzzles, but formed by identical rectangular pieces, or *tiles*. The literature for this kind of puzzle is somewhat recent [Cho et al. 2010b; Pomeranz et al. 2011; Gallagher 2012; Andaló et al. 2012; Fonseca 2012; Sholomon et al. 2013], although the problem is very important in practice. Besides the reconstruction of tile panels, puzzle solvers can generate solutions to other scientific problems: reassembling of broken archaeological artifacts [McBride and Kimia 2003; Brown et al. 2008; Koller et al. 2006], reconstruction of shredded documents [Justino et al. 2006; Zhu et al. 2008], speech recognition [Zhao et al. 2007], DNA/RNA modeling [Marande and Burger 2007], image editing [Cho et al. 2010a], among others.

Not every work considers the same *a priori* knowledge of the problem. Works by Cho et al. [2010b], Pomeranz et al. [2011], Andaló et al. [2012], and Sholomon et al. [2013] consider that the puzzle dimension and the orientation of each tile are known, opposed to the works by Gallagher [2012] and Fonseca [2012]. All of them accept only square tiles, except the method by Andaló et al. [2012] that can solve puzzles with arbitrary rectangular tiles, an useful characteristic when assembling shredded documents, for example.

Cho et al. [2010b] obtained an approximate reconstruction of the original image using graphical models and a global probabilistic function. However the method needs information about the layout of the original image, such as the correct location of some tiles informed by the user. Although being semi-automatic, this strategy allows the assembling of puzzles up to 432 tiles.

Pomeranz et al. [2011] presented a method that does not need user intervention. It is based in a greedy approach, in which a compatibility function is computed to measure the affinity between the tiles, and then the method solves three problems: positioning, segmentation, and translation. The positioning module put all tiles on the grid following a predetermined logic and considering randomly selected seeds; the segmentation module identifies the regions that are more likely to be assembled correctly; and the translation module reallocates regions and tiles to produce the final result. With this greedy strategy, they achieved the considerable improvement of solving puzzle with up to 3300 tiles.

Sholomon et al. [2013] proposed a greedy genetic algorithm to solve very large puzzles up to 22,834-puzzle pieces with known tile orientation and puzzle dimension.

The other works by Andaló et al. [2012], Fonseca [2012], and Gallagher [2012] are described in more details in the next section. We evaluate these methods using a standard dataset of natural images, showing that they provide good results in comparison with the other methods, considering the tested metrics and puzzle dimensions.

3. IMAGE PUZZLE SOLVERS

In this section, we describe three image puzzle solvers [Andaló et al. 2012; Fonseca 2012; Gallagher 2012]. The solver by Andaló et al. [2012] can be applied to puzzles with arbitrary rectangular tiles, with known panel dimension and tile orientation. The solvers by Fonseca [2012] and Gallagher [2012] can be applied to puzzles with square tiles and unknown panel dimension and tile orientation. The following subsections briefly describe each solver. For a more detailed explanation, please refer to the original publications.

3.1 Method by [Fonseca 2012]

The work by Fonseca [2012] was the first to apply the idea of image puzzles to panels of Portuguese tiles, although it was developed only for small panels.

The greedy method tries to minimize the distance between tile appearances at each iteration of the algorithm, as tiles are connected to the final solution. It begins by computing a Global Distance Matrix S, of size $4N \times 4N$, that comprehends the distance between all tiles in every possible tile orientation. The lowest value is chosen and the corresponding tiles are put together in the final solution as neighbors.

At this point, there are six available borders in the solution, so that a new tile can be connected, and $N - 2$ possible connections (tiles that have not been used yet). A $4N \times 4N$ mask is created and an element-wise product between the mask and S provides the minimum value corresponding to the best tile connection. The purpose of this mask is to disallow new connections with tiles that have already been used in the final solution.

This procedure is repeated until all tiles have been connected to the final solution (Figure 3).

To ensure a good quality result, an heuristic called *Lowe Scores* is also employed. When there are two tile candidates to be connected in the final solution, with close distance values according to a threshold, the connection is rejected. This heuristic was presented in [Lowe 2004] and suggests that a connection is not meaningful if the tile candidates have almost the same distance.

3.2 PSQP – Puzzle Solving by Quadratic Programming [Andaló et al. 2012]

The method presented in Andaló et al. [2012], named PSQP (Puzzle Solving by Quadratic Programming), is based in maximizing a global matching function which calculates the overall compatibility of a certain tile permutation.

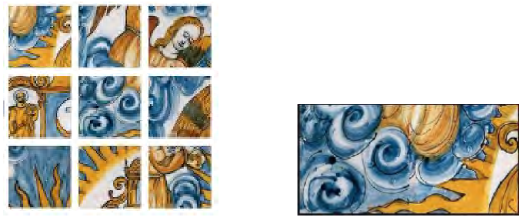

Fig. 3. The greedy method starts by chosen the lowest distance between two borders and put together the corresponding tiles. For the second iteration, six borders are available.

Consider an image partitioned into a regular 2D grid, forming N tiles of identical dimensions; and an empty grid of the same size as the previous one with N locations. The problem is to determine a one-to-one correspondence between the N tiles and the N locations, optimal with respect to a global compatibility function $\varepsilon(P)$ that sums up the compatibility of the neighboring tiles, considering P as a permutation matrix that assigns tiles to locations (Figure 4). Briefly, the compatibility between two tiles can be though of as a measure based on the color difference of the touching borders, when the tiles are considered as neighbors in a solution.

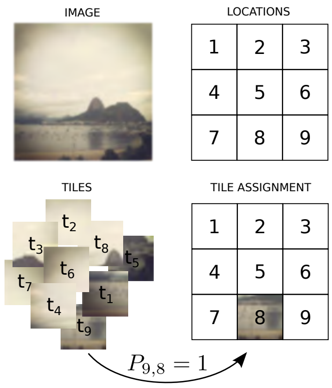

Fig. 4. Problem formulation. Each tile t_i is assigned to a location j. The final result is represent by a permutation matrix P.

The goal is to maximize $\varepsilon(P)$ over all permutation matrices P of size $N \times N$. Since this is a hard combinatorial optimization problem, it is necessary to extend the domain of the global compatibility function to the set of doubly stochastic matrices by relaxing the binary constraint of matrix P. The problem is then reformulated as a constrained continuous optimization problem, which can be solved by numerical methods.

The final global compatibility function to be maximized is

$$\begin{aligned} &\text{Maximize } f(p) = p^\top A p, \\ &\text{subject to } P\mathbb{1} = \mathbb{1}, P^\top \mathbb{1} = \mathbb{1}, \text{ and } p_{ij} \geq 0, \end{aligned} \tag{1}$$

where p is the column concatenation of P, A is the Hessian of $\varepsilon(P)$, $\mathbb{1}$ is a column vector of size N with all elements equal to one.

To search for the local maxima of the problem, which in practice is a permutation matrix representing a possible solution, the authors proposed a modified constrained gradient ascent algorithm, with gradient projection [Rosen 1960].

3.3 Method by Gallagher [2012]

The work by Gallagher [2012] was the first to introduce puzzles in which the orientation of the pieces is unknown. To solve this kind of puzzle, Gallagher proposed a new compatibility metric called the Mahalanobis Gradient Compatibility (MGC), that describes the local gradients near the boundary of a piece. This metric penalizes changes in intensity gradient, learns the covariance between the color channels and then uses the Mahalanobis distance.

The proposed method to assemble puzzles is inspired by the Minimum Spanning Tree (MST) algorithm for graphs. The problem is formulated as a graph where each piece is a vertex, and edge weights are the MGC computed for the corresponding pieces. The MST is the cheapest possible configuration that could be used to assemble the pieces into a single connected component, but some geometric constraints need to be applied so that the MST does not result in a puzzle that overlaps itself.

The proposed algorithm has three stages:

(1) **Constrained tree:** the method applies a constrained version of the MST algorithm to find a tree in the input graph, according to the geometric constraints of the problem.

(2) **Trimming:** if the resulting tree does not fit into a regular frame and the dimension of the puzzle is known, then the assembled tree is trimmed.

(3) **Filling:** after trimming, the puzzle frame can have unoccupied holes. At this stage, holes are filled by order of the number of occupied adjacent neighbors and, for each hole, the candidate piece is the one with the minimum total dissimilarity score across all neighbors.

3.4 Image puzzle solvers comparison

In order to understand the applicability and accuracy of the detailed image puzzle solvers, we compare them in a standard dataset of images. This dataset is composed of twenty natural images provided by [Cho et al. 2010b]. Each puzzle consists of 432 tiles of size 28×28 pixels.

The accuracy of the solutions are measured according to two different metrics previously proposed by Cho et al. [2010b] and Gallagher [2012]:

Neighbor comparison: for each tile, this metric computes the fraction of its neighboring tiles that are also its neighbors in the correct solution. The accuracy is the mean fraction of correctly assigned neighbors.

Perfect reconstruction: binary indication of whether every tile is assigned to the correct location in a puzzle.

Note that directly comparing the resulting puzzle with the ground-truth image is not a good metric because it is unable to cope with slightly shifted solutions [Sholomon et al. 2013].

First we present the results for the methods that can work with known puzzle dimension and tile orientation [Cho et al. 2010b; Pomeranz et al. 2011; Andaló et al. 2012; Gallagher 2012; Sholomon et al. 2013].

Table I summarizes the mean accuracy values for each method when run in the dataset of 20 images. Methods PSQP [Andaló et al. 2012] and by Sholomon et al. [2013], which employs a genetic-based greedy approach, attained the highest accuracy among all methods. PSQP, however, was able to perfectly reconstruct more puzzles.

Table I. Accuracy for each method, considering known panel dimension and tile orientation.

Methods/Metrics	**Neighbor (%)**	**# Perfect**
PSQP [Andaló et al. 2012]	96	13
Cho et al. [2010b]	55	0
Pomeranz et al. [2011]	94	13
Gallagher [2012]	95	12
Sholomon et al. [2013]	96	7

As PSQP, Cho et al. [2010b] also employs a global approach, but by maximizing a probabilistic function via Loopy Belief Propagation. It needs some tiles to be fixed in their right position, however it is not able to perform perfect reconstructions. Methods by Pomeranz et al. [2011] and Gallagher [2012] have similar accuracy.

Figure 5 shows one of the obtained results comparing PSQP [Andaló et al. 2012] with the method by Pomeranz et al. [2011], and Figure 6 shows one of the obtained results comparing PSQP [Andaló et al. 2012] with the method by Sholomon et al. [2013].

Fig. 5. From left to right: initial permutation, final result obtained with PSQP, and final result obtained with the method by Pomeranz et al. [2011].

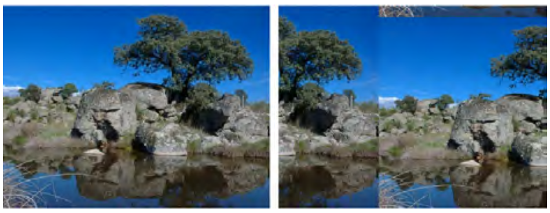

Fig. 6. Final results obtained with PSQP, and method by Sholomon et al. [2013].

Considering another puzzle formulation, with unknown tile orientation, almost all previous explored methods cannot be applied, because their formulations assumes that each tile is informed in its upright position. We compare two methods that allow this new formulation [Fonseca 2012; Gallagher 2012]. Note that both methods can solve puzzles with unknown panel dimension.

Table II summarizes the results. The accuracy of the methods in this dataset is almost equivalent.

Table II. Accuracy for each method, considering unknown panel dimension and tile orientation.

Methods/Metrics	**Neighbor (%)**	**# Perfect**
Fonseca [2012]	92.6	9
Gallagher [2012]	90.4	9

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Fig. 7. From left to right: original image, final result obtained with the method by Fonseca [2012], and final result obtained with the method by Gallagher [2012], with unknown panel dimension.

Figure 7 shows one of the obtained results comparing the method by Fonseca [2012] with the method by Gallagher [2012] with unknown panel dimension.

4. ASSEMBLY OF ANCIENT PORTUGUESE TILE PANELS

Despite the previously discussed difficulties inherent to image puzzles, the assembling of Portuguese tiles poses a new set of challenges: the advanced degradation state of some tiles. Other factors interfere in the process of comparing the tiles: variations in colors, cracks along the borders, non-continuous strokes from tile to tile, texture created by typically thorough strokes, etc.

The experiments were conducted with tile panels provided by the MNAz. The tiles were acquired by a simple tool [Fonseca 2012] that facilitates the acquisition process and the shape correction of the tiles. Each tile is captured by a camera and the corresponding image is automatically corrected: by using Hough transforms [R.O.Duda and P.E.Hart 1972], the corners of the acquired tile are found; and then the homography is calculated and applied to correct the perspective and size to match a square (Figure 8).

Fig. 8. Left: tile acquired by a camera. Right: corrected tile, considering perspective and size. Images provided by Fonseca [2012].

To reconstruct the tile panels, we considered the three image puzzle solvers detailed in Section 3. Except for the method by Fonseca [2012], the other two solvers [Andaló et al. 2012; Gallagher 2012] were chosen because of their good reported accuracy and available implementation. Results for the first solver are from [Fonseca 2012]. Because of the discussed issues inherent to Portuguese tile panels, the accuracy of their reconstruction is expected to be lower than in the previous experiment.

We considered three sets of panels provided by the MNAz:

—Twelve subsets of panels, with 25 tiles each (Figure 9);
—Four large panels, with 40, 48, 60 and 72 tiles each;
—Two mixes of 4 different tile panels, with 100 and 288 tiles each.

Experiments with each of the panel sets are described in the next subsections.

4.1 Experiment with small panels

In this first experiment, we used twelve subsets of panels provided by the MNAz, with 25 tiles each. Some examples are shown in Figure 9.

Fig. 9. Examples of Portuguese tile panels used in the experiments.

We consider unknown panel dimension and two conditioning scenarios: known and unknown tile orientation.

Known tile orientation
In this scenario the three methods [Andaló et al. 2012; Gallagher 2012; Fonseca 2012] can be used. Note that, because the panels are small (25 tiles each), PSQP can be applied to the possible three configurations and the one that yields the highest global compatibility is chosen as the solution. Table III summarizes the results.

Table III. Accuracy for each method, considering unknown panel dimension and known tile orientation.

Methods/Metrics	**Neighbor (%)**	**# Perfect**
PSQP [Andaló et al. 2012]	100.0	12
Gallagher [2012]	64.5	4
Fonseca [2012]	57.8	0

PSQP was able to reconstruct all the panels with 100% accuracy. Differently from PSQP, the other two methods assemble the panel taking into account the unknown dimensions and, for this reason, their lower accuracy is expected.

Unknown tile orientation
In this case only the methods by Fonseca [2012] and Gallagher [2012] can be applied, since PSQP needs the correct tile orientations to provide a solution. Table IV summarizes the results.
Although the accuracy in this scenario is low, note that the method by Gallagher [2012] is still able to correctly assign half of the neighboring tiles. In a real scenario, these results could help the user to assemble the entire panel.

Table IV. Accuracy for each method, considering unknown panel dimension and tile orientation.

Methods/Metrics	**Neighbor (%)**	**# Perfect**
Gallagher [2012]	49.4	3
Fonseca [2012]	35.9	0

To test the methods in solving panels with missing tiles, we conduct the same experiments but removing up to 30% of the tiles. Figure 10 shows the resulting accuracy for growing quantities of missing tiles.

Although PSQP gets better results when the orientation of the tiles is known, it is more affected by the missing tiles. Considering the scenario where the orientation of the tiles is not known, the method by [Gallagher 2012] can achieve the mean accuracy of 32% with 30% of missing tiles.

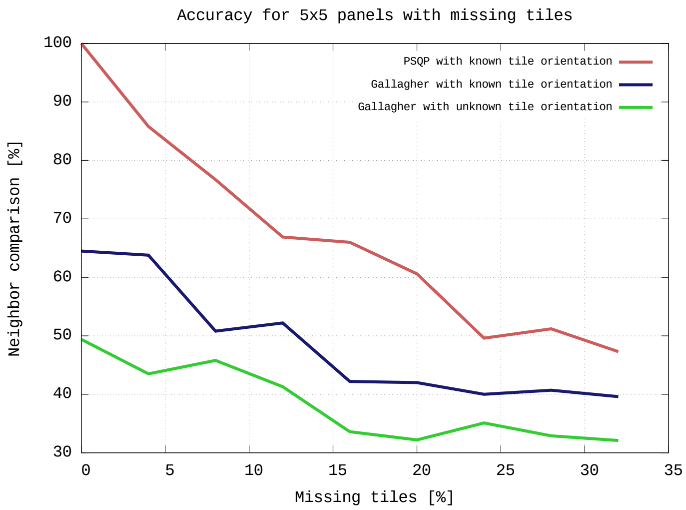

Fig. 10. Accuracy for 5×5 puzzles with missing tiles.

4.2 Experiment with larger panels

Considering the large panels provided by the MNAz, with 40, 48, 60, and 72 tiles each, we could only experiment with PSQP, as results for the method by Fonseca [2012] are not available.

PSQP was able to reconstruct the large panels with 100% accuracy, considering unknown panel dimension and know tile orientation. Some examples are shown in Figures 11 and 12.

The manually assembly of these panels, even with a few tiles, is an extremely difficult task, because often there is no information of what it is to be achieved, as would happen in a normal jigsaw puzzle problem. A computer method that is able to assemble an entire panel or parts of it can aid the restorers.

4.3 Experiment with mixed panels

In this third experiment, we mixed the tiles of four panels together, resulting in two mixed panels with 100 and 288 tiles each.

Fig. 11. Initial permutation and final result with 72 tiles.

Fig. 12. Initial permutation and final result with 60 tiles.

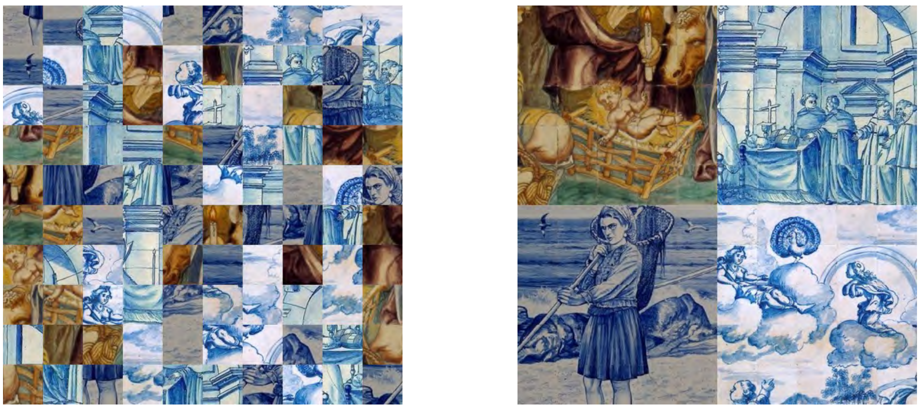

Fig. 13. Solving 4 mixed tile panels with PSQP, in a total of 100 tiles. Right: initial permutation. Left: final result.

Considering known tile orientation and unknown panel dimensions, PSQP was able to solve them with 100% accuracy. Figures 13 and 14 show some of these results.

A restorer could separate the mixed tiles prior to the assembly, taking into account, for instance, the appearance, color, and subject of the tiles. However, when these characteristics are similar, the

Fig. 14. Solving 4 mixed tile panels with PSQP, in a total of 288 tiles. Top: initial permutation. Bottom: final result.

automatic solver can separate them better and faster. The mixed panels (Figures 13 and 14) were assembled by PSQP in less than 1 minute each.

5. CONCLUSION

In this paper, we have studied two image puzzle solvers when applied to the reconstruction of ancient Portuguese tile panels and natural images divided as a jigsaw puzzle.

Preliminary experimental results showed that PSQP [Andaló et al. 2012] is promising in reconstructing images and panels when the tile orientations are known. Nevertheless, it is important to extend its application to panels with unknown dimensions, because as more tiles are considered, it is impossible to test every configuration.

The method by Fonseca [2012] can be applied when no *a priori* information about the panel is available. The application of such image puzzle solvers can aid restorers in reconstructing several parts of the panels, and visualizing the big picture.

Nevertheless, no work in the literature considers the reconstruction of panels with missing tiles. This challenge will be incorporated in a future study. We will also consider the study of several characteristics of the tiles, such as color pallete, stroke style, and material, that can aid the separation of mixed tiles prior to the overall assembly.

REFERENCES

F.A. Andaló, G. Taubin, and S. Goldenstein. 2012. Solving image puzzles with a simple quadratic programming formulation. In *XXV Conference on Graphics, Patterns and Images (SIBGRAPI)*.

B.J. Brown, C. Toler-Franklin, D. Nehab, M. Burns, D. Dobkin, A. Vlachopoulos, C. Doumas, S. Rusinkiewicz, and T. Weyrich. 2008. A system for high-volume acquisition and matching of fresco fragments: Reassembling Theran wall paintings. *ACM Transactions on Graphics (Proceeding of SIGGRAPH 2008)* 27, 3 (2008), 84:1–84:9.

A.P. Carvalho, M.F. Vaz, M.J. Samora, and J. Pires. 2006. Characterisation of ceramic pastes of Portuguese ancient tiles. 514 (2006), 1648–1652.

T.S. Cho, S. Avidan, and W.T. Freeman. 2010a. The patch transform. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 32, 8 (2010), 1489—1501.

T.S. Cho, S. Avidan, and W.T. Freeman. 2010b. A probabilistic image jigsaw puzzle solver. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*. 183–190.

N.P. da Silva, M. Marques, G. Carneiro, and J.P. Costeira. 2011. Explaining scene composition using kinematic chains of humans: application to Portuguese tiles history. In *Conference on Computer Vision and Analysis of Images of Art II*.

E.D. Demaine and M.L. Demaine. 2007. Jigsaw puzzles, edge matching, and polyomino packing: connections and complexity. *Graphs and Combinatorics* 23, 1 (2007), 195–208.

J. Fonseca. 2012. *Montagem automática de painéis de azulejos*. Master's thesis. Instituto Superior Técnico, Universidade Técnica de Lisboa.

H. Freeman and L. Garder. 1964. Apictorial jigsaw puzzles: The computer solution of a problem in pattern recognition. *IEEE Transactions on Electronic Computers* 2 (1964), 118–127.

A.C. Gallagher. 2012. Jigsaw puzzles with pieces of unknown orientation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*. 382–389.

E. Justino, L.S. Oliveira, and C. Freitas. 2006. Reconstructing shredded documents through feature matching. *Forensic Science International* 160, 2 (2006), 140–147.

D. Koller and M. Levoy. 2006. Computer-aided reconstruction and new matches in the Forma Urbis Romae. *Bullettino Della Commissione Archeologica Comunale di Roma, Supplementi 15* (2006), 103–125.

D. Koller, J. Trimble, T. Najbjerg, N. Gelfand, and M. Levoy. 2006. Fragments of the city: Stanford's digital Forma Urbis Romae project. In *Proceedings of the Third Williams Symposium on Classical Architecture, Journal of Roman Archaeology Suppl. 61*. 237–252.

D.A. Kosiba, P.M. Devaux, S. Balasubramanian, T.L. Gandhi, and K. Kasturi. 1994. An automatic jigsaw puzzle solver. In *12th International Conference on Pattern Recognition (IAPR)*, Vol. 1. 616–618.

D.G. Lowe. 2004. Distinctive image features from scale-invariant keypoints. *International Journal of Computer Vision* 60, 2 (2004), 91–110.

W. Marande and G. Burger. 2007. Mitochondrial DNA as a genomic jigsaw puzzle. *Science* 318, 5849 (2007), 415–415.

J.C. McBride and B.B. Kimia. 2003. Archaeological fragment reconstruction using curve-matching. In *Conference on Computer Vision and Pattern Recognition Workshop (CVPRW)*.

J.C.-B. Pereira, M.M.M. Gomes, and D.M. de S. Tavares. 1992. The treatment of ancient Portuguese tiles. *Studies in conservation* 37, 1 (1992), 112–115.

J.C. Pessoa, J.L.F. Antunes, M.O. Figueiredo, and M.A. Fortes. 1996. Removal and analysis of soluble salts from ancient tiles. *Studies in conservation* 41, 3 (1996), 153–160.

D. Pomeranz, M. Shemesh, and O. Ben-Shahar. 2011. A fully automated greedy square jigsaw puzzle solver. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*. 9–16.

R.O.Duda and P.E.Hart. 1972. Use of the Hough transformation to detect lines and curves in pictures. *Commun. ACM* 15, 1 (1972), 11–15.

J.B. Rosen. 1960. The gradient projection method for nonlinear programming. *J. Soc. Indust. Appl. Math.* 8, 1 (1960), 181–217.

H. Shin, C. Doumas, T. Funkhouser, S. Rusinkiewicz, K. Steiglitz, A. Vlachopoulos, and T. Weyrich. 2012. Analyzing and simulating fracture patterns of Theran wall paintings. *Journal on Computing and Cultural Heritage* 5, 3 (2012), 10:1–10:14.

D. Sholomon, O. David, and N.S. Nathan. 2013. A genetic algorithm-based solver for very large jigsaw puzzles. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*. 1767–1774.

T.P. Silva, M.-O. Figueiredo, M.-A. Barreiros, and M.-I. Prudêncio. 2013. Decorative 18th century blue-and-white Portuguese tile panels: a type-case of environmental degradation. *Journal of Materials* 2013 (2013).

Rede Temática. 2011. "A Blind Person's Cure" – an unknown tile panel from the collection of the National Tile Museum. http://redeazulejo.fl.ul.pt/redeazulejaria_en/noticias_arquivo.aspx?nid=522&op=detalhes&menu=290&id=332. (2011). Accessed: September 2014.

C. Toler-Franklin, B. Brown, T. Weyrich, T. Funkhouser, and S. Rusinkiewicz. 2010. Multi-feature matching of fresco fragments. In *ACM Transactions on Graphics (Proceedings of SIGGRAPH Asia)*.

Y.-X. Zhao, M.-C. Su, Z.-L. Chou, and J. Lee. 2007. A puzzle solver and its application in speech descrambling. In *Proceedings of the International Conference on Computer Engineering and Applications*. 171–176.

L. Zhu, Z. Zhou, and D. Hu. 2008. Globally consistent reconstruction of ripped-up documents. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 30, 1 (2008), 1–13.