

# MECHANISMS FOR OIL RECOVERY BY STEAM ALKANE CO-INJECTION

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## Abstract

Steam injection is a cheap and widely used method for enhanced oil recovery. However, some portion of oil is not recovered because it remains trapped in the pores of the reservoir after the passage of the injected steam. To increase recovery efficiency, Dietz proposed at the end of the 70's the co-injection of a small fraction of volatile oil. His intention was that the volatile oil injected ideally would condense with the steam, forming a bank of light oil capable of dissolving and displacing the heavy oil contained in reservoir, improving the recovery. The mechanism that describes this gain is therefore related to the formation of a volatile oil condensation bank that pushes the heavy oil inside the reservoir.

We establish a model capable of capturing these effects. It allows describing the behavior of the flow of oil in a uniform porous medium in local thermodynamic equilibrium. To understand the mechanisms that allow the improvement in oil recovery we solve Riemann problems for the conservation laws governing the injection of volatile oil in the reservoir.

In this work we will focus in the subproblem of volatile oil gas injection into a porous medium containing oil. The understanding gathered by means of this simplification constitutes a key step towards the understanding of the co-injection problem. Also the injection of light alkane possesses a relevance of its own: during off-shore production it is not always possible to utilize commercially the produced gas. Therefore, this gas can be reinjected into the reservoir to maintain the reservoir pressure.

For a combination of initial data and oil parameters we found a "cleaning wave" in which a volatile oil condensation wave arises when it encounters dead oil in the reservoir and pushes it. This is the first time this problem was solved exactly.

## 1. Introduction

Steam drive recovery of oil continues to be an economical way of producing oil and is used world wide. An overview of the last forty years of steam drive recovery in California is given in reference Hanzlik, Mims 2003. The main challenges are to improve sweep efficiency and to improve recovery from the steam swept zone. Our interest focusses on the latter issue.

In the late seventies Dietz *et al.* 1985 proposed to add small amounts of volatile oil to the steam. His view was that the volatile oil co-injected with the steam in almost infinitesimal amounts would displace the dead oil ahead of the steam condensation front leaving no oil behind in the steam swept zone. The stability of the steam displacement would guarantee an even distribution of volatile oil along the steam condensation front. Experiments investigating the mechanism are described in Dietz *et al.* 1985, Bruining *et al.* 1987, Farouqui, Abad 1976. Similar ideas were put forward independently by Farouq-Ali.

At the time the main criticism towards this idea was that volatile oil is also present naturally and therefore the virtue of adding additional volatile oil is not clear. What was required was a theoretical description of the displacement process such that the results could be upscaled from the laboratory scale to the field scale. The theoretical model must be able to establish the difference between steam drive recoveries with co-injection of volatile oil with respect to the recoveries from oil that contains a fraction of volatile oil. In spite of this fact these ideas have already found their way to field applications [6] where propane is used as low volatile hydrocarbon.

Due to the preliminarily stage of this work we will focus in the subproblem of volatile oil gas injection into a porous medium containing a mixture of oil and gas. The comprehension gathered with this simplification constitute a key step of the co-injection problem. Also the sole alkane injection posses a relevance of it's own: during off-shore production it is not always possible to store the produced gas. Therefore, this gas can be reinjected into the reservoir raising the reservoir pressure and so increasing the recovery factor. Our approach is to follow Bruining, Marchesin 2007

and so simplify the model equations in such a way that the essential elements are retained and yet avoids the complexities of solving the pressure equation.

In Section 2 the model for the flow of oil with volatile oil gas injected is presented. The thermodynamical phase relationships and fluid densities are described. The hydrodynamical and energy equations that govern the flow are also discussed in Section 3. The numerical scheme for the governing equations is presented in Section 4. Section 5 introduces the Riemann problem. The resulting solution is discussed in Section 6. Appendices A and B contain a plethora of empirical relationships needed.

## 2. Physical Model

We consider the injection of volatile oil into a linear horizontal core with constant porosity. The core is originally filled with oil. The oil consists of a mixture of dead oil and volatile oil.

Physical quantities are evaluated at a representative constant pressure throughout the core. Thermal expansion of liquids will be disregarded. All fluids are considered incompressible. We assume Darcy's law of multiphase flow. The tube diameter is considered sufficiently small so that gravity segregation does not occur and temperature is homogeneous radially.

### 2.1. Thermodynamical Fundamentals:

Let us describe the phase behavior. We always assume local thermodynamic equilibrium. We have two phases, i.e., gaseous and oleic. There are two components, viz. volatile oil ( $v$ ) and dead oil ( $d$ ). We define dead oil as an oil with zero vapor pressure and volatile oil as an oil with finite nonzero vapor pressure.

We use the following convention for subscripts: the first subscript ( $o, g$ ) refers to the phase, the second subscript ( $v, d$ ) refers to the component. The pure phase densities of liquid volatile oil and liquid dead oil are denoted  $\rho_V$  and  $\rho_D$  respectively. The pure phase densitie of volatile oil vapor is denoted by  $\rho_{gV}$ .

Gibbs' phase rule ( $f = c - p + 2$ ) yields two degrees of freedom for the thermodynamic variables. As in our model we fix an average pressure at which the the fluid and gas properties are evaluated, all concentrations are functions of the temperature only.

We derive a simple model for the thermodynamic behavior. We disregard any mixing heat between volatile oil and dead oil. Moreover we disregard any volume contraction effects resulting from mixing. The concentration [ $kg/m^3$ ] of (dead) volatile oil in the oleic phase is denoted as ( $\rho_{od}$ )  $\rho_{ov}$ . In the same way we define the concentration of the volatile oil in the gaseous phase as  $\rho_{gV}$ . For ideal fluids we obtain

$$\frac{\rho_{ov}}{\rho_V} + \frac{\rho_{od}}{\rho_D} = 1. \quad (1)$$

The pure liquid densities  $\rho_V, \rho_D$  [ $kg/m^3$ ] are considered to be independent of temperature, and the pure vapor density is considered to obey the ideal gas law, i.e.,  $\rho_{gV} = M_V P / RT$ , where  $M_V$  denote the molar weight of volatile oil.  $P$  is the prevailing pressure and the gas constant is  $R = 8.31 [J/mol/K]$ .

### 2.2. Two-phase Behavior:

We assume that the volatile oil vapor pressure  $P_v$  is determined by the Clausius-Clapeyron equation with Raoult's law (see Moore 1962), which states that the vapor pressure of volatile oil is equal to its pure vapor pressure times the mole fraction  $x_{ov}$  of volatile oil in the oil phase. Therefore we obtain

$$P_v(T) = x_{ov} P_o \exp\left(\frac{-M_V \Lambda_V(T_b^v)}{R} \left(\frac{1}{T} - \frac{1}{T_b^v}\right)\right), \quad (2)$$

where  $T_b^v$  is the normal boiling temperature of volatile oil,  $\Lambda_v(T_b^v)$  is the evaporation heat of volatile oil at the normal boiling temperature  $T_b^v$  of volatile oil and  $M_v$  is the molar weight of volatile oil. Furthermore we assume that the total pressure is the volatile oil vapor pressures, i.e.  $P^{tot} = P_v$ .

Finally we need to derive an equation that relates the oleic phase densities to the mole fraction  $x_{ov}$ . From the definition of mole fraction (moles volatile oil / total moles)

$$x_{ov} = \frac{\rho_{ov}/M_v}{\rho_{ov}/M_v + \rho_{od}/M_D}, \quad (3)$$

where  $M_D$  is the molar weight of the dead oil. Combining Eq. (3) with the first ideal mixing rule of Eq. (1) we find after some algebraic manipulation

$$\rho_{ov} = \frac{x_{ov}\rho_D\rho_v M_v}{x_{ov}\rho_D M_v + (1-x_{ov})\rho_v M_D}, \quad \rho_{od} = \frac{(1-x_{ov})\rho_D\rho_v M_D}{x_{ov}\rho_D M_v + (1-x_{ov})\rho_v M_D}. \quad (4)$$

The pure phase densities  $\rho_v, \rho_D$  are given in Table A.

### 3. Balance Equations

**3.1. Heat Balance Equations:** The conservation of enthalpy is given by

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(uf_o H_o + uf_g H_g) = 0, \quad (5)$$

Capital  $H$  is used for enthalpies per unit volume. It is convenient to define  $H_o = \rho_{ov}h_{ov} + \rho_{od}h_{oD}$ ,  $H_g = \rho_{gv}h_{gV}$ .

The enthalpies  $h$  are all per unit mass and depend on temperature (see Appendix A). The enthalpy of volatile oil in the gaseous phase is  $h_{gV}$ . Furthermore  $h_{ov}$  and  $h_{oD}$  are the enthalpies of liquid volatile oil and dead oil. Hence  $H = H_r + \varphi(s_o H_o + s_g H_g)$ .

The saturation of oleic and gaseous phases are denoted as  $s_o, s_g$  and  $f_o, f_g$  are the Buckley-Leverett fractional flow functions. We use  $u$  to denote the total Darcy flow velocity and  $\varphi$  the constant rock porosity.

Rock enthalpy  $H_r$  is a function of temperature. It can be shown that if the heat capacities with respect to volume of volatile and dead oil are equal, then  $H_o$  is linear in temperature and independent of composition. In this case we can reduce the liquid equation to a much simpler equation that can be solved explicitly in our numerical scheme. This excellent approximation simplifies the dependence of the total enthalpy  $H = H(s_g, T)$ , i.e.,  $H$  does not depend on the composition of the oleic phase.

**3.2. Conservation Equations:**

We can write for the mass conservation volatile oil and dead oil respectively as:

$$\begin{aligned} \varphi \frac{\partial}{\partial t}(\rho_{gv}s_g + \rho_v s_o) + \frac{\partial}{\partial x}u(\rho_{gv}f_g + \rho_v f_o) &= 0, \\ \varphi \frac{\partial}{\partial t}(\rho_{od}s_o) + \frac{\partial}{\partial x}u(\rho_{od}f_o) &= 0. \end{aligned} \quad (6)$$

Notice that  $f_g$  and  $f_o$  are functions of the variables  $s_g$  and  $T$ . The independent variables of Eq. (6) are  $s_g$ ,  $T$  and  $u$ . The equations (6) and (5) can be rewritten in condensed form as:

$$\frac{\partial G_l}{\partial t} + \frac{\partial u F_l}{\partial x} = 0, \quad \text{for } l = v, d, T; \quad (7)$$

where we use the subscript  $l$  to denote the components ( $v, d$ ) and the temperature  $T$ .

## 4. Numerical Method

To visualize the solutions we use an implicit, conservative numerical method. We discretize (7) with an implicit, backward in time, backward in space method on a uniform mesh. We write:

$$G_l^k(t + \Delta t) = G_l^k(t) - \frac{\Delta t}{\Delta x} \left( u^k(t + \Delta t) F_l^k(t + \Delta t) - u^{k-1}(t + \Delta t) F_l^{k-1}(t + \Delta t) \right), \quad (8)$$

where the superscript denotes the  $k^{\text{th}}$  grid cell. The unknowns are  $u^k(t + \Delta t)$  and  $V^k(t + \Delta t)$  where  $V = (s_g, T)$ . To shorten the notation we rewrite (8),  $u^k(t + \Delta t)$  as  $u$  and  $V^k(t + \Delta t)$  as  $V$ . We obtain

$$G_l(V) + \frac{\Delta t}{\Delta x} u F_l(V) = R_l^{k,k-1}, \quad (9)$$

where  $R_l^{k,k-1} = G_l^k(t) + u^{k-1}(t + \Delta t) F_l^{k-1}(t + \Delta t) \Delta t / \Delta x$ . Note that all unknowns lie in the left side.

We solve this implicit system of non-linear equations by the Newton-Raphson method. Given a solution at the  $n^{\text{th}}$  iteration  $V^n$  and  $u^n$  of (9) we want to guess an approximation to the solution in the  $(n + 1)^{\text{th}}$  iteration, in order to use the Newton-Raphson method to correct it. Substituting  $V^{n+1} = V^n + \Delta V$ ,  $u^{n+1} = u^n + \Delta u$  and neglecting second order terms we obtain a first order approximation

$$\left( \frac{\partial G_l}{\partial V}(V^n) + \frac{\Delta t}{\Delta x} u^n \frac{\partial F_l}{\partial V}(V^n) \right) \Delta V + \frac{\Delta t}{\Delta x} F_l(V^n) \Delta u = -R_l^n \quad (10)$$

where  $R_l^n = G_l(V^n) + u_l F_l(V^n) \Delta t / \Delta x - R_l^{k,k-1}$ . The system will be solvable if  $u \Delta t / \Delta x$  is not a characteristic speed, which can be achieved by controlling the time step.

## 5. The Riemann Problem

The building blocks for solutions of conservation laws are shocks and rarefactions, see Smoller 1980. They arise in a very important class of initial value problems, the so called Riemann problems. Mathematically, a Riemann problem is a powerful tool that can be used to solve more general problems or to test new numerical methods. For the oil industry Riemann problems can be interpreted as constant injection problems in very thin and long reservoirs.

### 5.1 Rarefaction Waves.

Rarefaction waves are smooth self-similar solutions of equation (7). In the new variable  $\xi = x/t$ , they satisfy:

$$\left( u \frac{\partial F(V)}{\partial V} - \xi \frac{\partial G(V)}{\partial V} \Big| F(V) \right) \left( \frac{dV}{d\xi}, \frac{du}{d\xi} \right)^t = 0. \quad (11)$$

Equation (11) is a generalized eigenvector problem. After easy calculations we arrive at the eigenvalues:

$$\begin{aligned}\lambda_b &= \frac{u}{\varphi} \frac{df_o}{ds_o}, \\ \lambda_e &= \frac{u}{\varphi} \frac{A(T)f_o + \rho_{od}\rho_{gV}\partial_T h_{gV}}{A(T)s_o + \rho_{od}(\rho_{gV}\partial_T h_{gV} + \partial_T H_r)},\end{aligned}\quad (12)$$

where  $A(T) = \rho_V(h_g - h_o)\partial_T \rho_{od} + \rho_{od}(\partial_T H_o - \rho_{gV}\partial_T h_{gV})$ . The eigenvectors readily follow and are written as  $r_b = (1,0,0)$ ,  $r_e = (r_{e1}, r_{e2}, r_{e3})$ , where:

$$\begin{aligned}r_{e1} &= \frac{1}{u}(m_{13}m_{22} - m_{12}m_{23}), \\ r_{e2} &= \frac{\varphi}{u}m_{23}(\lambda_b - \lambda_e), \\ r_{e3} &= -\frac{1}{m_{13}}[(\rho_V - \rho_{gV})(\lambda_b - \lambda_e)r_{e1} + m_{12}r_{e2}].\end{aligned}\quad (13)$$

Here  $m_{ij}$  stands for the  $i$ -th row and  $j$ -th column in the matrix given by equation (11) after Gauss-Seidel elimination. Each pair eigenvalue-eigenvector gives rise to a distinct family of waves, namely the Buckley-Leverett type waves  $(\lambda_b, r_b)$  and the evaporation type waves  $(\lambda_e, r_e)$ . When the solution of the Riemann problem is constructed only one family can be used at each step.

The rarefaction waves are subsets of the solutions of the ODE  $(\dot{V}(\xi), \dot{u}(\xi)) = r_l$ ,  $l \in (b, e)$  with  $V(0), u(0)$  given. These subsets are chosen as follows: given one point in state space and one family  $l \in (b, e)$ , we get a solution parametrized in  $\xi$  which we call  $\mathcal{V}(\xi)$ . The admissible rarefaction is the set of states which we can reach using  $\mathcal{V}(\xi)$  while the speed  $\lambda_l(\mathcal{V}(\xi))$  is non-decreasing. Figure 1 shows some admissible rarefaction curves for the evaporation family.

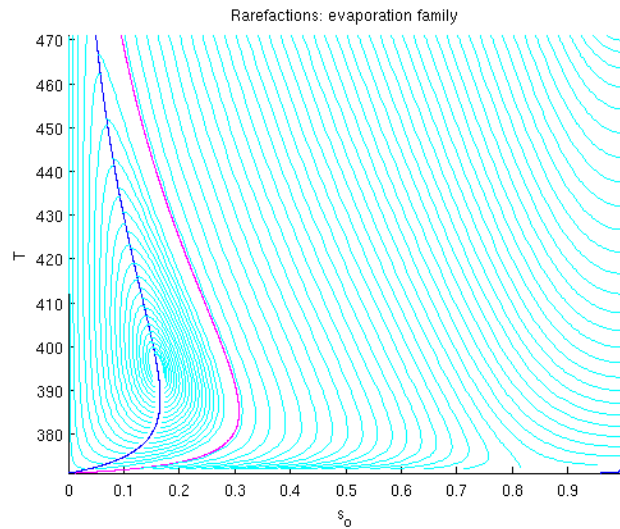


Figure 1: Rarefactions

## 5.2 Shock Waves.

The Riemann problem for the system in equation (7) possesses discontinuous solutions which are called shock waves. These solutions must satisfy the Rankine-Hugoniot condition which is written as:

$$\sigma(G(V^+) - G(V^-)) = u^+ F(V^+) - u^- F(V^-). \quad (14)$$

where  $\sigma$  is the shock speed and  $V^\pm = (s_o^\pm, T^\pm)$ ,  $u^\pm$  are the left (-) and right (+) states. Equation (14) can be solved to give the Darcy speed at one of the states:

$$u^+ = u^- \frac{F_v(V^-)[G_d(V^+) - G_d(V^-)] - F_d(V^-)[G_v(V^+) - G_v(V^-)]}{F_v(V^+)[G_d(V^+) - G_d(V^-)] - F_d(V^+)[G_v(V^+) - G_v(V^-)]}, \quad (15)$$

and the shock speed:

$$\sigma = u^- \frac{F_v(V^+)F_d(V^-) - F_d(V^+)F_v(V^-)}{F_d(V^+)[G_v(V^+) - G_v(V^-)] - F_v(V^+)[G_d(V^+) - G_d(V^-)]}. \quad (16)$$

A change of variables  $x \rightarrow x/u^-$  shows that the total Darcy velocity at injection only rescales the speed of waves, retaining their shape, and so for the purpose of visualization we can assume  $u^- = 1$ . Substituting equations (16) and (15) back in equation (14) we get an expression, which is satisfied by all right states (+) that can be joined to the left state (-) with a shock. Typically this set is a pair of curves which intersect at the left state (-) and is called the Hugoniot-Locus.

To construct the solution to the Riemann problem we use the Liu criterion (Liu 1975) to select the states within the Hugoniot-Locus that are admissible. Figure 2 shows a sample of Hugoniot-Locus and its corresponding admissible states for the evaporation branch highlighted.

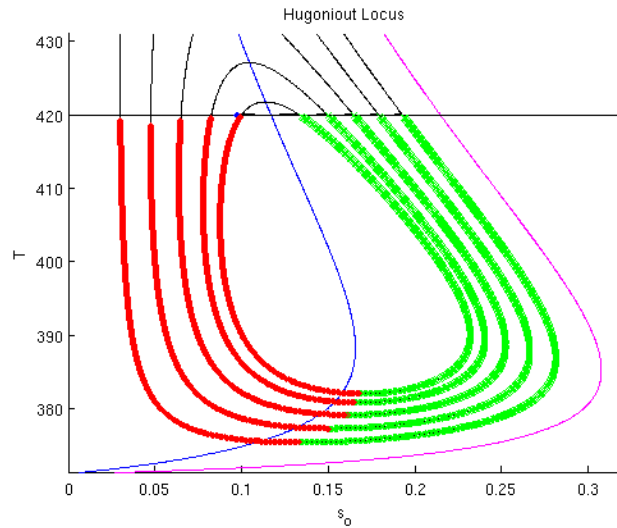


Figure 2: Shock curves

## 5.3 Solution

We consider gas injection into a reservoir containing a mixture of dead and light oil and gas in thermodynamical equilibrium. We write the saturation and temperature at injection as  $L = (s^{inj} = 0, T^{inj} > T_{bV})$  and the reservoir saturation and temperature as  $R = (s^{res} = 0, T^{inj} > T^{res} \geq T_{bV})$ . The oil concentration is given by equation (2).

From the  $L$  state, there is a Buckley-Leverett rarefaction wave which reaches a constant state, say  $M$ . The Buckley-Leverett family does not change temperature so the next wave (which must be a condensation wave) attains the reservoir temperature ( $T^{res}$ ). The admissibility criterion says that it must be a shock wave and the Liu criterion fixes the intermediate state ( $M$ ) so that, in state space, a thermal shock beginning from there touches tangentially the horizontal line that contains the  $R$  state (which means that it is a characteristic shock). The last wave is a fast composite (a rarefaction followed by a characteristic shock) Buckley-Leverett wave beginning from this tangency state.

Although the thermal shock leads to almost complete recovery, it is very slow in comparison to the composite Buckley-Leverett wave, typically there is a  $10^3$  ratio between speeds. For practical purposes the recovery is due to the first composite wave.

Figure 3 shows the construction of the solution in the state space.

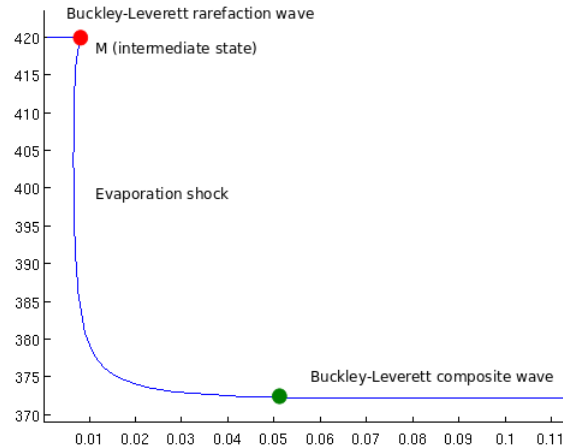


Figure 3: Riemann problem example

Figure 4 shows the same Riemann problem simulated.

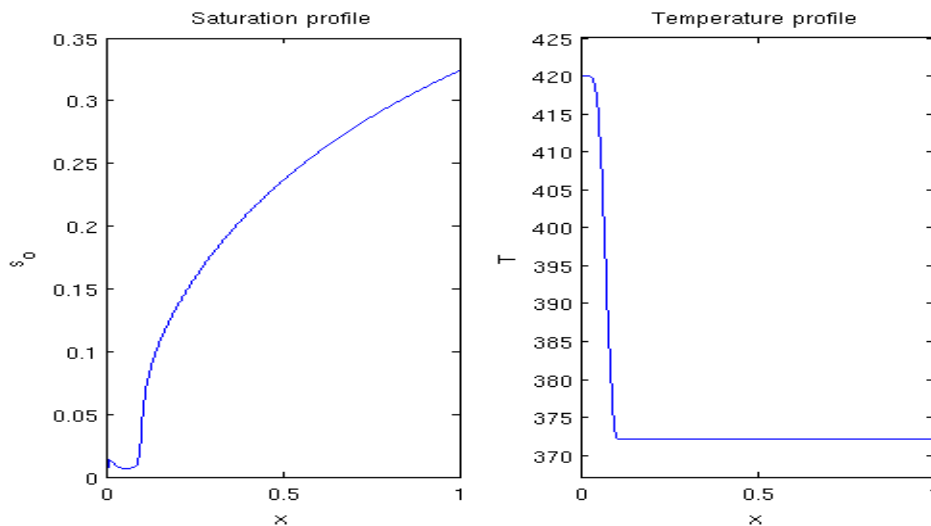


Figure 4: Simulation

## 6. Conclusions

We considered the problem of volatile oil injection into a porous medium containing a mixture of oil and volatile oil. Following [3] we used a simplified model in such a way that the key physical aspects were retained and yet the complexities of solving the pressure equation were avoided. We found the explicit solutions to the Riemann problem

of constant gas injection into a porous medium. The oil displacement was due a Buckley-Leverett composite wave. This solution is a fundamental step in the understanding of alkane steam co-injection into a porous medium and can be used to validate complex simulators.

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## Appendix A: Physical Quantities, Symbols and Values

In this appendix we summarize the values and units of the various quantities used in the computation and empirical expressions for the various parameter functions. For convenience we express the heat capacity of the rock  $C_r$  in terms of energy per unit volume of *porous medium* per unit temperature *i.e.* the factor  $1 - \phi$  is already included in the rock density. All other densities/concentrations are expressed in terms of mass per unit volume of the phase. All enthalpies per unit mass are with respect to the enthalpies at the reference temperature of the components in their standard form. All heat capacities are at constant pressure. All enthalpies in their standard form are zero at the reference temperature.

Table A. Summary of physical input parameters and variables

Physical quantity	Symbol	Value	Unit
Porous rock permeability	$k$	$1.0 \times 10^{-12}$	[m <sup>2</sup> ]
Molar weights	$M_V, M_D$	0.10021, 0.4	[kg/mole]
Total pressure	$p^{tot}$	$1.0135 \times 10^5$	[Pa]
Injection saturation	$S_g^{inj}$	input	[m <sup>3</sup> /m <sup>3</sup> ]
Reservoir, injection temperature	$T^{ref}, T^{inj}$	input	[K]
Boiling point of volatile, dead oil	$T_b^v, T_b^d$	371.57, $\infty$	[K]
Volatile, dead oil heat capacities	$c_{oV}, c_{oD}$	2121, $c_{oV} \rho_V / \rho_D$	[J/kg/K]



Volatile oil (log) viscosity	$\mu_{ov}$	$-11.145 + 981.25/T$	[Pa s]
Dead oil (log) viscosity	$\mu_{od}$	$-13.80 + 3780/T$	[Pa s]
Gas constant	$R$	8.31	[J/mole/K]
Pure vol-oil, dead oil densities	$\rho_v, \rho_D$	683, 800	[kg/m <sup>3</sup> ]
Rock porosity	$\phi$	0.38	[m <sup>3</sup> /m <sup>3</sup> ]

### A.1. Temperature Dependent Variables.

We use references Tortike, Farouq Ali 1989 and Weast 1978 to obtain all the temperature dependent properties below. The rock enthalpy  $C_r$  can be expressed as

$$H_r = (1 - \phi)C_r(T - \bar{T}), \quad C_r = (1 - \phi) \times 3.274 \times 10^6 = 2.03 \times 10^6 \quad J/m^3/K. \quad (17)$$

A conventional choice for the reference temperature is  $\bar{T} = 298.15K$ . The volatile oil enthalpy  $h_{ov}[J/kg]$  and the dead oil enthalpy  $h_{od}[J/kg]$  as a function of temperature is approximated by  $h_{ov}(T) = c_{ov}(T - \bar{T})$ ,  $h_{od}(T) = c_{od}(T - \bar{T})$ . The values of  $c_{ov}$  and  $c_{od}$  can be found in Table A. The enthalpies are chosen so that the enthalpy of oil per unit volume is independently of composition. Therefore the heat capacity of the oleic phase per unit volume can also be defined independent of composition.

The volatile oil vapor enthalpy  $h_{gv}[J/kg]$  as function of temperature is approximated by  $h_{gv}(T) = c_{gv}(T - \bar{T}) + \Lambda_v(\bar{T})$ . The enthalpies  $h_{ov}(T)$ ,  $h_{od}(T)$  vanish at the reference temperature  $\bar{T} = 298.15K$ . For the evaporation heat  $\Lambda_v(T)[J/kg]$  we use Trouton's rule

$$\Lambda_v(T) = 88.0 \times T_b^v / M_v - (c_{ov} - c_{gv})(T - T_b^v). \quad (18)$$

The viscosities of liquid volatile oil  $\mu_{ov}$ , and liquid dead oil  $\mu_{od}$  can be found from Table A. The viscosity of the oil mixture is approximated by  $\mu_{mix} = (\rho_{ov} / \rho_v)\mu_{ov} + (\rho_{od} / \rho_D)\mu_{od}$ . We assume that the viscosity of the gas is independent of composition  $\mu_g = 1.8264 \times 10^{-5} (T/300)^{0.6}$ .

## Appendix B: Relative Permeabilities for Three-phase Flow

We assume Darcy's law for multi-phase flow. Without the gravity terms and ignoring capillary effects it is

$$u_\alpha = \frac{k_\alpha}{\mu_\alpha} \frac{\partial P}{\partial x}, \quad \alpha = o, g. \quad (19)$$

The total Darcy flow velocity is written  $u = u_o + u_g$ . The simplest expressions for relative permeability functions are power functions of their respective saturations i.e.  $k_o = s_o^2$ ,  $k_g = s_g^2$ . Considering the mobilities for the oleic phase and the gaseous phase as given by  $\lambda_o = k_o / \mu_{mix}$  (where  $\mu_{mix}$  is in (19)) and  $\lambda_g = k_g / \mu_g$  we can express the Buckley-Leverett fractional flow functions as  $f_\alpha = \lambda_\alpha / (\lambda_o + \lambda_g)$ ,  $\alpha = o, g$ .  
g., v. 14, n. 2, p. 137-158, 1996.