A Comparison of GO-GARCH Estimation Methods

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Abstract

Abstract. Time series of financial returns are characterized by presenting heavy tails, gain/loss asymmetry and volatility clusters. These characteristics make Generalized Orthogonal GARCH (GO-GARCH) models an excellent option for the modeling of such series, as it is a conditional volatility model for multivariate returns that can incorporate heavy tails and asymmetric returns quite naturally. In this report, we review the definition of GO-GARCH models and compare two different estimation strategies that are based on the Method of Moments and on Independent Component Analysis using Brazilian market data.

Keywords. GO-GARCH, independent component analysis, method of moments, simultaneous diagonalization, principal component analysis.

Contents

1 Introduction

Generalized autoregressive conditional heteroskedasticity (GARCH) models have proven to be quite valuable in practice for the modeling of financial returns. Not surprisingly, GARCH models are also of great importance in the multivariate case, as many applications in mathematical finance rely on multivariate probabilistic models to represent the joint dynamics of asset returns. Among multivariate models, the Generalized Orthogonal GARCH (GO-GARCH) model has distinguished itself from other multivariate models for incorporating features commonly found in time series of financial returns, and for the simplicity and robustness of the procedures available for the estimation of model parameters.

We start by reviewing the definition of the GO-GARCH model in Section 2 and the estimation of model parameters using Independent Component Analysis (ICA) [3] and the Method of Moments (MM) [2] in Section 3. In the sequence, we present our test methodology. In Section 4.1, we describe the historical data used in the experiments, and the implemented software in Section 4.2. Then, we compare the estimation methods by comparing estimation errors and *Value at Risk* (VaR) forecasts for a portfolio of Brazilian stocks in Sections 4.3.2 and 4.3.1. Finally, in Section 5, we present our conclusions.

2 The GO-GARCH Model

The GO-GARCH model, introduced by Van der Weide [20], is a multivariate conditional volatility model that supports random vectors with probability distributions that are asymmetric and heavy tailed, making it an excellent choice to represent financial returns. As shown by Cont [6], these features are commonly found in time series of financial returns.

At time t, the random vector x_t is considered to be a linear combination of the hidden factors y_t:

$$x_t = Z\, y_t, \tag{1}$$

where $Z \in \mathbb{R}^{m\times m}$ is a non singular matrix and the m-dimensional stochastic process $\{y_t\}_{t\geq 1}$ is stationary, ergodic and with finite kurtosis. Factors are also considered to have zero conditional mean,

$$\mathbb{E}\left[y_t|\mathcal{F}_{t-1}\right] = 0, \tag{2}$$

and uncorrelated coordinates,

$$\text{Cov}\left(y_t|\mathcal{F}_{t-1}\right) = \text{diag}\left(h_{1t},\ldots,h_{mt}\right). \tag{3}$$

The structure of $\{y_t\}_{t\geq 1}$ is also specified in the GO-GARCH model:

$$y_t = H_t^{1/2}\epsilon_t, \tag{4}$$

where

$$H_t^{1/2} = \text{diag}\left(\sqrt{h_{1t}},\ldots,\sqrt{h_{mt}}\right), \tag{5}$$

with h_{it} following an univariate conditional volatility model with unitary variance,

$$\mathbb{E}\left[h_{it}\right] = 1,\ h_{it} \geq 0. \tag{6}$$

The coordinates of the innovation series $\{\epsilon_t\}_{t\geq 1}$ are considered to form sequences of iid random variables with zero conditional mean,

$$\mathbb{E}\left[\epsilon_{it}|\mathcal{F}_{t-1}\right] = 0, \tag{7}$$

and with unitary conditional variance,

$$\mathrm{Var}\left(\epsilon_{it}|\mathcal{F}_{t-1}\right) = 1. \tag{8}$$

Usually, GARCH(1,1) models are adopted for h_{it} [3]. That is,

$$h_{it} = (1 - \alpha_i - \beta_i) + \alpha_i y_{i,t-1}^2 + \beta_i h_{i,t-1}, \tag{9}$$

but it is possible to use different univariate models as well. This flexibility explains how GO-GARCH models with different features can be easily specified. In our case, the innovations of the univariate GARCH(1,1) processes are specified as random variables with skewed Student's-t distributions [7], resulting in a multivariate model for asymmetric returns with heavy-tails. Gaussian innovations, for example, would result in a GO-GARCH model without asymmetry and without heavy-tails. Notice that it is also possible to mix different univariate models by choosing different models for the components of y_t.

2.1 Covariance Matrices

The conditional covariance matrix is of particular interest to many applications in finance, such as the the computation of VaR forecasts and the construction of minimum risk portfolios.

By the previous definitions, the conditional covariance Σ_t of the GO-GARCH model is given by:

$$\Sigma_t = \mathbb{E}\left[x_t x_t' | \mathcal{F}_{t-1}\right] = \mathbb{E}\left[Z y_t y_t' Z' | \mathcal{F}_{t-1}\right] = Z H_t Z', \tag{10}$$

and its unconditional counterpart, Σ, is:

$$\Sigma = \mathbb{E}\left[\Sigma_t\right] = \mathbb{E}\left[Z H_t Z'\right] = Z\ \mathbb{E}\left[H_t\right]\ Z' = Z Z'. \tag{11}$$

3 Estimation of Parameters

Decomposing the financial returns $\{r_t\}_{t\geq 1}$ into components related to its expected value $\{\mu_t\}_{t\geq 1}$ and its volatility $\{x_t\}_{t\geq 1}$,

$$r_t = \mu_t + x_t, \tag{12}$$

model parameters are estimated in two stages, as illustrated in Figure 1.

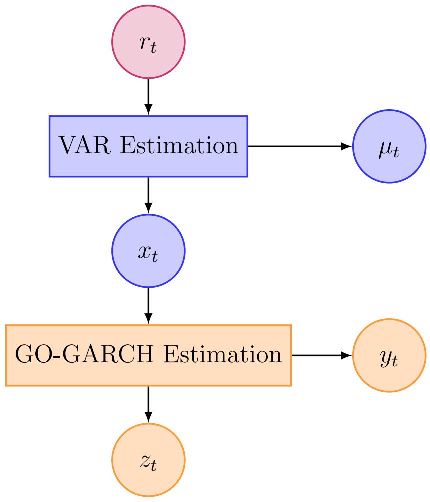

Figure 1: Procedure for parameter estimation

In the first stage, the parameters of a *Vector Autoregressive* (VAR) [11] process are estimated using *Ordinary Least Squares* (OLS). Using the OLS approach, instead of the maximization of a likelihood function, there is no need for the specification of probability distributions for the returns. Therefore, for consistency, we have chosen this approach when using the GO-GARCH model, as probability distributions are already chosen for the components of y_t.

In the second stage, the parameters of the GO-GARCH model are estimated using the time series of the VAR residuals. In this stage, estimating the mixing matrix Z is the first step to fully specify a GO-GARCH model. Once the estimate $\widehat{Z}$ is available, the y_t factors can be extracted from the VAR residuals using the inverse $\widehat{Z}^{-1}$:

$$\hat{y}_t = \widehat{Z}^{-1} x_t. \tag{13}$$

In the sequence, the parameters of the univariate processes driving the volatility of the returns can be estimated. Following the definitions in Section 2, that means estimating a GARCH(1,1) process for each component of $\hat{y}_t$.

To estimate the mixing matrix Z, we use a three step procedure based on its polar decomposition, $Z = S\,U$:

1. Estimate S

2. Estimate U
3. Compute $\widehat{Z} = \widehat{S}\,\widehat{U}$

Since S is related to the unconditional covariance Σ,

$$\Sigma = ZZ' = (SU)\,(SU)' = SS' = S^2, \tag{14}$$

it can be estimated using the spectral decomposition of the sample covariance matrix:

$$\widehat{\Sigma} = PLP' \longrightarrow \widehat{S} = PL^{1/2}P'. \tag{15}$$

For the computation of $\widehat{U}$, two alternatives were considered in this study: ICA [3] and MM [2].

Both alternatives use the standardized returns $\{s_t\}_{t\geq 1}$, which are defined as

$$s_t = \Sigma^{-1/2} x_t = S^{-1} x_t. \tag{16}$$

These returns provide a relation to the hidden factors $\{y_t\}_{t\geq 1}$ that can be explored in estimation procedures:

$$s_t = S^{-1}\,x_t = S^{-1}\,\left(S\,U\right) y_t = U\,y_t. \tag{17}$$

3.1 Using Independent Component Analysis (ICA)

In ICA [14], a signal $\{w_t\}_{t\geq 1}$ is considered to be a linear combination of other independent signals $\{v_t\}_{t\geq 1}$. That is,

$$w_t = A\,v_t. \tag{18}$$

The estimation of the A matrix, therefore, is performed by maximizing a metric of probabilistic independence. As an example, the mutual information of a random vector x, which is defined as

$$I\left(x\right) = \sum_{i=1}^{m} H\left(x_i\right) - H\left(x\right), \tag{19}$$

where $H\left(x\right)$ is the differential entropy of x.

Comparing Equation 1 and Equation 18, it is easy to notice that the ICA can be applied directly over $\{s_t\}_{t\geq 1}$ to compute an estimate for U [3].

3.2 Using the Method of Moments (MM)

U can also be regarded as the orthogonal matrix that diagonalizes the covariance $\Gamma(s)$ of $\{s_t\}_{t\geq 1}$, since

$$\Gamma(s) = U\Gamma(y)\,U' = U\,I_m U'. \tag{20}$$

Similarly, for the correlation $\Phi(s)$ of $\{s_t\}_{t\geq 1}$:

$$\Phi(s) = U\,\Phi(y)\,U'. \tag{21}$$

For robustness, a set of autocorrelation matrices from $\{s_t\}_{t\geq 1}$ can be used to estimate $\widehat{U}$. For such a set,

$$C_U = \left\{\widetilde{\Phi}_1, \ldots, \widetilde{\Phi}_p\right\}, \tag{22}$$

$\widehat{U}$ is then defined as the solution of a simultaneous diagonalization problem:

$$\widehat{U} = \underset{B\in\mathcal{O}_m}{\operatorname{argmin}} \sum_{k=1}^{p}\sum_{\substack{i,j=1\\ i\neq j}}^{m}\left(B\widetilde{\Phi}_k B'\right)^2_{ij}. \tag{23}$$

Notice that, following Boswijk & van der Weide [2], the regular sample autocorrelation matrices $\widehat{\Phi}_k$ were replaced by a symmetric version,

$$\widetilde{\Phi}_k = \frac{1}{2}\left(\widehat{\Phi}_k + \widehat{\Phi}'_k\right), \tag{24}$$

for additional robustness.

4 Methodology

In order to compare the estimations methods, two strategies were chosen. In the first strategy, we conduct a series of simulation/estimation experiments, using GO-GARCH processes with known parameters and evaluating the estimation error in each experiment. In order to compare our results with previously published ones, we use the metric for estimation errors used by Boswijk & van der Weide [2] and the same model parameters defined by Zheng & Wei [21], which can found in Figure 4.

In the second testing strategy, we use the ICA and MM estimators with the same time series of historical prices and compare the VaR forecasts obtained with each estimator. We also use VaR forecasts computed using Principal Component Analysis (PCA).

The comparison of VaR forecasts is performed using the Kupiec test [15], which checks the frequency of VaR violations, and two independence tests introduced by Christoffersen [5] and by Christoffersen & Pelletier [4]. Appropriate VaR forecasts should exhibit a series of independent violations, indicating that the risk model is properly adjusted from one day to another.

4.1 The Data

In the experiments, we have used historical stock prices from BM&F Bovespa in the period between 2006-Jan-01 and 2013-Jan-14. Five important companies were chosen (PETR4, VALE5, GGBR4, USIM5, CSNA3) and their historical prices adjusted for different quoting factors and corporate events (dividends, splits, etc.). The importance of adjusting stock prices is illustrated in Figure 2, where it is possible to notice how different adjusted prices can be. In Figure 3, we show the historical log returns for the selected companies.

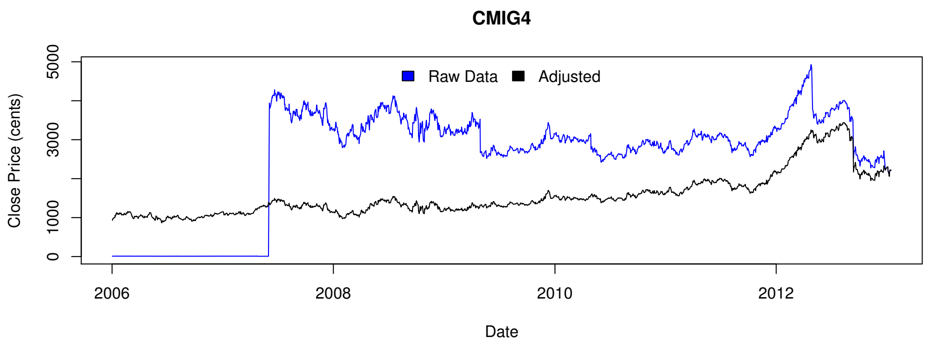

Figure 2: CMIG4 stock prices before and after correction

4.2 The Software

The software used in the experiments, described in Section 4, has been written in R [18], using several of the available packages. For example: *vars* for the estimation of VAR models; *rugarch* [10] for the estimation of GARCH(1,1) processes with skewed Student's-t innovations; *fastICA* [17] to estimate U using the FastICA [13] algorithm; and *portes* [16] for the multivariate autocorrelation tests.

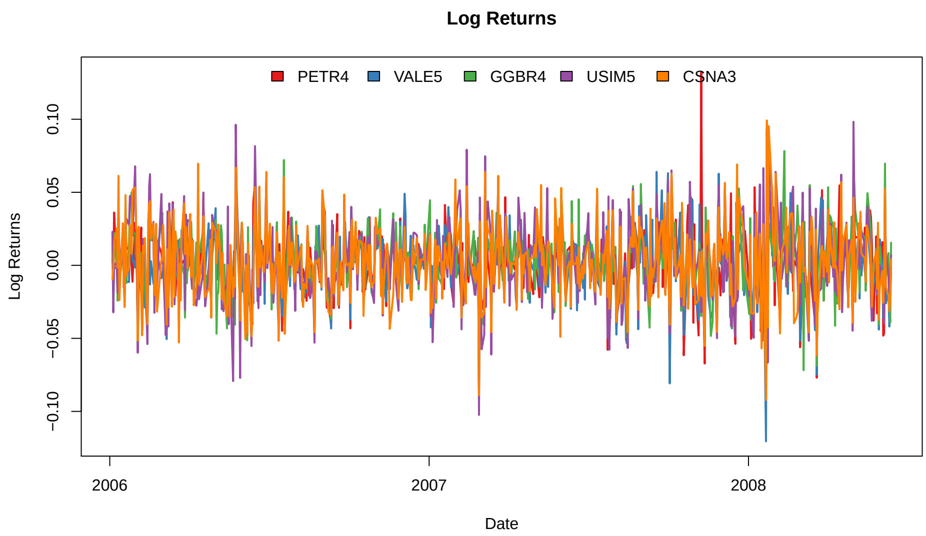

Figure 3: Historical log returns for the selected companies

To estimate U using the method of moments, we have implemented the FG algorithm [9] for the simultaneous diagonalization of a set of matrices, which provides a solution for Problem 23. We have also implemented the VaR/CVaR estimator introduced by Rockafellar & Uryasev [19]. Finally, the backtesting procedures used to check the Value at Risk forecasts in Section 4.3.2 were implemented by Ferreira [8].

4.3 Results

4.3.1 Comparing Estimation Errors

Since the estimation methods being tested only differ in the computation of the estimate for U, the estimation errors can be computed as the distance between U and an estimate $\widehat{U}$. As Boswijk & van der Weide [2], we use the following distance between orthogonal matrices:

$$d(\widehat{U}_i, U) = \sqrt{\frac{1}{2}\left[D(U, \widehat{U}_i) + D(\widehat{U}_i, U)\right]}, \tag{25}$$

where

$$D(\widehat{U}, U) = 1 - \frac{1}{m}\sum_{i=1}^{m} \max_{1\leq j\leq m} |u_i' \hat{u}_j|. \tag{26}$$

For a sample of N time series the average estimation error can be computed as the square root of the mean quadratic distance (RMSD) between U and its estimates $\widehat{U}_i$:

$$RMSD = \sqrt{\frac{1}{N}\sum_{i=1}^{N} d(U, \widehat{U}_i)^2}. \tag{27}$$

In the experiments, $N = 500$ time series were simulated with varying length (from 1000 to 8000 days). As mentioned before, we have used the same GO-GARCH processes specified by Zheng & Wei [21], which can be found in Figure 4. Figure 5 contains the results obtained.

$$\alpha_3 = \begin{bmatrix} 0.03 \\ 0.08 \\ 0.12 \end{bmatrix} \quad \beta_3 = \begin{bmatrix} 0.94 \\ 0.89 \\ 0.85 \end{bmatrix} \quad Z_3 = \begin{bmatrix} -0.6300 & 3.2030 & -4.4890 \\ 0.3470 & 4.6790 & -1.4470 \\ 2.3780 & 1.4940 & 1.7820 \end{bmatrix}$$

$$\alpha_4 = \begin{bmatrix} 0.03 \\ 0.08 \\ 0.12 \\ 0.15 \end{bmatrix} \quad \beta_4 = \begin{bmatrix} 0.94 \\ 0.89 \\ 0.85 \\ 0.82 \end{bmatrix} \quad Z_4 = \begin{bmatrix} -0.9860 & -3.5920 & 4.7760 & -6.7020 \\ -1.6110 & 0.9870 & -0.6420 & -4.6880 \\ 0.7470 & 0.0700 & -3.1620 & -4.3350 \\ -1.1560 & 4.5710 & -0.4010 & -2.5800 \end{bmatrix}$$

$$\alpha_6 = \begin{bmatrix} 0.03 \\ 0.08 \\ 0.11 \\ 0.14 \\ 0.16 \\ 0.18 \end{bmatrix} \quad \beta_6 = \begin{bmatrix} 0.95 \\ 0.90 \\ 0.87 \\ 0.84 \\ 0.82 \\ 0.80 \end{bmatrix} \quad Z_6 = \begin{bmatrix} 1.8680 & -2.4810 & 2.1760 & -5.2500 & -4.1570 & 1.4740 \\ 0.6240 & 5.0450 & 4.4930 & 3.8410 & -4.0520 & -1.1120 \\ 4.7850 & 2.0860 & 0.2510 & -2.8270 & 8.0870 & -3.6840 \\ -0.1670 & 0.9070 & 0.1060 & -2.2990 & 1.7230 & 3.1190 \\ -4.0590 & -1.6580 & -2.0360 & -11.4510 & -4.3180 & -1.5050 \\ 6.3010 & -4.8140 & 7.1500 & 3.8200 & -2.8200 & 2.4580 \end{bmatrix}$$

$$\alpha_8 = \begin{bmatrix} 0.02 \\ 0.06 \\ 0.09 \\ 0.11 \\ 0.13 \\ 0.15 \\ 0.17 \\ 0.19 \end{bmatrix} \quad \beta_8 = \begin{bmatrix} 0.96 \\ 0.92 \\ 0.89 \\ 0.87 \\ 0.85 \\ 0.83 \\ 0.81 \\ 0.79 \end{bmatrix} \quad Z_8 = \begin{bmatrix} -8.4960 & 4.5640 & 1.5260 & -2.8840 & -0.3660 & 1.4390 & -5.7900 & 3.0480 \\ -1.3430 & -5.4180 & 6.4020 & -4.2590 & 0.9180 & -10.7550 & -1.1990 & -1.9340 \\ -2.4820 & -5.7960 & -3.5670 & -6.3800 & 1.8280 & -8.1560 & -3.3280 & 7.5840 \\ -4.6440 & -5.6890 & 0.2640 & -0.6390 & 5.7650 & 1.8350 & 8.4050 & -0.4790 \\ 3.3600 & -3.2280 & -3.0830 & -2.1630 & 13.2690 & 3.4750 & -3.9330 & 5.2420 \\ -1.1210 & 1.6190 & -0.8550 & 5.2890 & -2.2670 & -6.5510 & -3.2760 & -1.3080 \\ 2.6160 & 6.6740 & 3.4010 & -2.3740 & 4.1780 & 7.9270 & 2.9450 & -4.7090 \\ 4.8540 & -2.1450 & -3.2710 & 3.9950 & -4.8070 & -1.9060 & -5.5120 & 5.4640 \end{bmatrix}$$

Figure 4: Parameters for the simulation of GO-GARCH processes

By examining the results in Figure 5, we can notice that estimation errors diminish as the length of the time series increase. Estimation errors also seem to increase with an increase in the dimension of the GO-GARCH process. The errors presented by the process with dimension three, however, seem higher than expected when using the ICA estimator. This result could be an indication of high variance in the estimation errors. Finally, it is important to note that, in general, smaller estimation errors are found using the ICA estimator.

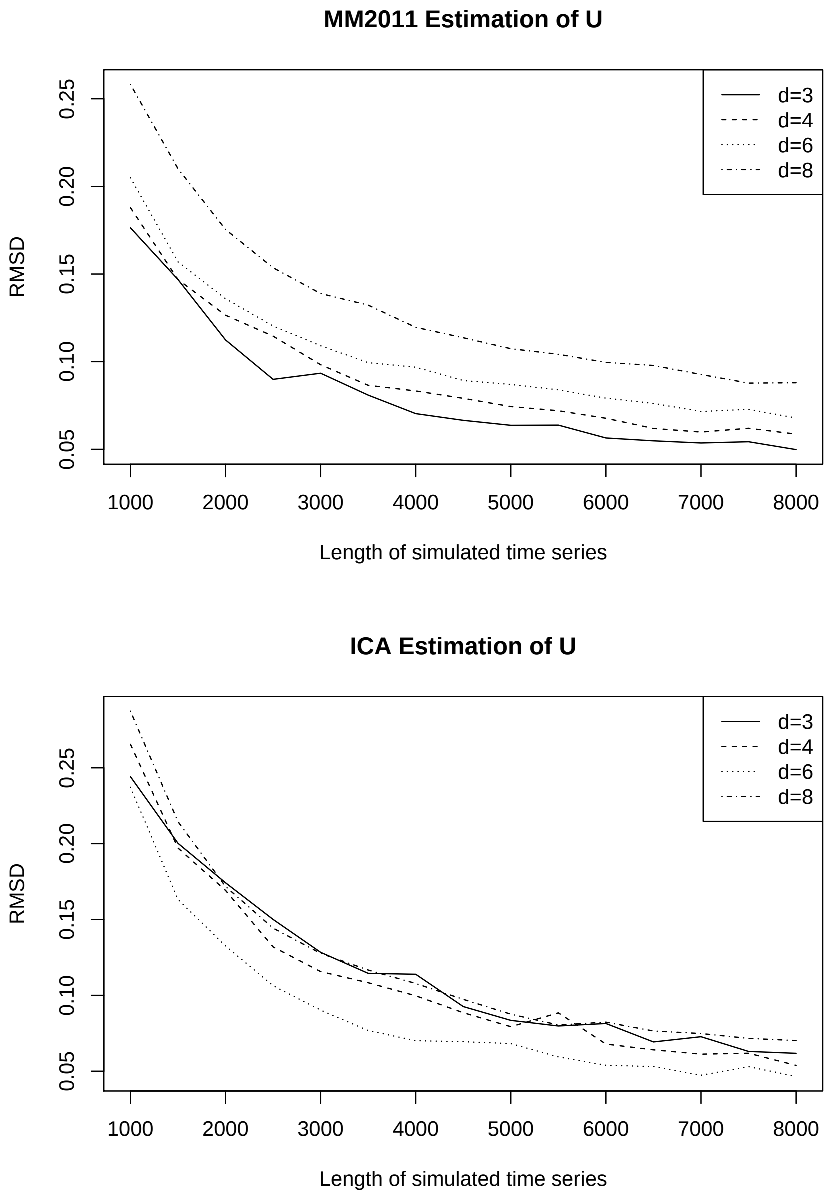

Figure 5: Estimation errors

4.3.2 Comparing VaR Forecasts

Before computing VaR forecasts for a portfolio comprised of the selected stocks, we verified if the GO-GARCH model could capture their joint dynamics correctly. Therefore, we followed the estimation procedure illustrated in Figure 1 using the first 600 historical stock prices available in our data set. First, the parameters for a vector autoregressive process of order 5 were computed and then the parameters of a GO-GARCH model using ICA.

Using the multivariate portmanteau test introduced by Hosking [12], we verified that the VAR and GO-GARCH models captured the joint dynamics of the stocks correctly. More specifically, as illustrated in Figure 6, no autocorrelations were found in the residuals $\{z_t\}_{t\geq 1}$ and $\{z_t^2\}_{t\geq 1}$. Notice in the same figure that autocorrelations were detected in the returns series $\{r_t\}_{t\geq 1}$ and in its square (indicated by the low p-values). However, as the historical returns were filtered by the VAR and GO-GARCH models, the autocorrelations disappear, indicating that an appropriate model was estimated.

Lags	Statistic	df	p-value
6	192.3920	150	0.0111
10	309.9618	250	0.0058
15	427.7906	375	0.0309
20	578.5314	500	0.0085

(a) Hosking test for r_t

Lags	Statistic	df	p-value
6	354.7336	150	0
10	464.2409	250	0
15	645.1586	375	0
20	784.1894	500	0

(b) Hosking test for r_t^2

Lags	Statistic	df	p-value
6	22.0671	25	0.6319
10	140.5794	125	0.1614
15	245.1438	250	0.5748
20	403.0684	375	0.1528

(c) Hosking test for x_t

Lags	Statistic	df	p-value
5	279.7014	125	0
10	398.5589	250	0
15	562.6849	375	0
20	680.7541	500	0

(d) Hosking test for x_t^2

Lags	Statistic	df	p-value
5	13.7861	125	1.0000
10	150.8289	250	1.0000
15	247.7314	375	1.0000
20	393.7500	500	0.9998

(e) Hosking test for z_t

Lags	Statistic	df	p-value
5	118.6954	125	0.6418
10	257.9895	250	0.3507
15	388.3685	375	0.3062
20	540.3178	500	0.1033

(f) Hosking test for z_t^2

Figure 6: Analysis of residuals - Autocorrelation tests

The QQ-plots of the residuals $\{z_t\}_{t\geq 1}$ also confirm the adequacy of the model, as illustrated in Figure 7. In Figure 8, the estimated conditional variances and covariances are shown.

Finally, after the estimation of an appropriate model for the returns $\{r_t\}_{t\geq 1}$, we compute Value at Risk forecasts with the horizon of a single day for portfolio Π (defined in Table 1) using Monte Carlo simulations and the VaR/CVaR estimator introduced by Rockafellar

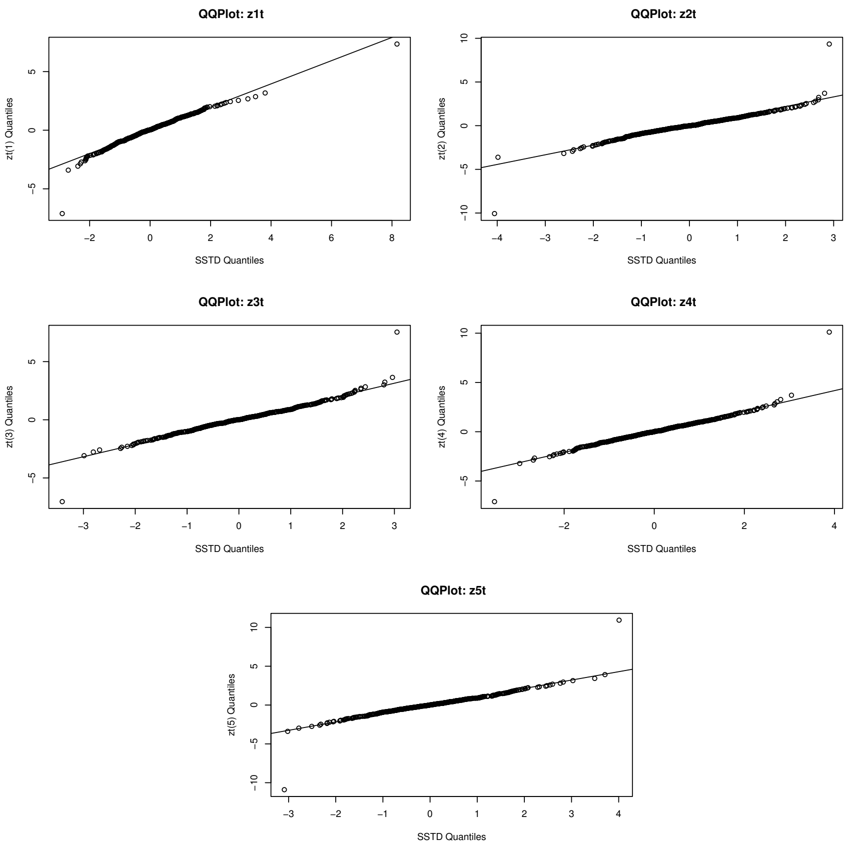

Figure 7: Analysis of residuals - QQ-plots

& Uryasev [19]. Using a sliding window approach, we compute each forecast using the previous 600 returns to estimate the GO-GARCH model and a total of 50000 simulated scenarios for the log-return of the following date. The forecasts obtained using the ICA estimator for GO-GARCH parameters are shown in Figure 9.

After computing the VaR forecasts, a time series of violations is computed and tested for their frequency and independence (using the tests described in Section 4). In Figure 10, we show an example of the VaR/CVaR forecasts along with the VaR violations.

The same procedure was performed with the MM estimator and with the O-GARCH model [1], which is based on the PCA. The compiled results can be found in Figure 11.

Inspecting Figure 11, it is easy to verify that both GO-GARCH models have passed all of the frequency and independence tests. Forecasts computed using the O-GARCH model, however, failed two of the frequency tests. Checking the number of violations, it is easy to see that this model overestimates the financial losses (it has a small number

Stock	*Proportion*
PETR4	0.25
VALE5	0.20
GGBR4	0.10
USIM5	0.15
CSNA3	0.30

Table 1: Composition of portfolio Π

of violations). Comparing only the GO-GARCH models, one could say that the MM estimator also overestimates the losses when compared to the ICA estimator.

5 Conclusion

In this study, we compared two estimators for GO-GARCH models, which are multivariate models for asymmetric returns with heavy tails.

Simulation/estimation tests have shown that estimation errors, using both estimators, decrease as the time-series length increases. Also, the errors increase as the dimension of the model increases.

The analysis of residuals, performed after the estimation of the parameters using historical data, has shown that the model is capable of capturing the dynamics of the financial returns correctly. The analyzed residuals were found to be uncorrelated and with the expected properties.

While comparing the ICA and MM estimators, we found that smaller estimation errors are obtained when using the ICA estimator.

The estimators were also tested using an application. VaR forecasts were computed and evaluated using common tests for their frequency and independence. The results seem to indicate that both estimators would be appropriate for managing the risk of a portfolio of financial assets.

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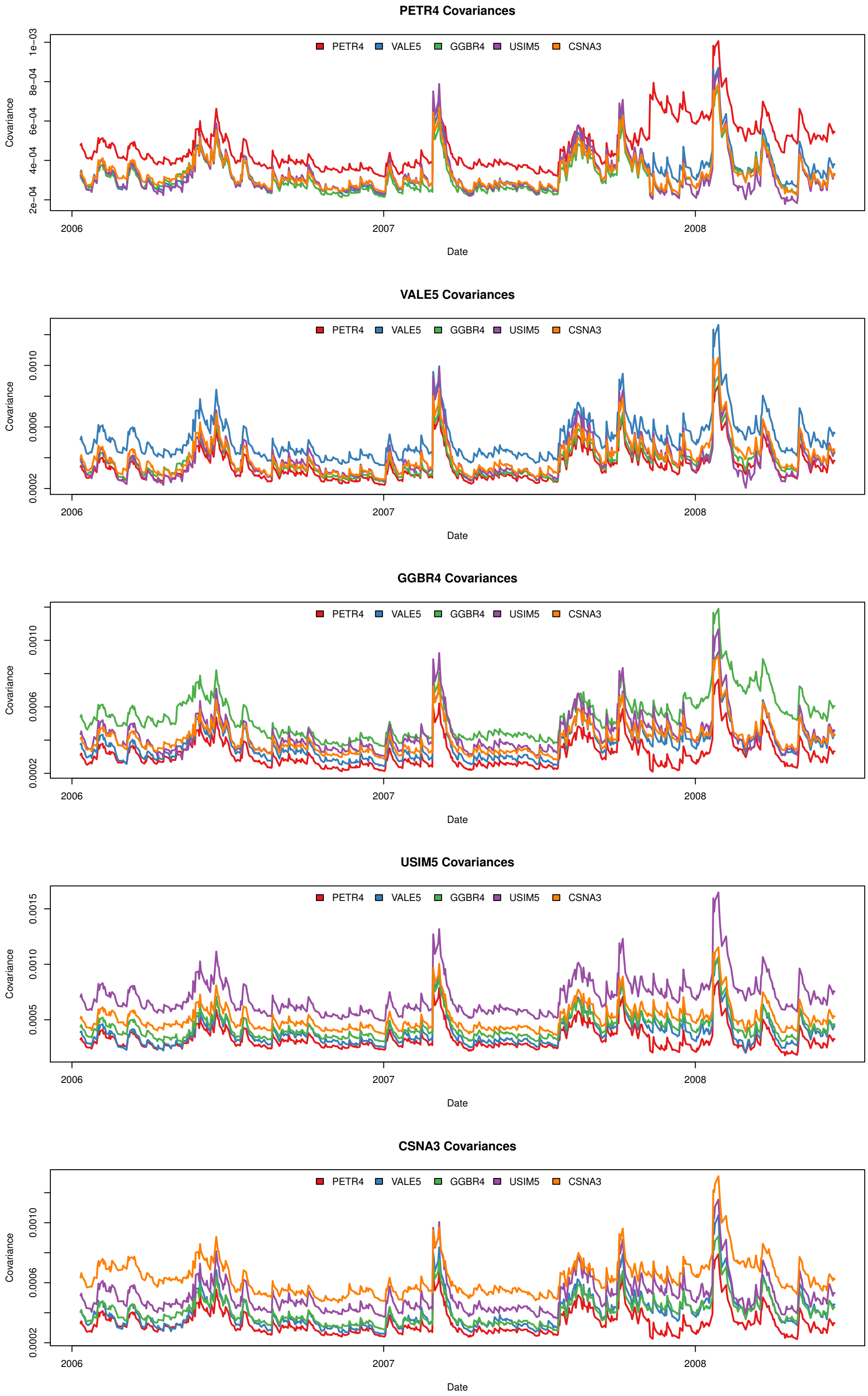

Figure 8: GO-GARCH conditional variances and covariances

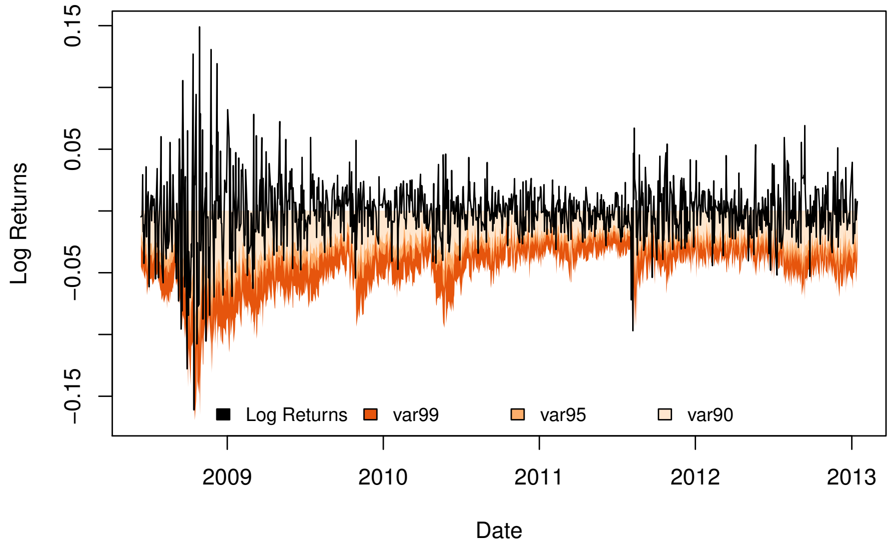

Figure 9: VaR forecasts

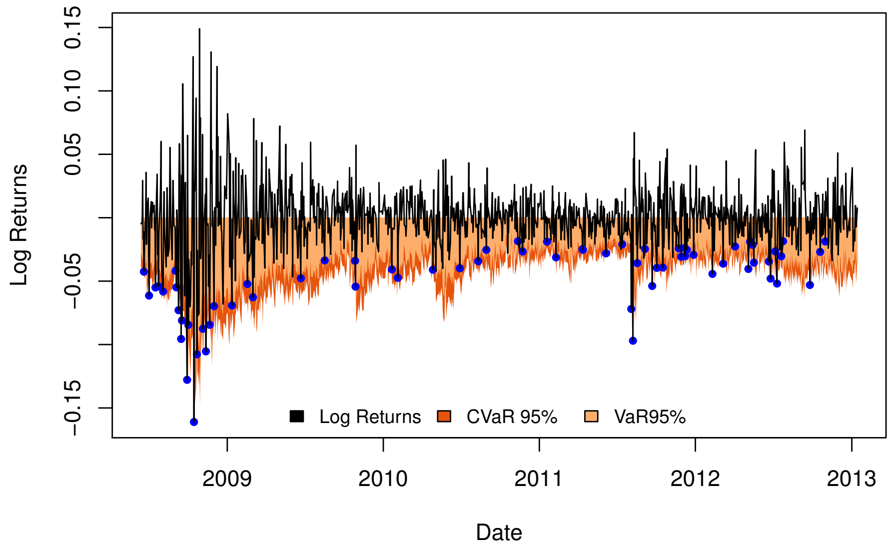

Figure 10: VaR and CVaR forecasts at 95% with violations

from	to	test	VaR.level	p.value	samples	violations	non.violations.ratio
2010-01-26	2013-01-14	Kupiec	90	0.9412	736	73	90.0815
2010-01-26	2013-01-14	IT	90	0.9174	736	73	90.0815
2010-01-26	2013-01-14	DBIT	90	1.0000	736	73	90.0815
2010-01-26	2013-01-14	Kupiec	95	0.2354	736	30	95.9239
2010-01-26	2013-01-14	IT	95	0.1100	736	30	95.9239
2010-01-26	2013-01-14	DBIT	95	1.0000	736	30	95.9239
2010-01-26	2013-01-14	Kupiec	99	0.8930	736	7	99.0489
2010-01-26	2013-01-14	IT	99	0.7137	736	7	99.0489
2010-01-26	2013-01-14	DBIT	99	0.2361	736	7	99.0489

(a) GO-GARCH (ICA)

from	to	test	VaR.level	p.value	samples	violations	non.violations.ratio
2010-01-26	2013-01-14	Kupiec	90	0.2820	736	65	91.1685
2010-01-26	2013-01-14	IT	90	0.1629	736	65	91.1685
2010-01-26	2013-01-14	DBIT	90	1.0000	736	65	91.1685
2010-01-26	2013-01-14	Kupiec	95	0.0544	736	26	96.4674
2010-01-26	2013-01-14	IT	95	0.1673	736	26	96.4674
2010-01-26	2013-01-14	DBIT	95	1.0000	736	26	96.4674
2010-01-26	2013-01-14	Kupiec	99	0.8930	736	7	99.0489
2010-01-26	2013-01-14	IT	99	0.7137	736	7	99.0489
2010-01-26	2013-01-14	DBIT	99	0.3924	736	7	99.0489

(b) GO-GARCH (MM)

from	to	test	VaR.level	p.value	samples	violations	non.violations.ratio
2010-01-26	2013-01-14	Kupiec	90	0.0003	736	46	93.7500
2010-01-26	2013-01-14	IT	90	0.5612	736	46	93.7500
2010-01-26	2013-01-14	DBIT	90	1.0000	736	46	93.7500
2010-01-26	2013-01-14	Kupiec	95	0.0038	736	21	97.1467
2010-01-26	2013-01-14	IT	95	0.2663	736	21	97.1467
2010-01-26	2013-01-14	DBIT	95	0.3513	736	21	97.1467
2010-01-26	2013-01-14	Kupiec	99	0.1729	736	4	99.4565
2010-01-26	2013-01-14	IT	99	0.8343	736	4	99.4565
2010-01-26	2013-01-14	DBIT	99	1.0000	736	4	99.4565

(c) O-GARCH (PCA)

Figure 11: VaR violations testing