A SIMPLE INVERSE SOLVER FOR THE PERMEABILITY FUNCTION FOR FLOW OF WATER WITH PARTICLES IN POROUS MEDIA

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Abstract. In secondary oil recovery, water is injected into the petroleum reservatory to push out the oil. This procedure accounts for most of the oil a reservoir produces. The efficiency of an injection well is measured in terms of its *injectivity*: the ratio between the injection flow rate and the injection pressure required to maintain this rate. Injection of unclean water causes the well injectivity to decline, because the porous medium acts as a filter to the particles suspended in the water. The capture of suspended particles and consequent loss of permeability characterize a phenomenon called *deep bed filtration with formation damage*. Retention of particles on the surface of the porous medium, where injection takes place, also causes injectivity decline, and is known as *cake formation*.

Models for deep bed filtration correlate the pressure drop to particle deposition through an empirical *permeability reduction function*, which represents the macroscopic effect of particle retention on the permeability.

Different laboratory experiments using rock samples from oil reservoirs can be performed to determine this function. We describe a practical method for recovering the permeability reduction function from experimental data consisting of deposition profiles and pressure measurements during the injection of a suspension of solid particles in a linear rock core.

Inverse problem, Formation damage, Deep bed filtration, Convection-reaction equations

1. Introduction

Oil wells go through different production stages before they are abandoned. Primary oil recovery is due to natural reservatory pressure, which causes the oil to rise to the surface naturally through the production well. In secondary oil recovery, water is injected into the reservatory to push out the oil through another well. This procedure accounts for most of the oil a reservoir produces. The efficiency of an injection well is measured in terms of its *injectivity*: the ratio between the injection flow rate and the injection pressure required to maintain this rate. The injection rate is proportional to the recovery rate, and the injection pressure represents operational cost.

The recovered oil is mixed with water, which has to be separated. This produced water accounts for around three quarters of a production well output: even after separation it contains solid particles and oil droplets, and must be disposed of in an environmentally safe way. A common practice is to use the produced water itself for injection, hence the engineering term *produced water re-injection*.

However, injection of unclean water causes the well injectivity to decline, because the porous medium acts as a filter to the particles suspended in the water. The capture of suspended particles inside the porous medium and consequent loss of permeability characterize a phenomenon called *deep bed formation damage*. Experience shows that another phenomenon takes place on the injection surface of the porous medium, called *cake formation*. The "cake" is an aglomeration of particles outside the porous medium, and thus it is altogether separate from the formation damage. It causes injectivity decline as well.

Models for deep bed filtration correlate the pressure drop to particle deposition through an empirical *permeability reduction function*, which represents the macroscopic effect of particle retention on the permeability. Our work utilizes a model for deep bed filtration based on the fundamental work of Hertzig et al. [5], which consists of equations expressing the particle mass conservation and the particle retention process. Models for cake formation are still incipient. However, experimental methodologies to separately measure the pressure drops due to the cake and to the formation damage are available.

We describe a practical method for recovering the permeability reduction function from experimental data consisting of deposition profiles and pressure measurements during the injection of a suspension of solid particles in a linear rock core. We extend the method to the case were cake formation occurs and is measured.

The paper is organized as follows. In Section 2 we present the relation between injectivity decline and particle deposition. In Section 3, we present the inverse problem and the type of solution we seek to obtain. In Section 4 the numerical results are shown, and in Section 5 a summary of our results is presented.

2. Global solution for the direct problem

We use a physical model for the phenomenon called *deep bed filtration with formation damage* based on [5]. Such models, further detailed in Appendix A, can be used to predict the deposition profile $\sigma(x,t)$ along the core, where σ is a dimensionless quantity between 0 and 1, the fraction of the pore volume occupied by retained particles. The physical domain is $t > 0$ and $0 \leq x \leq L$, where L is the length of the core. This deposition information can then be related to the pressure drop along the core by:

$$U = -\frac{k_0\, k(\sigma)}{\mu}\, \frac{\partial p}{\partial x}. \tag{2.1}$$

which is a form of Darcy's law relating the flow rate U to the pressure p. Here, k_0 is the absolute rock permeability and $k(\sigma)$ is the permeability reduction due to the retained particles; when expressed as a function of σ, it is called the *formation damage function.* It is normalized so that $k(0) = 1$, i.e. it is one for clean porous rock. In general, the water viscosity μ can be considered constant for small particle concentrations.

We assume that permeability reduction $k(\sigma)$ is a decreasing function of the retained concentration. For one-dimensional flow in a rock core, we multiply Eq. (2.1) by $[k(\sigma(x,t))U]^{-1}$, and integrate the resulting equation in some interval $[a,b]$ to obtain the following relationship between deposited particle distribution and pressure drop:

$$\int_a^b \frac{dx}{k(\sigma(x,t))} = \frac{k_0}{\mu U} \Delta p_{\sigma[a,b]}(t), \quad t \geq 0, \quad 0 \leq a < b \leq L. \tag{2.2}$$

Notice that we have arbitrarily changed the sign of the pressure drop so as to compute $\Delta p_{[a,b]} = p(a) - p(b) > 0$ for $b > a$, and not the other way around. The pressure drop computed in Eq. (2.2) represents formation damage due to retained particles, and so we subscript it Δp_σ. The pressure drop due to cake formation Δp_{cake} can only be inferred experimentally, since there are no good models for it.

Next, we transform the physical domain into dimensionless coordinates by the formulae

$$X = \frac{x}{L} \quad \text{and} \quad T = \frac{U}{\phi L} t, \tag{2.3}$$

so X is in the $[0,1]$ range; one unit of dimensionless time T is the time it takes to inject a volume of fluid corresponding to the porous volume of the sample, hence the term *porous volume injected* (PVI).

In the following sections, we assume the deposition profiles $\sigma(X,T)$ are given along with the corresponding pressure drop measurements, i.e., $\Delta p_{\text{exp}}(T) = p_{\text{inlet}}(T) - p_{\text{outlet}}(T)$. If cake formation is to be considered, a second pressure drop measurement $\Delta p_{\text{exp}}(T) = p_{X_{\text{mid}}}(T) - p_{\text{outlet}}(T)$ is required. Since deposition is most significant near the injection face, $X_{\text{mid}} > 0$ should be as close to 0 as experimentally possible without compromising the phenomena under study. Previous experience in [6] with deposition data obtained by Al-Abduwani in [1] suggests careful preprocessing of the profiles before attempting any parameter recovery procedure.

3. The Recovery Method

We wish to determine a function $k(\sigma)$ from input data consisting of σ_{exp} and Δp_{exp}, either synthetic or experimental, such that Eq. (2.2) holds. We assume that these data are a set of numerical values $0 \le \sigma_{\text{exp}}(X_i, T_j) \ll 1$ and $\Delta p_{\text{exp}}(T_j) > 0$, for $i = 1, \ldots, m$, $X_1 > 0$, $X_m = 1$, and $j = 1, \ldots, n$. Where required, we use $X_0 = 0$ and $T_0 = 0$.

3.1. **Case 1: One pressure drop measurement, no cake discrimination.**

If cake formation is not taken into account during the experiments, i.e., Δp_{cake} is assumed negligible, then only $\Delta p_{\text{exp}}(T)$ is available, i.e., the difference between the inlet and outlet pressures. We assume $\Delta p_{\sigma}(T_j) = \Delta p_{\text{exp}}(T_j)$.

First, we choose a parametrization $k(\sigma;\beta)$ for the permeability reduction function, where $\beta = (\beta_1, \ldots, \beta_p)$ is a set of p parameters that we will determine from the input data. From physical properties of the permeability reduction function, e.g., it is a decreasing function of σ, we select the following form for $k(\sigma;\beta)$:

$$k(\sigma;\beta) = \left(1 + \sum_{\ell=1}^{p} \beta_\ell \sigma^\ell\right)^{-1}. \tag{3.1}$$

Using the parametrization above and the change of variables (2.3), Eq. (2.2) can be rewritten in the interval $[0,L]$ as:

$$\int_0^1 \left(1 + \sum_{\ell=1}^{p} \beta_\ell \sigma(X,T)^\ell\right)^{-1} dX = \frac{k_0}{\mu U} \Delta p_\sigma(T), \tag{3.2}$$

Applying the discrete input data we obtain:

$$\sum_{\ell=1}^{p} M^{\ell}_{[0,1]}(T_j)\beta_\ell = \frac{k_0}{\mu U} \Delta p_\sigma(T_j) - 1. \tag{3.3}$$

where $M^{\ell}_{[0,1]}(T)$ are the integrals of the moments of σ_{exp}, which we be computed using any given quadrature, such as trapezoidal rule:

$$M^{\ell}_{[0,1]}(T_j) = \int_0^1 \big(\sigma_{\text{exp}}(X,T_j)\big)^\ell dX \cong \frac{1}{2} \sum_{i=0}^{m-1} \Big[\big(\sigma_{\text{exp}}(X_i,T_j)\big)^\ell + \big(\sigma_{\text{exp}}(X_{i+1},T_j)\big)^\ell\Big]\big(X_{i+1} - X_i\big). \tag{3.4}$$

The expression above can clearly be improved to take advantage of an uniform partition of the X interval:

$$M^{\ell}_{[0,1]}(T_j) \cong \left(\frac{\big(\sigma_{\exp}(X_0, T_j)\big)^{\ell} + \big(\sigma_{\exp}(X_m, T_j)\big)^{\ell}}{2} + \sum_{i=1}^{m-1} \big(\sigma_{\exp}(X_i, T_j)\big)^{\ell} \right) \Delta X. \tag{3.5}$$

Let $A = (a_{ji}) = \big(M^{i}_{[0,1]}(T_j)\big)$ be a $n \times p$ matrix and $\gamma = \big(\frac{k_0}{\mu U}\Delta p(T_j)\big)$ be a n-vector. This reduces Eq. (3.3) to a $n \times p$ system of linear equations $A\beta = \gamma$ that can be solved for the p-vector β of unknowns using standard procedures for overdetermined linear systems.

For example, one can transform the $n \times p$ system into a $p \times p$ system of the form $A^{\star}A\beta = A^{\star}\gamma$, where $\star$ denotes matrix transpose. These procedures are found in standard packages such as Matlab.

3.2. **Case 2: Two pressure drop measurements, cake discrimination.** One way of taking cake formation into account during experiments is to perform a separate pressure drop measurement near the injection face. Given two pressure drop histories $\Delta p^1_{\exp}(T)$ and $\Delta p^2_{\exp}(T)$ as in Figure 3.1, we know that $\Delta p_{\sigma}(T) + \Delta p_{\text{cake}}(T) = \Delta p^1_{\exp}(T) + \Delta p^2_{\exp}(T)$, and we assume that $\Delta p_{\text{cake}}(T)$ is fully contained in $\Delta p^1_{\exp}(T)$.

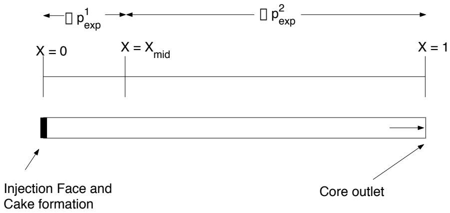


FIGURE 3.1. Two pressure drop measurements obtained from inlet, outlet and middle pressure taps.

Little change is required to the method described in the previous subsection in order to recover the β coefficients and the values of Δp_{cake}. The whole procedure in 3.1 can be executed in the interval $[X_{\text{mid}}, 1]$, instead of $[0, 1]$, which amounts to simple changes in Eqs. (3.3) and (3.4). The system (3.3) takes the form

$$\sum_{\ell=1}^{p} M^{\ell}_{[X_{\text{mid}},1]}(T_j)\beta_{\ell} = \frac{k_0}{\mu U}\Delta p^2_{\exp}(T_j) - (1 - X_{\text{mid}}), \tag{3.6}$$

where $M^{\ell}_{[X_{\text{mid}},1]}(T_j)$ are computed as $M^{\ell}_{[0,1]}(T_j)$ in Eq. (3.4), but in the $[X_{\text{mid}}, 1]$ interval instead.

Once the set of parameters β has been determined, it can be applied to Eq. (2.2) to determine $\Delta p_{\text{cake}}(T_j) = \Delta p^1_{\exp}(T) + \Delta p^2_{\exp}(T) - \Delta p_{\sigma}(T_j)$. In practice, we are solving the same problem as before in the $[X_{\text{mid}}, 1]$ interval, thus failing to take into account the deposition data available in the $[0, X_{\text{mid}}]$ interval. In the absence of a model for the cake, the amount of pressure drop due to formation damage in this interval is unknown. A compromise is made in this approach, since the deposition data is more reliable near the injection face.

The simplest way of taking into account the deposition data in the $[0, X_{\text{mid}}]$ interval is to expand the system to accomodate the pressure drops $\Delta p^1_{\exp}(T)$ as data and the values of

$\Delta p_{\mathrm{cake}}(T_j)$ as unknowns, obtaining another linear system of $2n$ equations and $p+n$ unknowns. The system (3.3) takes the form:

$$\begin{aligned} \sum_{\ell=1}^{p} M^{\ell}_{[X_{\mathrm{mid}},1]}(T_j)\beta_\ell &= \frac{k_0}{\mu U}\Delta p^2_{\mathrm{exp}}(T_j) - (1 - X_{\mathrm{mid}}), \\ \Delta p_{\mathrm{cake}}(T_j) + \sum_{\ell=1}^{p} M^{\ell}_{[0,1]}(T_j)\beta_\ell &= \frac{k_0}{\mu U}\big(\Delta p^1_{\mathrm{exp}}(T_j) + \Delta p^2_{\mathrm{exp}}(T_j)\big) - 1. \end{aligned} \tag{3.7}$$

3.3. **Case 3: Two experiments, two measurements, cake discrimination.** Another way of determining the permeability reduction function when cake forms is to perform two "identical" experiments, with two cores of different lengths, as identical as possible in every other aspect. Assuming that the two cores have lengths $L_1 > L_2$, we use L_1 in Eq. (2.3) so that σ in the second data set will be restricted to the $[0, X_{\max}]$ interval. If we assume that the second experiment is merely a truncation of the first, then cake formation and deposition in the $[0, X_{\max}]$ interval should be the same in both runs. Thus, $\Delta p^1_{\mathrm{cake}} = \Delta p^2_{\mathrm{cake}}$ and, in the $[0, X_{\max}]$ interval, $\Delta p^1_\sigma = \Delta p^2_\sigma$. This assumption must be verified by comparing the deposition profiles and other quantitative results from both experiments. It follows that $\Delta p^1_{\mathrm{exp}} - \Delta p^2_{\mathrm{exp}}$ is the pressure drop due to deposition in the $[X_{\mathrm{mid}}, 1]$ interval, which was labeled $\Delta p^2_{\mathrm{exp}}$ in the previous case. In a sense, this is a trick to measure the pressure value at $X_{\mathrm{mid}} = X_{\max}$ in the first experiment, by repeating the experiment with a truncated core. The shorter L_2 is when compared to L_1, the closer we are to the situation where X_{mid} was close to 0 in the previous subsection.

4. Numerical examples

We present an experiment using synthetic data that includes cake formation. The data were generated prescribing $\lambda(\sigma) = \max\{0, 1 - 171\sigma\}$ and $k(\sigma) = [1 + 300\sigma + 10^5\sigma^2]^{-1}$, solving the direct problem using the methods described in Appendix A and Section 2, and introducing random noise of relative magnitude of 10%.

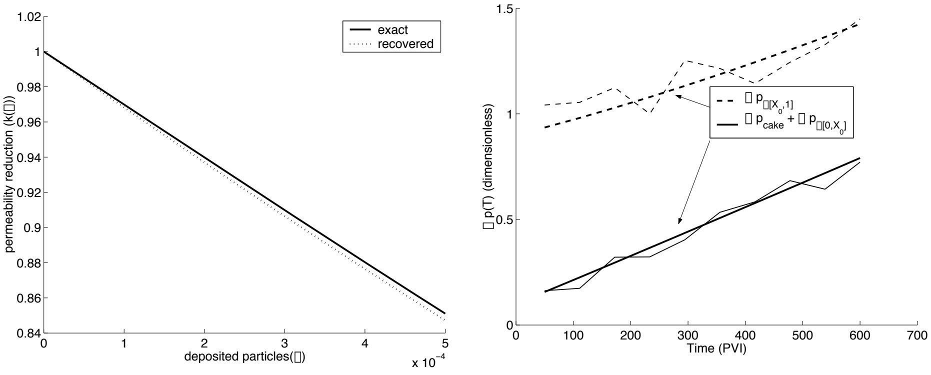
The data mimics the experiment described in Section 3.2: we split the pressure drop integral in two at $X_0 = 0.1$, and added an arbitrary cake to the first part. Consistently with empirical knowledge of cake formation, we chose a cake that grows linearly with time, as can be seen in Figure 4.1b, but it could have any form as it is not relevant for the determination of the parameters of $k(\sigma)$.

Solving the system (3.6) yields $k(\sigma) = [1 + 319\sigma + 8.27 \times 10^4\sigma^2]^{-1}$, with errors of 6% and 18% for the first and second coefficient, respectively, for a 10% perturbation. The use of the augmented system (3.7) recovered the parameters with similar accuracy.

Notice that we picked a value for β_2 three orders of magnitude above that of β_1. Since $\max(\sigma) \cong 10^{-3}$ in this experiment, a smaller coefficient would have made the quadratic term negligible when compared to the linear one. However, this led to a unbalanced matrix A; since this lack of balance did not seem to cause problems in the solution, we did not attempt to balance it.

We can now use this filtration function to determine the fraction of the pressure drop that is to be attributed to cake: one can perform a similar regression on this data once it has been isolated from the pressure drop due to formation damage. In this synthetic example, we can recover the cake growth rate using any linear least squares routine, such as `LSQLIN` in Matlab.

Figure 4.1a shows the prescribed and recovered permeability functions; Figure 4.1b shows the solution of the direct problem using the recovered permeability reduction function, compared with the noisy data.



a) $k(\sigma)$ b) $\Delta p(T)$

FIGURE 4.1. Synthetic data corresponding to the case described in Subsection 3.2, and solution of the system (3.6): on the left, the prescribed and recovered permeability functions; on the right, the pressure from the solution of the direct problem using the recovered permeability function, plotted over the noisy pressure drop input data.

5. CONCLUSIONS

This method recovers the coefficients of a parametric permeability reduction function from pressure drop and deposition measurements. It works well for synthetic data, and can be adapted to different experimental procedures. It is easy to implement and computationally light.

The aim of this procedure is to recover de permeability reduction function, which is related to pressure drop due to deep bed filtration. However, cake formation may occur, and it is of interest to quantify it when it is present. If one assumes the model for deep bed filtration to be valid, this procedure can provide good experimental data about cake formation, which cannot be measured directly with satisfactory accuracy.

Alternatively, the procedure described herein can be complemented by an analogous parametric model for the cake formation, allowing simultaneous recovery of both permeability reduction as a function of the deposition and of cake formation as a function of other information such as deposition at the injection face, injection pressure, elapsed time and cake thickness. Such investigation is motivation for further research both experimental and theoretical.

APPENDIX A. DEEP BED FILTRATION MODEL

We now summarize the model for beed bed filtration with formation damage. Further detail can be found in [3] and [2]. Assuming for simplicity that:

- the mass densities of solid particles in suspended and entrapped states are equal,

- water is incompressible,
- injection takes place at constant volumetric rate,

the mass conservation equation proposed by Herzig et Al. in [5], neglecting diffusive and suspended particle mass accumulation terms, can be written as:

$$\phi \frac{\partial \sigma}{\partial t} + U \frac{\partial c}{\partial x} = 0, \tag{A.1}$$

where

- ϕ is the rock porosity,
- $c(x,t)$ and $\sigma(x,t)$ are the (nondimensional) concentrations of the particles suspended in the water and entrapped in the rock pores, respectively,
- $0 \leq x \leq L$ is the position,
- L is the length of the core,
- $t > 0$ is the time value,
- U is the fluid injection rate.

The quantities c and σ have values between 0 and 1, but usually $c \cong 10^{-4}$, while σ grows up to 10^{-2}. Thus ϕc and $\phi \sigma$ are the concentration of suspended and retained particle per unit volume of core rock. X-ray measurements capture the sum of these quantities. However, because c is much smaller than σ most of the time, we often neglect c in such a sum. (See Remark A.1.)

The model requires a law for particle deposition rate, usually written as:

$$\frac{\partial \sigma}{\partial t} = \lambda(\sigma) c U. \tag{A.2}$$

The form of the right hand side of Eq. (A.2) cannot be determined from first principles, so this is a heuristic equation. The non-negative $\lambda(\sigma)$ is an empirical coefficient known as the *filtration function*, which cannot be measured directly.

Remark A.1. Except at initial times, it turns out that c is much smaller than σ. Because of this fact, the simplified mass conservation equation (A.1) was proposed in [5], while the correct form of the mass conservation equation is the given in [4], for negligible dispersive effects:

$$\phi \frac{\partial}{\partial t}(c + \sigma) + U \frac{\partial c}{\partial x} = 0.$$

The simplified equation (A.1) is appropriate for our purposes since usually there are no accurate data measurements at early times. Indeed, at such times both c and σ are so small that experimental error predominates in the measured data.

A.1. **Boundary and initial conditions.** We denote the solid particle concentration entering into the porous medium as follows:

$$x = 0: \quad c = c_i(t) > 0, \quad t > 0. \tag{A.3}$$

In the absence of cake formation, c_i is the volumetric solid particle concentration in the injection fluid. As initial data at $t = 0$, we assume that the rock contains water with no particles:

$$t = 0: \quad \sigma = 0 \quad \text{and} \quad c = 0 \quad \text{for} \quad x \in [0, L]. \tag{A.4}$$

Using the change of variables given by Eq. (2.3), the system (A.1)–(A.2) and the definition (A.3) can be rewritten in non-dimensional form, in terms of $c(X,T)$ and $\sigma(X,T)$ with $X \in [0,1]$, and $T \geq 0$:

$$\frac{\partial \sigma}{\partial T} + \frac{\partial c}{\partial X} = 0, \tag{A.5}$$

$$\frac{\partial \sigma}{\partial T} = \lambda(\sigma) c, \tag{A.6}$$

$$c(0,T) = c_i \left(\frac{\phi L}{U} T \right), \tag{A.7}$$

where the non-dimensional $\lambda(\sigma)$ in (A.6) is the dimensional $\lambda(\sigma)$ in (A.2) multiplied by ϕL. Along the line $X = 0$, equation (A.5) becomes:

$$\frac{d\sigma(0,T)}{dT} = \lambda(\sigma(0,T))c(0,T), \quad \text{and} \quad \sigma(0,0) = 0. \tag{A.8}$$

Integrating Eq. (A.8) provides $\sigma(0,T)$, which is positive and increasing.

A.2. **Solution of the direct deposition problem.** In [3] the following is proved:

Theorem A.2. *Assume that $\lambda(\sigma)$ is a known, non-negative function, piecewise differentiable with uniformly bounded derivative, and that $c_i(T)$ is also known. Then the system of PDEs* (A.1)–(A.2) *with data* (A.3)–(A.4) *has a unique solution, and given the family of ordinary differential equations*

$$\frac{\partial \sigma}{\partial X} = -\lambda(\sigma)\sigma \quad \text{and} \quad \frac{\partial c}{\partial X} = -\lambda(\sigma)c, \tag{A.9}$$

this solution can be obtained by solving Eq. (A.9a) *with data* (A.8) *and Eq.* (A.9b) *with data* (A.7)*.*

In Eq. (A.9) the time T is fixed, so these should be regarded as a system of ordinary differential equations in X, which determine $\sigma(X,T)$ and $c(X,T)$ in terms of X provided the values of $\sigma(0,T)$ and $c(0,T)$ are available. The value of $\sigma(0,T)$ can be calculated by solving Eq. (A.8), and $c(0,T)$ is given by Eq. (A.7).

REFERENCES

[1] F. A. H. Al-Abduwani, R. Farajzadeh, Broek W. M. G. T. van den, and Currie P. K. Utilizing static filtration experiments to test existing filtration theories for conformance. In *Second Produced Water Workshop. Aberdeen, UK*, 2004.

[2] A. C. Alvarez. *Inverse problems for deep bed filtration in porous media.* PhD thesis, IMPA. Brasil, 2005.

[3] A. C. Alvarez and D. Marchesin. A simple inverse solver for the filtration function for flow in porous media. Internal report, 2005.

[4] P. G. Bedrikovetsky, D. Marchesin, G. Hime, A. G. Siqueira, A. L. Serra, J. R. P. Rodriguez, A. O. Marchesin, and M. Vinicius. Inverse problem for treatment of laboratory data on injectivity impairment. In *International Symposium and Exhibition on Formation Damage Control. Society of Petroleum Engineers. SPE 86523*, 2004.

[5] J. P. Herzig, D. M. Leclerc, and P. Le. Goff. Flow of suspensions through porous media-application to deep filtration. *Industrial and Engineering Chemistry*, 65(5):8–35, 1970.

[6] D. Marchesin, A. C. Alvarez, G. Hime, P. G. Bedrikovetsky, and F. A. H Al-Abduwani. The inverse problem od determining filtration function and permeability reduction in porous media. In *XXV CILAMCE. Iberian Latin American Congress on Computational Methods.Recife, Brasil*, 2004.