ROBUST MANAGEMENT AND PRICING OF LNG CONTRACTS WITH CANCELLATION OPTIONS

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ABSTRACT. Liquefied Natural Gas contracts offer cancellation options that make their pricing difficult, especially if many gas storages need to be taken into account. We develop a valuation mechanism for such contracts from the buyer's perspective, a large gas company whose main interest in these contracts is to provide a reliable supply of gas to its clients. The approach combines valuation with hedging, taking into account that price-risk is driven by international markets, while volume-risk depends on local weather and is stagewise dependent. The methodology is based on setting risk-averse multistage stochastic mixed 0-1 programs, for different contract configurations. These difficult problems are solved with light computational effort, thanks to a robust rolling-horizon approach. The resulting pricing mechanism not only shows how a specific set of contracts will impact the company business, but also provides the manager with alternative contract configurations to counter-propose to the contract seller.

AMS subject classifications: 90C15, 91B30.

1. INTRODUCTION AND BACKGROUND

For large gas companies, *Liquefied Natural Gas* (LNG) appears as a convenient complement to their own *natural gas* (NG) resources. A proper mix of NG and LNG contracts allows a gas company not only to diversify its portfolio, but to hedge risk better. In the gas sector, risk concerns are related to the volatility of gas prices and also to the obligation of providing faultless delivery, especially to "uninterruptible" clients, usually crucial customers for the business.

We consider a problem faced by large companies dealing with NG storage, wishing to contract shipments of LNG, and having the ability to choose among various contract configurations (prices, delivery dates, fees). Typically, LNG contracts offer the possibility to ship LNG loads at pre-specified dates, with a cancellation option. In our application, LNG loads are seen by the company as a mechanism ensuring that clients will be delivered *only gas*, without resorting to alternative, more expensive,

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fuels. But the clients demand is subject to seasonality, making uncertain the future gas consumption. For this reason, to hedge against *volume-risk*, we adopt a robust approach ensuring that the company will always have a sufficiently high storage of NG.

As robust approaches tend to be exceedingly conservative, it is crucial to dispose of corrective measures, to adapt better the actual NG volume to the uncertainty realization. The cancellation options of LNG contracts help precisely in this matter, by making LNG delivery flexible and, hence, rendering NG storage more adaptable. Admittedly, versatileness does not come for free: cancellations involve paying a fee, which gets higher as the forecasted shipment date approaches. For a given LNG contract, it is therefore important to determine if some loads might be cancelled, and if so, when cancellations might occur. Both the price of each load and the cancellation fees depend on the NG spot price at the time the load is delivered or cancelled. Since future spot prices are highly uncertain, we hedge out the corresponding *price-risk* by means of a Conditional Value-at-Risk. As cancellations are modelled by using binary variables, the optimal management of a portfolio of various NG storages and NG and LNG contracts is a multistage stochastic linear program with mixed-integer variables.

The model considered in this work, for large NG companies dealing with storage, represents the buyers' point of view in a nonequilibrium context. More precisely, suppose that a manager who operates in a vertically integrated system has the offer of different LNG contracts with cancellation options. We give an answer to the following question: *what are the acceptable contract prices in such a setting?* This viewpoint brings into consideration two challenging issues, *hedging* and *valuation*, both of which must be handled adequately: the company's NG portfolio needs to be managed in an optimal manner, controlling the volume- and price-risks. From the buyer's perspective, the value of each LNG contract is given by the benefits the contract brings to the company business. Such value is translated into a price that the company manager considers acceptable for contracting LNG shipments. On the other hand, to measure the benefits of a contract for company operations, the contracted price should be specified.

The intertwining between valuation and hedging makes the problem complex and difficult to solve. We adopt a simple but rather realistic framework (all uncertain parameters depend affinely on the NG spot price in a reference market) and proceed in two steps. First, we define the optimal portfolio management problem, for a portfolio composed of NG storages, NG contracts, and a set of LNG contracts with fixed configuration. By *fixed configuration*, we mean that both the purchasing price and cancellation fees of the LNG contracts are known affine functions of the spot price (cf. (2) below). The associated mixed-integer multistage stochastic program is solved by a heuristic based on a *robust rolling-horizon* approach that considers a sequence of risk-averse multi-stage programs, defined over shorter and shorter horizons. The heuristic provides a policy that is implementable, feasible, and time-consistent in the sense of [19]. The LNG pricing problem is addressed in a second step, by simulating the optimal management of the portfolio, over a set of different contract configurations that are considered acceptable by the seller.

Our paper is organized as follows: Section 2 reviews some relevant literature and Section 3 describes the company gas network and its dynamics. The stochastic modelling of uncertainty for this particular problem is given in Section 4. Section 5 explains the robust rolling-horizon approach employed to make the stochastic programs tractable using the solution method in Section 6. The contract pricing mechanism is described in Section 7. Numerical testing on a Brazilian case-study, reported in Section 8, confirms the interest of the approach and validates the methodology.

2. Relation to existing literature

In Europe and North America energy derivatives and physical markets monetize capital investment in commodity storage assets. Such is not the case for our application, because the large oil and energy company under consideration distributes NG in the Brazilian network. Price-risk is international, as it is driven by the NG spot price in some reference market. By contrast, volume-risk is mostly national, and varies with the pluvial regime, because in Brazil electrical power is mostly hydraulic. When hydro-reservoirs are full, there is no thermal generation and the thermal power plants practically do not consume any NG. When reservoirs are nearly empty, thermal power plants are dispatched at full capacity and the NG consumption peaks out.

We refer to [9] for an overview of gas related problems and to [11] and [12] for a survey on contract portfolio management problems. Albeit not straighforwardly applicable to our problem, for the sake of completeness we now review some of the NG asset valuation literature.

For gas sale retailers operating in the Italian market, the model considered in [3] finds optimal commercial policies in a deterministic framework; a risk-neutral stochastic formulation is developed in [16]. Since retailer companies are of reduced size, they do not have the capacity of storing gas, which considerably simplifies the optimization process describing company operations. But when storage is involved, any multistage stochastic NG model is doomed to suffer from the curse of dimensionality, due to the presence of many state variables.

As explained in [18], valuation and hedging of real option cash flows rely on the risk-neutral dynamics of state variables (the various gas reservoirs in our application). For this reason, usual approaches for NG valuation from the Real Option literature, such as in [8, 22], are not applicable in our setting, because decisions on the cancellations have to be robust with respect to demand fluctuation. Furthermore, we are taking the manager's point of view under a highly incomplete market context.

Since NG storages are not traded in liquid markets, in [5] the valuation of working capacity of NG storages is modeled by replicating nontraded components of the storage option with traded instruments. The approach is justified by the fact that owning storage combined with the ability to transact in the physical market is similar to owning straddles. The optimization model is a deterministic linear programming problem. Uncertainty in the bid and ask prices is dealt with by Monte Carlo simulation, using historical records to run different linear programs, and by making a sensitivity analysis. Only one NG storage is valued, without hedging, and without a cancellation option, because LNG contracts are not considered. In [1] the problem of evaluating an investment related to NG is addressed. Since the focus is on valuing the investment, no hedging is considered in the work. Furthermore, the model does not include operational constraints, neither demand satisfaction by various NG storages nor LNG contracts.

Large-scale NG storage valuation with uncertain price dynamics is considered in the work [14]. To overcome the curse of dimensionality inherent to NG storage problems, the authors define an intrinsic rolling policy that is close to our rolling-horizon approach. The work assesses different heuristics from a trader's perspective and does not include cancellation options for LNG contracts. Moreover, for our application, a basic assumption of the model, requiring NG storage facilities to be located near a liquid wholesale spot market where the company performs its physical trading, is not satisfied.

We finish this short review by mentioning the valuation proposal in [6], which also views NG storages as straddles on gas prices. However, unlike the previous references, the optimal control model from [6] considers 0-1 variables, to model the ability to inject or withdraw gas. Another distinctive feature of the approach is that it can handle the so-called path-dependence of NG inventory. Such wording refers to the fact that the actual NG level in the reservoir depends on the history of past injections and withdrawals. In our context, this model would amount to having one reservoir with the full contracted LNG, to be depleted as loads are being shipped, without injections, using only withdrawals. The *optimal switching* of [6] would therefore correspond to determining cancellation dates for the contract under consideration. Unfortunately, in our problem, the company manager needs to choose contracts in a *set*, thus increasing the dimensionality of the state variable, to represent a "vectorial" reservoir. The state dimension is further increased because uncertainty dynamics (defining the aforementioned paths) is not Markovian. For our application, the stochastic process representing the clients' demand is not timewise memoryless. This is due to the fact that streamflows into the hydro-reservoirs are periodically autoregressive; we refer to Section 4.2 for more details.

3. MODELLING NG ASSETS IN THE COMPANY PORTFOLIO

For a major company operating in a vertically integrated context of gas and energy, we now formalize the problem of choosing, among various LNG contracts, which ones suit the company's needs better. In what follows, $t = 1, \ldots, T$ is a time step, while n and ℓ correspond to LNG contracts and LNG loads, respectively.

3.1. Network representation and customers.

Network representation and customers. The company owns a gas distribution network whose operation is described schematically in Figure 1.

In addition to several NG gas storage units (corresponding to the box labelled "C" in the figure), the company has an NG contract (box B), an alternative source of energy modelled as a reservoir (F in the figure), and some potential future LNG loads, which might be cancelled (box E). Once a ship arrives with a LNG load, it is docked and constitutes a temporary above-ground storage unit until fully depleted (box D). Alternative energy, typically fuel, is delivered directly to the clients, which are segmented into four different groups (G in the figure). Finally, gas is delivered to the clients through the network of pipelines (A in the figure). Pipelines and storages have known minimum and maximum capacities.

Table 3 in the Appendix summarizes the model notation. Basically, to each reservoir corresponds a (continuous) state variable, denoted by s. Injections and withdrawals in the reservoirs are represented

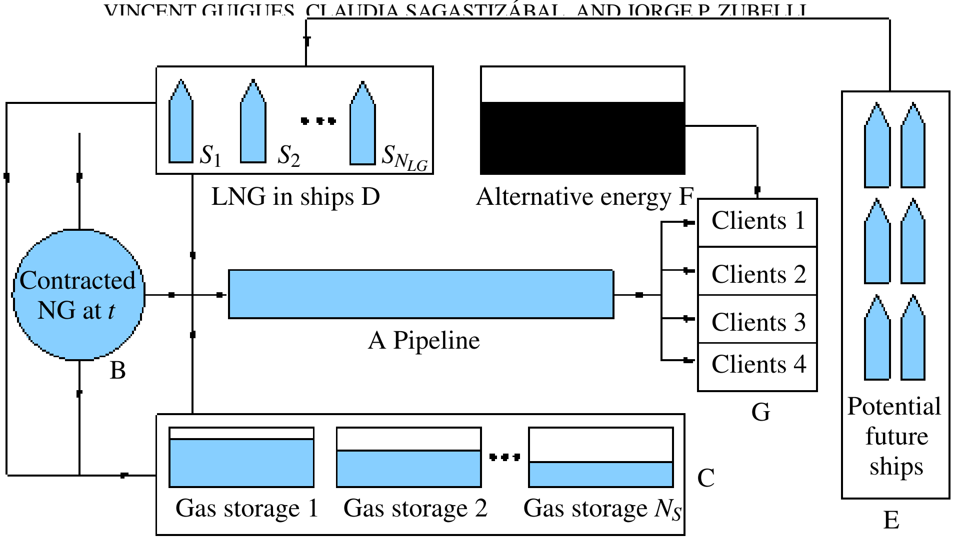

FIGURE 1. Gas supply network

by (continuous) control variables, denoted by u. Finally, there are binary control variables for each LNG contract, denoted by y. Figure 2 shows how the reservoir control variables modify the NG flow in the network from Figure 1.

The nodes with capital letters A to G in Figure 2 correspond to the references in the network from Figure 1. For example, the incoming arrows for node G stand by the fact that, at each time step t, clients receive either gas from the pipeline (A), or the alternative source of energy (F). The situation is similar for node C in the figure, although more involved. There is one outgoing arrow, u_{Si}^{-t}, for withdrawals from the i-th NG storage into the pipeline. The three incoming arrows, $u_{Lj}^{i\,t}$, $u_{n\ell}^{i\,t}$, and u_{Si}^{+t}, show that the level of the i-th NG storage increases with injections from:

– regasified LNG from the docked ships, ($u_{Lj}^{i\,t}$ is a fraction of the load of the j-th ship);

– regasified LNG from potential ships, corresponding to the ℓ-th load of contract n; and

– gas from the NG contract (u_{Si}^{+t}).

Finally, demand is segmented into four groups of clients, with firm contracts having different prices and penalties (depending affinely on the NG spot price):

Group 1: Customers whose gas can be replaced by an alternative energy source (usually more expensive than NG). These clients pay the contracted price even if they get the alternative energy source

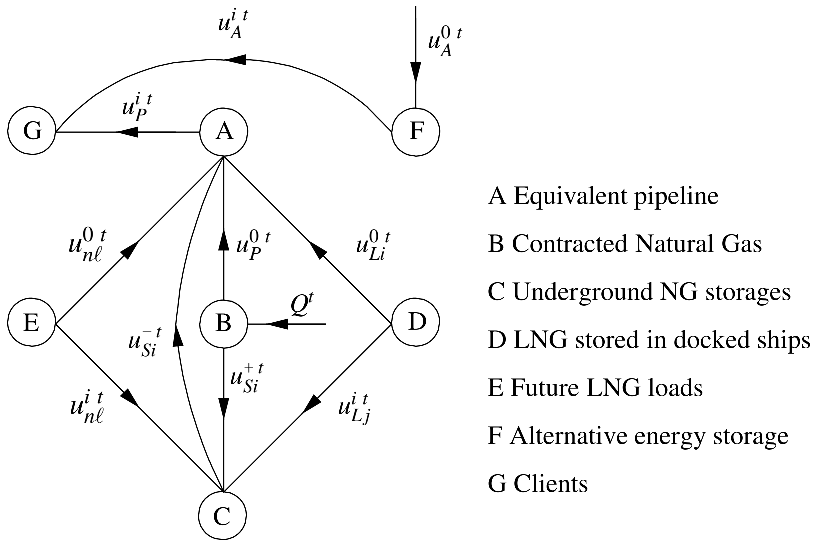

FIGURE 2. Variables controlling the multi-commodity flow in the network

and their demand is always satisfied.

Group 2: Customers whose gas can be left undelivered without paying any penalty.

Group 3: Customers whose unsatisfied demand involves a high penalty cost.

Group 4: Thermal power plants fueled by gas with high penalties to pay when their demand, which is highly seasonal, is left unsatisfied.

3.2. **LNG contracts with cancellation options.** As shown in Figure 2, at each time step a deterministic quantity Q^t of NG is injected into the system. The amount of LNG gasified and injected into the pipelines is uncertain and depends on each LNG contract.

Suppose there are N different contracts. For $n = 1, \ldots, N$, the n-th contract offers the possibility to receive $\ell_n \geq 1$ gas loads of volume $Q_{n\ell}$. The starting time step for the contract, after which cancellations are possible, is $t_{n0} \geq 0$.

Let $y^t_{n\ell}$ denote a binary variable which is set to 1 if the load is cancelled at time step t or before, and which is 0 otherwise. Loads are scheduled for arrival at pre-specified dates, $t_{n1} \leq t_{n2} \leq \ldots \leq t_{n\ell_n}$. If some load is not to be sent, it must be cancelled no later than time $t_{n\ell} - 1$, that is, at least one time step earlier than the forecasted delivery date. As a result, the relations

$$\begin{cases} y_{n\ell}^t \geq y_{n\ell}^{t-1}, & t = t_{n0}+2, \ldots, t_{n\ell}-1, \\ y_{n\ell}^t \in \{0,1\}, & t = t_{n0}+1, \ldots, t_{n\ell}-1 \end{cases}$$

hold.

Remark 3.1. *For future use, note that at time step t the number of different subvectors with 0-1 components will be equal to*

$$2^{I^t} \quad \textit{where} \quad I^t := \sum_{n=1}^{N} \sum_{\ell=1}^{\ell_n} \Big| \{t\} \cap [t_{n0}+1, t_{n\ell}-1] \Big| . \tag{1}$$

When $I^t = 0$, there are no 0-1 variables at time step t. □

The dynamic depletion of the ship is modeled with state and control variables $s_{n\ell}^t$ and $u_{n\ell}^{0\,t}$, representing, respectively, the gas available on the ship at the end of time step t, and the gas taken from the ship and injected into the pipelines at time step t. The state variable is null if the load is cancelled before the delivery date. The affine transition equations involve the binary control variable $y_{n\ell}^{t_{n\ell}-1}$:

$$\begin{cases} s_{n\ell}^t = \left(1 - y_{n\ell}^{t_{n\ell}-1}\right) Q_{n\ell} & \text{if } t = t_{n\ell}-1\,, \\ s_{n\ell}^t = s_{n\ell}^{t-1} - \sum_{i=0}^{N_S} u_{n\ell}^{i\,t} - \left(\left(1 - y_{n\ell}^{t_{n\ell}-1}\right) Q_{n\ell} - w_{n\ell}^t\right) \mathrm{Evap}^t\,, & \text{if } t = t_{n\ell}, \ldots, T\,. \end{cases}$$

Here, $\mathrm{Evap}^t \in [0,1)$ stands for the fraction of the initial load that evaporated at time t, and N_S denotes the number of underground storage units, with associated control variables $u_{n\ell}^{i\,t}$. Slack variables $w_{n\ell}^t$, introduced for feasibility reasons only if there is evaporation, are penalized in the objective function.

Each LNG shipment involves cancellation, purchasing, and docking costs, as shown by Figure 3.

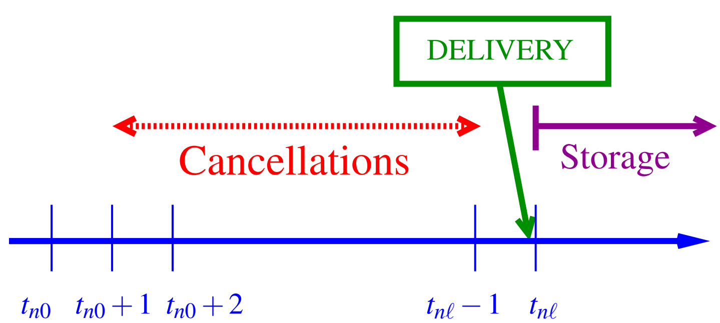

FIGURE 3. LNG contract dynamics

Cancellation fees and unit purchasing prices are uncertain and depend affinely on some index spot price on the delivery date, $\mathscr{S}^t$, or on a stochastic variable proxied by such an index. The corresponding slopes and intercepts are defined in the contract. We denote such costs by $f_{n\ell}(\mathscr{S}^t)$ and $p_n(\mathscr{S}^t)$, respectively. Likewise, for LNG loads, unit buying prices and cancellation fees take the form

$$\text{(2)} \qquad \begin{cases} p_n(\mathscr{S}^t) &= \mathscr{S}^t K_n + k_n^t, \\ f_{n\ell}(\mathscr{S}^t) &= \beta_{n\ell}^t\,(\mathscr{S}^t K_n + k_n^t)\,Q_{n\ell}, \end{cases}$$

for known coefficients $(K_n, k_n^t, \beta_{n\ell}^t)$, with K_n and k_n^t such that $p_n(\mathscr{S}^t) > \mathscr{S}^t$ and $\beta_{n\ell}^t \in (0,1)$ a non-decreasing function of time. In particular, the fee to be paid is proportional (increasing with time) to the value of the load at the cancellation time. If for some contract fees are deterministic, the corresponding coefficient K_n is null. The interest of the relations in (2) is that uncertainty in the objective function is fully captured by the stochastic process of spot prices along the optimization period, that is by $(\mathscr{S}^1, \dots, \mathscr{S}^T)$.

If there were a cancellation, the corresponding fee would be paid at cancellation time and would result in the cost:

$$\text{(3)} \qquad \mathsf{C}_{n\ell}^t = \begin{cases} y_{n\ell}^t f_{n\ell}(\mathscr{S}^t) & \text{if } t = t_{n0}+1 \\ (y_{n\ell}^t - y_{n\ell}^{t-1}) f_{n\ell}(\mathscr{S}^t) & \text{if } t = t_{n0}+2, \dots, t_{n\ell}-1\,. \end{cases}$$

When the load is delivered, it involves a purchasing cost. Once the ship arrives, it stays in place until all of its LNG load has been used, paying a docking cost, linear in time, to discourage a too long gas confinement on ships. These operations induce the costs below:

$$\text{(4)} \qquad \mathsf{C}_{n\ell}^t = \begin{cases} \left(1 - y_{n\ell}^{t_{n\ell}-1}\right) p_n(\mathscr{S}^t) Q_{n\ell} + \alpha_{n\ell} s_{n\ell}^t & \text{if } t = t_{n\ell}\,, \\ \alpha_{n\ell} s_{n\ell}^t & \text{if } t = t_{n\ell}+1, \dots, T\,; \end{cases}$$

where $\alpha_{n\ell}$ is the storage unit cost.

Remark 3.2. *For $t = t_{n\ell}-1$, we see from* (3) *and* (4) *that the portion of the objective function depending on the cancellation variable is $y_{n\ell}^t(f_{n\ell}(\mathscr{S}^t) - p_n(\mathscr{S}^{t+1})Q_{n\ell})$. This is an affine function of both $\mathscr{S}^t$ and $\mathscr{S}^{t+1}$. Likewise for the cost prior to the cancellation deadline, that is for $t \in \{t_{n0}+1, \dots, t_{n\ell}-2\}$.*

For these reasons, the costs before the delivery date depend on **current and future** *realizations of the uncertainty:* $\mathtt{C}^t_{n\ell} = \mathtt{C}^t_{n\ell}\left(\mathscr{S}^t, \mathscr{S}^{t+1}\right)$ *for* $t \in \{t_{n0}+1, \ldots, t_{n\ell}-1\}$. □

3.3. **Non-LNG storage dynamics.** The LNG reservoir for future shipments was modelled in Section 3.2. We now consider the remaining reservoirs, with associated nonnegative state and control variables related by affine transition equations.

Gas can be stored in four types of reservoirs: LNG loads available at $t = 0$ waiting in ships, the alternative energy storage, underground NG storage units, and the equivalent pipeline. Managing these reservoirs has a cost that is linear in their state and control variables. In particular, the unit cost for alternative energy depends affinely on the NG spot price: $p^t_A = p_A(\mathscr{S}_t)$.

3.3.1. *Alternative energy and initial ships.* The state equations for the alternative source of energy and for the N_{LG} ships available at the beginning of the optimization period are given by

$$s^t_{Li} = s^{t-1}_{Li} - \sum_{j=0}^{N_S} u^{j\,t}_{Li} - \mathrm{Evap}^t(Q_{Li} - w^t_{Li}), \quad s^t_A = s^{t-1}_A - \sum_{i=1}^{4} u^{i\,t}_A + u^{0\,t}_A,$$

for $t = 1, \ldots, T$. Here, s^t_{Li} is the storage level in the i-th of the N_{LG} ships at the end of time step t, Q_{Li} its load, $u^{0\,t}_{Li}$ the gas taken from ship i and injected into the pipeline, and $u^{j\,t}_{Li}$ the gas transferred from ship i to underground storage $j \in \{1, \ldots, N_S\}$. Similarly, s^t_A is the available alternative energy at the end of time step t, $u^{0\,t}_A$ the alternative energy bought, and for $i = 1, \ldots, 4$, $u^{i\,t}_A$ is the alternative energy supplied to clients from segment i, with $u^{i\,t}_A = 0$ for $i = 2, 3, 4$ (only clients in Group 1 receive alternative energy).

3.3.2. *Underground NG storage units.* The transition equation for the i-th underground storage reflects gas transferred to the pipeline (u^{-t}_{Si}), as well as gas injected into the storage (NG, u^{+t}_{Si}, or regasified LNG coming from the ships):

$$s^t_{Si} = s^{t-1}_{Si} - u^{-t}_{Si} + u^{+t}_{Si} + \sum_{j=1}^{N_{LG}} u^{i\,t}_{Lj} + \sum_{\{(n\ell)\,|\,t_{n\ell} \le t\}} u^{i\,t}_{n\ell},$$

for $t = 1, \ldots, T$, and $i = 1, \ldots, N_S$. There are "tunneling" constraints, to keep the storage state in predefined lower and upper bounds, that can vary with the season. Losses can be modeled too, multiplying the incoming control variables $(u^{+t}_{Si}, u^{i\,t}_{Lj}, u^{i\,t}_{n\ell})$ by a factor, possibly varying with time.

3.3.3. *Equivalent pipeline.* If $u_P^{0\,t}$ is the contracted NG injected into the pipeline, the injections in the pipeline at time step t are

$$In^t = u_P^{0\,t} + \sum_{\{(n\ell)\,|\,t_{n\ell}\le t\}} u_{n\ell}^{0\,t} + \sum_{i=1}^{N_S} u_{Si}^{-t} + \sum_{i=1}^{N_{LG}} u_{Li}^{0\,t}\,.$$

Finally, the (more involved) transition equation of the equivalent pipeline, with state s_P^t at the end of this time step, is

$$s_P^t = s_P^{t-1} - \sum_{i=1}^{4} u_P^{i\,t} + In^t \tag{5}$$

where $u_P^{i\,t}$ for $i = 1,\dots,4$ is the gas delivered to clients in segment i. If, in addition, $\underline{\mathrm{F}}_+^t$ (resp. $\underline{\mathrm{F}}_-^t$) and $\overline{\mathrm{F}}_+^t$ (resp. $\overline{\mathrm{F}}_-^t$) are the minimal and maximal entrance (resp. exit) flows; and $\mathrm{G}_{\max}^t$ is the maximal quantity of gas that can be regasified at time step t, the flow constraints for the pipeline are of the form

$$\begin{cases} \underline{\mathrm{F}}_+^t \le In^t \le \overline{\mathrm{F}}_+^t\,, \quad \underline{\mathrm{F}}_-^t \le \sum_{i=1}^4 u_P^{i\,t} \le \overline{\mathrm{F}}_-^t\,, \\ \sum_{i=1}^{N_{LG}}\sum_{j=0}^{N_S} u_{Li}^{j\,t} + \sum_{\{(n\ell)\,|\,t_{n\ell}\le t\}} \sum_{i=0}^{N_S} u_{n\ell}^{i\,t} \le \mathrm{G}_{\max}^t\,, \quad u_P^{0\,t} + \sum_{i=1}^{N_S} u_{Si}^{+t} = Q^t\,. \end{cases} \tag{6}$$

3.4. **Demand satisfaction and objective function.** At time step t, for clients in segment i, we let $u_{NS}^{i\,t}$ denote the demand left unsatisfied (equal to 0 if $i = 1$); $\mathscr{D}^{i\,t}$ the gas demand, so that demand satisfaction induces the constraints

$$u_{NS}^{i\,t} + u_P^{i\,t} + u_A^{i\,t} = \mathscr{D}^{i\,t}, \tag{7}$$

for $i = 1,2,3,4$ and $t = 1,\dots,T$, with $u_{NS}^{1\,t} = 0$. Finally, we suppose that for each client, the unit selling price $\pi^{i\,t}$ and shortage cost $\delta^{i\,t}$ depend affinely on the spot price:

$$(\pi^{1\,t},\pi^{2\,t},\pi^{3\,t},\pi^{4\,t},\delta^{1\,t},\delta^{2\,t},\delta^{3\,t},\delta^{4\,t},p_A^t)^\top = \mathscr{S}^t K_0 + k_0^t\,.$$

Putting together all the elements above, the gas portfolio problem has an objective function given by some penalties involving the slack variables and the terms

$$\sum_{n=1}^{N}\sum_{\ell=1}^{\ell_n}\sum_{t=1}^{T} \frac{1}{(1+r)^t}\mathtt{C}_{n\ell}^t + \sum_{t=1}^{T}\frac{1}{(1+r)^t} p_A(\mathscr{S}^t)u_A^{0\,t} + \sum_{i=1}^{4}\sum_{t=1}^{T}\frac{1}{(1+r)^t}\Big(\delta^{i\,t}u_{NS}^{i\,t} - \pi^{i\,t}\left(u_P^{i\,t} + u_A^{i\,t}\right)\Big)$$

where the constat interest rate is denoted by r and the costs $\mathtt{C}_{n\ell}^t$ are those in Section 3.

As explained in Remark 3.2, at some time steps the LNG contracts have a cost depending on future realizations of the stochastic process $\mathscr{S}$. For this reason, and for notational simplicity, in what follows the gas portfolio objective function will be denoted in the shorter form

$$\sum_{t=1}^{T} \mathtt{C}^t(\mathscr{S}^t, \mathscr{S}^{t+1})^\top z^t\,, \tag{8}$$

for a decision vector z^t resulting from joining the various subvectors s^t, y^t, u^t from Section 3, and for vectors $\mathtt{C}^t$ of appropriate dimensions, depending on current and future realizations of the spot price.

4. Stochastic modelling of uncertainty

As mentioned, uncertainty has two distinct sources: volatility of gas prices on the international market and fluctuations in clients' demand for gas on the local market (Brazil in our specific example). In view of our modelling, price-risk appears only in the objective function (8), while volume-risk is given by the contraints (7). We now consider how to model and hedge each type of uncertainty.

4.1. **Natural Gas spot prices.** In our specific application, the stochastic process of the NG spot price is modelled by the two-factor Schwartz and Smith model [17]. This model has been used for modelling oil prices [17], natural gas [7] and commodities such as corn, soybeans, and wheat [20]. The Schwartz and Smith (S&S) model decomposes the logarithm of the spot price $\ln \mathscr{S}^t = se^t + \xi^t + \chi^t$ into a seasonal part se^t and two unobserved processes, ξ^t and χ^t. The first process is modelled as a geometric Brownian motion $d\xi^t = \mu_\xi dt + \sigma_\xi dW_\xi$, while the second one is an Ornstein-Uhlenbeck process $d\chi^t = -\kappa \chi^t dt + \sigma_\chi dW_\chi$, with W_ξ and W_χ correlated Brownian motions, i.e., satisfying the relation $dW_\chi dW_\xi = \rho_{\chi\xi} dt$. In [17] it is shown that the corresponding expectation, given χ_0 and ξ_0, has the explicit expression

$$\begin{aligned} \ln(\mathbb{E}[\mathscr{S}^t | \chi^0, \xi^0]) \quad = \quad & \exp(-\kappa t)\chi^0 + \xi^0 + \mu_\xi t + \tfrac{1}{2}(1 - \exp(-2\kappa t))\tfrac{\sigma_\chi^2}{2\kappa} \\ & + \tfrac{1}{2}\left(\sigma_\xi^2 t + 2(1 - \exp(-\kappa t))\tfrac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right). \end{aligned} \tag{9}$$

Model calibration. For commodities such as NG, it is generally assumed that the spot price is not available (there is in general a lack of reliable spot price time series and there are different markets with different transaction processes and prices). There can also be local peak prices in local markets and a lack of liquidity. The usual way of dealing with the non-observable nature of spot prices is to use the closest future prices as an approximation. With the S&S model, such prices (observable) are known functions of the unobserved processes χ^t and ξ^t. The calibration of the S&S model is then done based on maximum likelihood, by using the Kalman filter [13] or particle filters [2] to restore the unobserved processes. We refer to [18] for a different approach, focusing on the valuation and hedging performance of a family of multifactor models in the context of a specific real option.

Hedging price-risk. Price-risk appears in the objective function (8), so uncertain future values are controlled by their Conditional Value-at-Risk (CVaR). If the continuous random variable $\mathscr{X}$ represents a cost, [23] shows that its Conditional Value-at-Risk of confidence level ε is given by

$$CVaR_\varepsilon(\mathscr{X}) = \min_{w_0\in\mathbb{R}}\left(w_0 + \frac{1}{\varepsilon}\mathbb{E}(\mathscr{X} - w_0)^+\right) \tag{10}$$

where $(z)^+ = \max(z,0)$ is the positive-part function. After calibration of the S&S model, by generating NP scenarios for the spot price, (9) gives an empirical estimate for the expectation in (10); see (16) below.

4.2. **Natural Gas demand.** The volume-risk, given by the contraints (7), is modelled by generating future demand scenarios over the optimization period. To generate such scenarios for each client segment, we consider separately the first three groups of clients, for which no historical data is available in our application, and the fourth segment, composed of thermal power plants.

For the first three segments, future demand slowly increases with a seasonal component, according to a Buys-Ballot model:

$$\mathscr{D}^{i\ t} = \mathscr{D}^i_{init} + \mu_i t + \mathscr{D}_S^{i\ t} + \varepsilon^{i\ t}, \tag{11}$$

where $\mathscr{D}_S^{i\ t}$ is one year periodic on t and noises $\varepsilon^{i\ t} \sim \mathscr{N}(0,\sigma_i^2)$ are independent (we assume the parameters of the model are known). These processes are discretized to sample $ND_i = 200$ scenarios from the distribution of $(\varepsilon^{i\ 1},\ldots,\varepsilon^{i\ T})$ for Group i. As a result, for all $i = 1,2,3$,

(12)

$$\forall t = 1,\ldots,T\ ,\ \text{given demand realizations } \widetilde{\mathscr{D}}^{i\ t}, \text{ future uncertain demands } \mathscr{D}_t^{i\ \tau} \text{ remain bounded.}$$

In this expression, time steps t and τ have different meanings, related to our rolling-horizon methodology, described in Section 5 below. For the moment, we just mention that the approach solves at each time step t a multi-stage problem with future stage variables in the horizon $[t+1,T]$. So at each time step t the clients demand $\widetilde{\mathscr{D}}^t$ is known and conditions uncertainty, corresponding to future time stages, $\tau = t+1,\dots,T$. The dependence of future demand on the current realization is made explicit by the subindex t in the notation $\mathscr{D}_t^{i\ \tau}$. Actually, the stochastic processes for the first three segments of clients is Markovian, making future uncertain demands in (12) independent of t. Such is not the case of the fourth group, corresponding to thermal power plants. The reason is that in a hydro-dominated power system like Brazil's, thermal power mainly complements hydro-power. So, depending on the streamflows arriving into the reservoirs over a given year, thermal energy may vary from zero up to the maximal thermal generation capacity. Accordingly, the fourth segment of demand depends on the streamflows' stochastic process, that is interstage dependent, usually periodic autoregressive and multivariate [15]. In our application we first discretize the autoregressive process of inflows into a tree with ND_4 scenarios. Then, the optimal thermal power generation is simulated over the tree, using a model described in [10]. The computed thermal dispatch scenarios are finally transformed into demand scenarios of gas by means of a suitable conversion factor.

With the construction above, relation (12) also holds when $i = 4$. Since we will make use of this boundedness property to write down the robust counterpart of the demand constraint, we state it as the following assumption

$$(A1) \qquad \begin{cases} \text{for all } t = 1,\dots,T \text{ and each } \tau = t+1,\dots,T\,, \\ \text{given a realization } \widetilde{\mathscr{D}}^t \text{, the future demand } \mathscr{D}^\tau \text{ remains in a box:} \\ \mathscr{D}^\tau \in \left[0, \overline{\mathscr{D}}_t^\tau\right] \text{ almost surely.} \end{cases}$$

Hedging volume-risk. As mentioned, the NG network should be designed to satisfy, most of the time, all of the clients' demand by delivering gas only, without using the expensive alternative energy. This rule applies even when customer demand is high. In such a context, the machinery of robust optimization [4] is at first sight suitable to control risk. For robust optimization to apply, constraints (7) must be reformulated so that uncertainty (the demand) appears only in inequality constraints.

A first observation is that, due to the particular structure of objective function (8) in the current setting, the reformulation with (7) written as inequality constraints can result in an optimum for which some of such inequality constraints are not binding. This indicates that the inequality reformulation is not equivalent to the original problem. More seriously, this means that the robust constraints have the pernicious effect of *over-delivering* some customers. This is because the model does succeed in delivering as much gas as possible, without using alternative energy. But then, since each group of clients only sets lower bounds on gas demand and since the gas buying price is lower than the gas selling price, a client can get more gas than requested.

Instead of the traditional approach, our robust counterparts for future demand constraints are

$$u_P^\tau + u_A^\tau + u_{NS}^\tau = \overline{\mathscr{D}}_t^\tau\,, \tau = t+1,\ldots,T\,, \tag{13}$$

where the vectorial equality holds for all the clients. By assumption (A1), the relation above is well defined. As for its physical meaning, it designs an NG portfolio able to respond to demand even at the highest peaks. This feature responds to the company manager's desire of providing to the clients only gas. But, since highest peaks are not frequent, NG storage units may be too full. As mentioned, the cancellation options of LNG contracts compensate for such conservatism.

Remark 4.1. *Alternative robust reformulations of the demand constraint, unsastisfactory for the physics of our problem, can be obtained by using* (7) *written for a future time step* $\tau \in [t+1,T]$ *to eliminate some variables. Suppose we choose the amount of gas delivered to clients and write the identity* $u_P^\tau = \mathscr{D}^\tau - u_{NS}^\tau - u_A^\tau$*. The right-hand side is then used in all inequality constraints involving* u_P^τ*, replacing* $\mathscr{D}^\tau$ *by its extreme values* 0 *and* $\overline{\mathscr{D}}_t^\tau$ *from Assumption (A1). More specifically, consider constraints* (6)*, written for the future period* $[t+1,T]$ *for an equivalent pipeline with capacity limited in* $[\underline{s}_P, \bar{s}_P]$*. Consider* $\tau = t+1,\ldots,T$*, then, because* $u_P^\tau \geq 0$*, the robust counterpart of the uncertain constraints amounts to setting*

$$\left\{\begin{array}{ll} 0 \geq u_A^{i\,\tau} + u_{NS}^{i\,\tau},\ i = 1,\ldots,4 & (a)\\ \sum_{i=1}^{4} \overline{\mathscr{D}}_t^{i\tau} - \overline{F}_-^\tau \leq \sum_{i=1}^{4}\left(u_A^{i\,\tau} + u_{NS}^{i\,\tau}\right) \leq -\underline{F}_-^\tau & (b)\\ \underline{s}_P + \sum_{k=t+1}^{\tau}\sum_{i=1}^{4} \overline{\mathscr{D}}_t^{ik} \leq \widehat{s}_P^{\,\tau} \leq \bar{s}_P\,, & (c)\\ \textit{where } \widehat{s}_P^{\,\tau} = \widehat{s}_P^{\,\tau-1} + In^\tau + \sum_{i=1}^{4}\left(u_A^{i\,\tau} + u_{NS}^{i\,\tau}\right) \textit{ if } \tau > t+1, \textit{ starting with } \widehat{s}_P^{\,t+1} = s_P^{t+1}. & \end{array}\right. \tag{14}$$

Constraint (c) forces the network to "keep" as much gas as possible, even in the case when the demand attains its upper bound for all the clients. Whenever $\underline{F}_-$ *is null (a realistic situation), satisfaction of* *(a) and (b) forces $u_{NS}^{\tau} = u_A^{\tau} = 0$, that is, there is no shortage and only gas can be delivered to the customers, as desired. However, by the inequalities in (c), the robust setting* (14) *also imposes the (unreasonable) relation*

$$\sum_{k=t+1}^{\tau} \sum_{i=1}^{4} \overline{\mathscr{D}}_t^{ik} \leq \overline{s}_P - \underline{s}_P \,.$$

This condition is unlikely to hold for large T, taking into account that pipelines have often a fixed capacity, so $\overline{s}_P = \underline{s}_P$. Other robust alternatives, for instance eliminating shortage variables u_{NS} in (7)*, also give robust models that are unsatisfactory for the physics of the problem.* □

5. THE ROLLING-HORIZON RISK-AVERSE OPTIMAL PORTFOLIO PROBLEM

For convenience, we write down the optimal portfolio problem in a more compact form. We start by recalling that, as explained in Remark 3.1, the decision variable z^t has 2^{I^t} binary components. The remaining components are continous and belong to a polyhedron, that we denote by $\mathscr{X}^t$. Furthermore, since all state equations and constraints in Section 3 are affine, for matrices A^t, B^t, E^t and a vector a^t of appropriate dimensions, the constraint set has the form

$$z^t \in \mathscr{X}^t \times \{0,1\}^{I^t} \,, \quad A^t z^t \geq B^t z^{t-1} + a^t \,, \quad E^t z^t = \mathscr{D}^t \,.$$

This representation makes explicit the fact that in our application inequality constraints are all deterministic and that the uncertain rightmost equality refers to the demand constraint (7).

With respect to uncertainty, we assume that the stochastic processes $\mathscr{S}$ and $\mathscr{D}$ are independent and have been discretized to define a multi-stage scenario tree.

We now explain how to build a rolling-horizon policy that is both feasible and risk-averse for the problem under consideration.

5.1. **Rolling horizon methodology.**

The rolling-horizon model is similar (but not identical) to the one in [10]. In the Stochastic Programming terminology, the methodology could be labeled “here-and-now looking-forward”. For comparison purposes, we first consider how a dynamic programming approach would look for a solution to a stochastic program defined over a multi-stage scenario tree.

Basically, dynamic programming makes a nested stagewise decomposition, using the Bellman equation. However, when uncertainty is not memoryless, but interstage dependent, state variables need to be augmented with the uncertainty history. Such is precisely the case for the stochastic process of the demand of thermal power plants and for the gas spot price process.

Instead, the rolling-horizon approach considers $T-1$ successive multi-stage problems, each one defined for a time $t = 1,2,\ldots,T-1$. In Figure 4, the left and right graphs correspond, respectively,

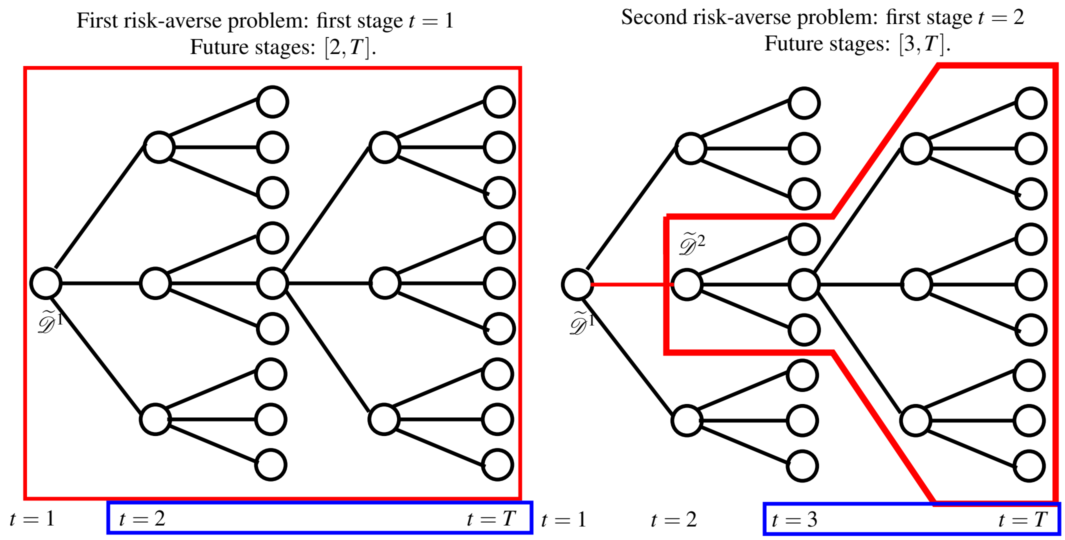

FIGURE 4. Successive multi-stage programs in the rolling-horizon approach.

to the first and second of such problems. For $t<T$, the t-th problem is a multi-stage program defined over the horizon $[t,T]$, with the first stage given by present time t (with deterministic constraints), and with $T-t$ subsequent stages covering the future $[t+1,T]$ (with uncertain constraints). In the figure, future stages are shown with a blue box. Similarly, the considered subtrees are surrounded by a red polyhedron, "descending" from t-th stage node until final stage T. Regarding Assumption (A1), the dependence of the bounds therein with respect to t is now made clear. For each $\tau = 3,\ldots,T$, the right graph in Figure 4 shows that (for the fourth group of clients) the maximal demand $\overline{\mathscr{D}}_t^\tau$ is the maximum over all nodes at time stage τ of the subtree in red, a subtree which depends on t via $\widetilde{\mathscr{D}}^{[t]}$, the history of realizations until time t.

5.2. **Risk-averse subproblems and policy.** For the t-th subproblem, z_t^t and $(z_t^{t+1},\ldots,z_t^T)$ denote the first and subsequent stage variables, respectively. Since the current constraint is deterministic and future constraints are uncertain, the feasible set is defined by

$$
\left\{
\begin{array}{llll}
z_t^t \in \mathscr{Z}^t \times \{0,1\}^{I^t}, & A^t z_t^t \geq B^t z_t^{t-1} + a^t, & E^t z_t^t = \widetilde{\mathscr{D}}^t, & \\
z_t^\tau \in \mathscr{Z}^\tau \times \{0,1\}^{I^\tau}, & A^\tau z_t^\tau \geq B^\tau z_t^{\tau-1} + a^\tau, & E^\tau z_t^\tau = \overline{\mathscr{D}}_t^\tau, & \text{for } \tau = t+1, \ldots, T.
\end{array}
\right. \tag{15}
$$

The initial state is known and is given by $z_t^{t-1} := \bar{z}_{t-1}^{t-1}$, the $(t-1)$-th component of the solution to the previous risk-averse program. As explained, constraint uncertainty is hedged with a worst-case approach for future demand, using the upper bound in Assumption (A1).

Also as mentioned, for handling uncertainty in the objective function (8), we set a conditional value-at-risk considering only the times steps relevant for the t-th problem. We obtain the following t-th risk-averse subproblem, depending on given parameters $\theta \in [0,1]$ and $\varepsilon \in (0,1)$:

$$
\left\{
\begin{array}{ll}
\min & \theta \mathbb{E}\Big(\sum_{\tau=t}^{T} \mathtt{C}^\tau(\mathscr{S}^\tau, \mathscr{S}^{\tau+1})^\top z_t^\tau\Big) + (1-\theta) CVaR_\varepsilon \Big(\sum_{\tau=t}^{T} \mathtt{C}^\tau(\mathscr{S}^\tau, \mathscr{S}^{\tau+1})^\top z_t^\tau\Big) \\
\text{s.t.} & (z_t^t, z_t^{t+1}, \ldots, z_t^T) \text{ satisfies } (15).
\end{array}
\right. \tag{16}
$$

In what follows, a particular realization of the NG spot price at time t is denoted by $\widetilde{\mathscr{S}^t}$, and a trajectory until time step t is denoted by $\widetilde{\mathscr{S}^{[t]}}$. Note that both the expectation and the CVaR in (16) are conditioned to the history $\widetilde{\mathscr{S}^{[t]}}$.

Each risk-averse problem is a deterministic MILP, with the important property that its decision variables $(z_t^t, \ldots, z_t^T)$ are of type “here-and-now”: they do not depend on future realizations of the underlying stochastic processes. With this modelling choice, basically done for tractability reasons, to define a feasible policy, only the t-th component of an optimal decision vector is used. Specifically, if $(\bar{z}_t^t, \ldots, \bar{z}_t^T)$ denotes a solution to the t-th risk-averse problem, then, after solving the T MILPs (16), the policy is defined by the decisions $(\bar{z}_1^1, \bar{z}_2^2, \ldots, \bar{z}_T^T)$. Under the assumption of relatively complete recourse, such a rolling-horizon risk-averse policy is also feasible.

6. Approximations and the algorithm

We now present a heuristic algorithm for solving approximations of the risk-averse problems (16). Such approximations refer not only to the estimation of the S&S model parameters, but also to the actual computation of the objective function terms in (16).

6.1. **Expectation and CVaR estimation.** Since in the model all relations are affine, each individual cost vector in problem (16) has the expression

$$\mathtt{C}^\tau(\mathscr{S}^\tau, \mathscr{S}^{\tau+1}) = a_1 \mathscr{S}^\tau + a_2 \mathscr{S}^{\tau+1} + a_3\,, \tag{17}$$

for certain vectors $a_1\,, a_2\,, a_3$, depending on both τ and t. By (9), an estimate $\widehat{\mathbb{E}}$ for the expected values in (17) is available. Since, in addition, the expectation operator is linear, the first term in the objective function in (16) has the following empirical estimate

$$\widehat{\mathbb{E}}\Big(\sum_{\tau=t}^{T} \mathtt{C}^\tau(\mathscr{S}^\tau, \mathscr{S}^{\tau+1})^\top z_t^\tau\Big) = \sum_{\tau=t}^{T}\Big(a_1 \widehat{\mathbb{E}}\left(\mathscr{S}^\tau | \widetilde{\mathscr{S}}^{[t]}\right) + a_2 \widehat{\mathbb{E}}\left(\mathscr{S}^{\tau+1} | \widetilde{\mathscr{S}}^{[t]}\right) + a_3\Big)^\top z_t^\tau\,. \tag{18}$$

The CVaR estimation requires an additional approximation, based on the equivalent minimization formulation in (10). Specifically, given a trajectory realization until time step t, say $\widetilde{\mathscr{S}}^{[t]}$, the full spot price process is simulated by generating NP realizations of prices between times $t+1$ and T:

$$\widetilde{\mathscr{S}}_1^{[T]}, \widetilde{\mathscr{S}}_2^{[T]}, \ldots, \widetilde{\mathscr{S}}_{NP}^{[T]}\,.$$

By relation (17), price scenarios also sample the cost terms, yielding in (10) the estimation

$$\begin{aligned} CVaR_\varepsilon\Big(\sum_{\tau=t}^{T} \mathtt{C}^\tau(\mathscr{S}^\tau, \mathscr{S}^{\tau+1})^\top z_t^\tau\Big) \quad &\approx \quad \min_{w_0 \in \mathbb{R}} \left(w_0 + \frac{1}{\varepsilon} \frac{1}{NP} \sum_{i=1}^{NP} (\mathtt{C}_i^{\tau\,\top} z_t^\tau - w_0)^+ \right) \\ &\qquad \text{with } \mathtt{C}_i^\tau := a_1 \widetilde{\mathscr{S}}_i^\tau + a_2 \widetilde{\mathscr{S}}_i^{\tau+1} + a_3 \end{aligned} \tag{19}$$

for $i = 1, \ldots, NP$.

6.2. **Rolling Horizon Risk-Averse Algorithm.** The algorithm is initialized by selecting risk aversion coefficients $\theta \in [0,1]$ and confidence levels $\varepsilon \in (0,1)$, possibly depending on $t = 1, \ldots, T$. Initial storage levels s^0 and cancellation variables y^0 are given. For the first three groups of clients, $ND_{1,2,3}$ gas demand scenarios are generated, and the corresponding components for the upper bound in Assumption (A1) are computed for each time step. The time step counter is set to $t = 1$.

Algorithm 1 (ROLLING HORIZON RISK-AVERSE CANCELLATION STRATEGY)**.**

Step 1 (Statistical toolbox) Having the history of uncertainty realization until time step t (streamflows for the fourth segment of clients and NG spot prices $\widetilde{\mathscr{S}}^{[t]}$):

- generate ND_4 future scenarios of gas demand for the fourth segment to obtain the maximal demand $\overline{\mathscr{D}}_t^{\tau}$ for $\tau = t+1,\ldots,T$;

- calibrate the spot price model, to estimate the S&S parameters from Section 4.1; and

- generate NP spot price scenarios $\widetilde{\mathscr{S}}_i^{[T]}$, for $i = 1,\ldots,NP$.

Step 2 (Approximate risk-averse subproblem) For $\tau = t,\ldots,T$ compute the empirical expectation (18) and generate the samples $\mathtt{C}_i^{\tau}$, $i = 1,\ldots,NP$ from (19). Solve the following approximation of (16), on variables $(z_t^t, z_t^{t+1},\ldots,z_t^T)$ and $(w_0, w_1,\ldots,w_{NP})$:

$$
(20)\qquad \left\{
\begin{array}{ll}
\min & \theta\widehat{\mathbb{E}}\Big(\sum_{\tau=t}^{T}\mathtt{C}^{\tau}(\mathscr{S}^{\tau},\mathscr{S}^{\tau+1})^{\top}z_t^{\tau}\Big)+(1-\theta)\left(w_0+\frac{1}{\varepsilon}\frac{1}{NP}\sum_{i=1}^{NP}w_i\right)\\
\text{s.t.} & (z_t^t, z_t^{t+1},\ldots,z_t^T)\text{ satisfies (15)}, w_0\geq 0\\
 & w_i\geq 0 \text{ and } w_i\geq \mathtt{C}_i^{\tau\,\top}z_t^{\tau}-w_0 \text{ for } i=1,\ldots,NP.
\end{array}
\right.
$$

Step 3 (Policy definition) Let $\bar{z}_t$ be a solution to (20), split into subvectors $\bar{s}_t, \bar{y}_t, \bar{u}_t$. Save $\bar{u}_t^t$, the t-th component of the computed optimal controls for time step t.

Step 4 (Load cancellation) Apply any of the rules below:

(A) Cancel at time t all loads (n,ℓ) such that the component of $\bar{y}_t^t$ corresponding to load (n,ℓ) is equal to 1.

(B) For any time $\tau \in \{t,\ldots,T\}$, cancel at time τ all loads (n,ℓ) such that the component of $\bar{y}_t^{\tau}$ corresponding to load (n,ℓ) is equal to 1.

(C) Apply B above only at the first time step, when $t = 1$, thus fixing 0-1 variables for the subsequent problems (20). With this rule, Step 4 is skipped for all $t > 1$.

Step 5 (Update and Loop) If $t = T$, stop. Otherwise, increase t by 1, update reservoir levels s^t, and loop to Step 1. □

Some comments regarding Algorithm 6.2 are now in order. Step 4 applies three alternative heuristics to deal with 0-1 variables in an economic, yet reasonable, manner. Variant A uses the rolling-horizon philosophy from Section 5 for the 0-1 variables: the cancellation strategy at time step t is defined by the t-th component of a solution to (20). Variant B offers the possibility of eliminating more 0-1 variables, making the subsequent risk-averse problems (20) easier. As for Variant C, it is the simplest to implement, because it solves (20) with 0-1 variables *only* at the first time step. For $t > 1$, cancellation decisions are those obtained for $t = 1$ (seen as "here-and-now" decision variables) and problems (20) are linear programs.

The feasible sets considered by Variant A contain those considered by Variant B, which are in turn larger than the ones used with Variant C. Thus, the corresponding problems (20) are easiest to solve with Variant C, but they also give higher costs. However, because the robust counterpart of the demand constraint is formulated as an equality, even the most costly Variant C avoids the pernicious effect of over-delivery, mentioned in Section 4.2,

We finish by mentioning that when $\theta = 1$ in (16), the number of 0-1 variables can be substantially reduced. By linearity of the corresponding objective function in (16), without CVaR term, it is possible to reformulate the problem using only one 0-1 variable per load, at the time step $t_{n\ell}^{\min}$, when the mean fee $\mathbb{E}[f_{n\ell}(\mathscr{S}^\tau|\widetilde{\mathscr{S}}^{[t]})]$ is minimal among all time steps $\tau = t+1,\ldots,T$. In this context, the sets I^τ in (15) have the expression $I^\tau := \sum_{n=1}^{N}\sum_{\ell=1}^{\ell_n} |\{\tau\}\cap\{t_{n\ell}^{\min}\}|$, to be compared with (1).

7. Pricing mechanism

We now explain a heuristic procedure for the company manager to determine a set of admissible prices for the portfolio of LNG contracts.

Pricing a contract amounts to establishing a fair price to pay for each load and the cancellation fees. As mentioned, these are affine functions of the NG spot price, with slope and intercepts in (2) specified by the LNG seller in the contract. It is therefore natural for the manager to request from the seller of the n-th contract a set of parameters considered acceptable from the seller's point of view. The seller then proposes a set with O_n configurations for the parameters

$$\Delta_n := \bigcup_{i=1}^{O_n} \left\{ \left(K_n^i, k_n^{i\,t}, \beta_{n\ell}^{i\,t} \text{ for } t = t_{n0}+1,\ldots,t_{n\ell}-1\,, \ell = 1,\ldots,\ell_n \right) \right\}. \tag{21}$$

The manager can then modify the optimal portofolio problem (16), written for $t = 1$, to price a set of contracts, determining which sellers and which of the configurations offered by the sellers are the most suitable for the company. Basically, any element in Δ_n becomes a decision variable, introducing a binary variable r_n^i, set equal to 1 if contract n with configuration i is taken, and set to 0 otherwise. The relation $r_n^i = 0$ is equivalent to $r_n^i = 1$, if all the contract loads are cancelled without paying a fee.

For this reason, the new feasible set in (16) has the additional constraints

$$\begin{cases} r_n^i \in \{0,1\} \text{ for } i=1,\ldots,O_n\,, n=1,\ldots,N \\ \sum_{i=1}^{O_n} r_n^i \le 1 \text{ for } n=1,\ldots,N \\ (r_n^i-1)(y_{n\ell}^t-1)=0, \text{ for } \ t=t_{n0}+1,\ldots,t_{n\ell}-1\,, \ell=1,\ldots,\ell_n\,, n=1,\ldots,N\,. \end{cases}$$

As for the objective function, terms (3) and (4) now include new binary variables:

$$\mathtt{C}_{n\ell}^t = \sum_{i=1}^{O_n} r_n^i \mathtt{C}_{n\ell}^{i\,t}$$

where $\mathtt{C}_{n\ell}^{i\,t}$ is the cost induced at time step t by load (n,ℓ) if configuration i of contract n is taken. Such a cost is obtained by replacing $f_{n\ell}(\mathscr{S}^t)$ and $p_n(\mathscr{S}^t)$ in (3) and (4) by the respective expressions in (2), written with coefficients $(K_n^i, k_n^{i\,t}, \beta_{n\ell}^{i\,t})$.

Suppose that the manager applies Algorithm 6.2 and uses Variant C as a cancellation strategy. The modified problem (16), written with $t=1$, is a bilinear program with mixed 0-1 variables, difficult to solve. We use a heuristic valuation, based on Monte Carlo simulation, in which the manager chooses a set R of *fixed* values $\bar{r}_n^i$ satisfying the first two constraints above for all the contracts and configurations. The corresponding 0-1 variables are now fixed, and the manager can generate a set of scenarios for both the spot prices and future demand, and apply Algorithm 6.2 to determine for each scenario the corresponding optimal management cost (as well as the cancellation dates and gas/alternative energy transfers). The process is repeated for as many different sets R as the manager desires to examine.

Furthermore, if desired, the manager can set an upper bound $\mathtt{C}^{\max}$ on the mean LNG management cost, induced by load purchases and/or cancellations. To compute such a mean cost, the probability $\Pi_{n\ell}^t(R)$, of cancelling load $(n\ell)$ at time step t under the set of configurations R, needs to be estimated. This can be done by simulating the optimal management strategy with Algorithm 6.2. Then, once the empirical estimations $\hat{\Pi}_{n\ell}^t$ are computed for each set R, the manager can then choose the set R with maximal mean income satisfying the desired bound

$$\sum_{n=1}^{N}\sum_{\ell=1}^{\ell_n}\left(\sum_{t=t_{n0}+1}^{t_{n\ell}-1}\frac{\hat{\Pi}_{n\ell}^t(R)}{(1+r)^t}\beta_{n\ell}^t\left(\mathbb{E}(\mathscr{S}^t)K_n+k_n^t\right)Q_{n\ell}\right.$$
$$\left.+\,(1-\sum_{t=t_{n0}+1}^{t_{n\ell}-1}\hat{\Pi}_{n\ell}^t(R))\left(\mathbb{E}(\mathscr{S}^{t_{n\ell}})K_n+k_n^{t_{n\ell}}\right)\frac{Q_{n\ell}}{(1+r)^{t_{n\ell}}}\right)\le \mathtt{C}^{\max}\,.$$

The left-hand side is the empirical mean cost (computed over the simulated scenarios) induced by the LNG contracts. When at Step 4 in Algorithm 6.2 loads are cancelled using Variant C, the probabilities $\Pi^t_{n\ell}$ are known, because cancellation decisions are taken at the first time step. Then, either $\Pi^t_{n\ell}(R) = 1$ if, under configuration R, load $(n\ell)$ is cancelled at time t, or the probability is 0 otherwise.

8. Numerical assessment

In this section we illustrate the methodology on an example based on realistic data for the Brazilian gas industry, for a company manager using Algorithm 6.2 with Variant C in Step 4. Specifically, for a time horizon of 2 years with monthly time steps ($T = 24$) and two LNG contracts ($N = 2$), we analyze the sensitivity of the model with respect to variations of the different parameters. In Section 8.2, we study how cancellation strategies vary with different slope and intercept configurations for the contracts, as well as with volatility in demand. Section 8.3 reports the cost distributions for different risk-aversion parameters, as well as a cost sensitivity study on the fees and buying prices (2).

The model was implemented in MATLAB with the Mosek optimization toolbox `http://mosek.com/`. This is a reliable general purpose solver that is free for academic use, with a built-in mixed-integer solver available only for evaluation purposes (combining Branch and Bound with relaxation techniques). In our simulations, a solution to (20) was often rapidly found with Mosek default settings. If, with these settings, the method did not converge, Algorithm 6.2 proceeded ahead, with the best feasible point found by Mosek.

8.1. Data and simulation protocol.

The LNG and NG contracts data is given in the third column in Table 3. There are two LNG contracts ($N = 2$), each one with $\ell_n = 22$ loads. The delivery dates for the first (resp. second) contract are $2i$ (resp. $2i+1$) for $i = 1,\dots,11$.

The equivalent pipeline has no initial gas, neither limits ($\underline{s}_P = 0$ and $\bar{s}_P = +\infty$). The maximal quantity of gas that can be regasified per day is 20 million cubic meters ($G^t_{\max} = 30.44 \times 20 \times 10^6$ cubic meters). The alternative energy source is not stored, so $s^t_A \equiv 0$.

For the first three groups of clients, parameters in (11) are given in Table 1 in the Appendix. To compute the maximal demands for the first three groups, a set of $ND_{1,2,3} = 200$ scenarios for each client is generated. The fourth segment, corresponding to thermal power plants, uses a scenario tree of inflows with $ND_4 = 5000$ scenarios and data from [10]. For illustration, some demand scenarios for Groups 3 and 4 are represented in Figure 5.

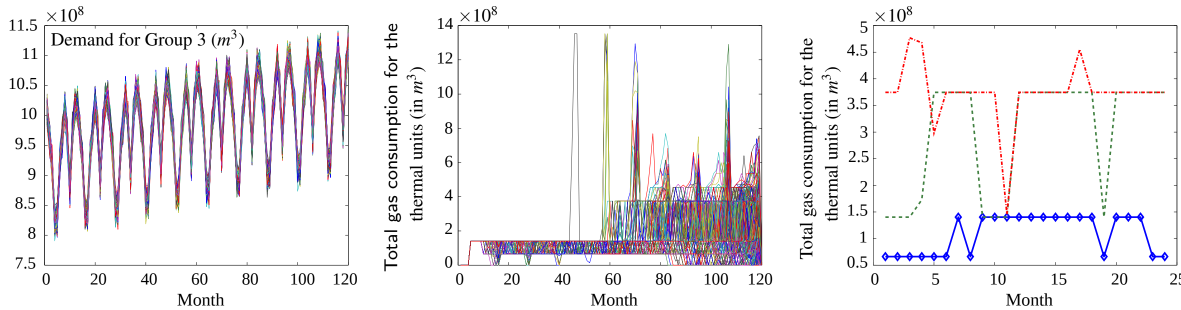

FIGURE 5. Monthly gas scenarios of demand, for customers in Group 3 (on the left), in Group 4 over 10 years (middle), and in Group 4 for the two years of the optimization period (right).

The S&S model for spot prices was calibrated for January 2nd, 2008 by using 6 years of 8 future prices and taking a seasonal part se^t as in Table 2 in the Appendix.

Figure 6 shows scenarios of Henry Hub spot prices, generated after the first time step, as well as an estimation of the maximal total demand.

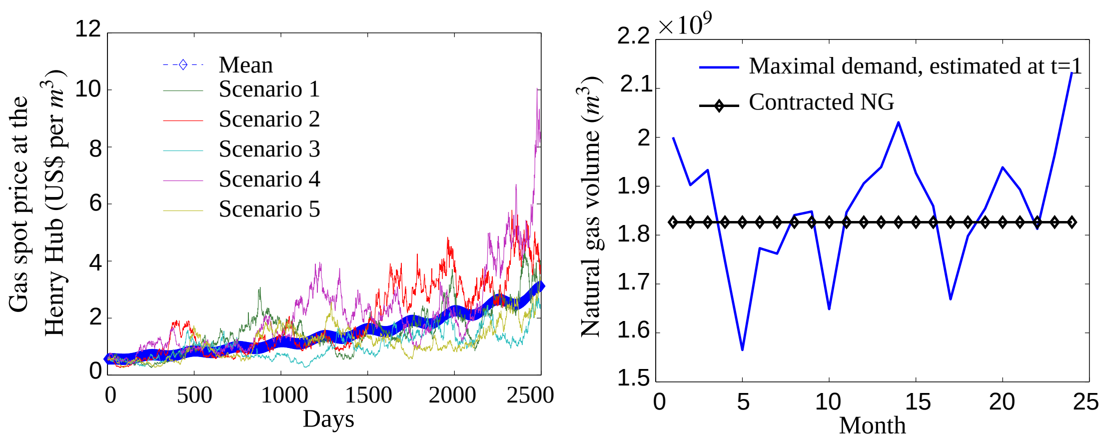

FIGURE 6. Spot price scenarios (left) and estimated maximal demand and contracted NG (right).

For a confidence level $\varepsilon = 0.1$, the CVaR estimation (19) uses $NP \in \{20, 100, 200\}$ scenarios depending on the study.

With this data, using $NP = 100$ scenarios to approximate the $CVaR_\varepsilon$, when $t = 1$ problem (20) has 253 binary and 915 continuous decision variables. There are 798 affine constraints (barring box constraints). The runs were done on a Dell PowerEdge 2900 server with 2 CPUs Intel Xeon E5345 (2.33 GHz, 8M of cache memory, 1333 MHz FSB), running under CentOS release 5, with 48 GB of RAM. The average CPU time to solve (exactly or approximately, depending on the instance) such an MILP was a few minutes, with large variability (for some instances, only a few seconds were needed, while others took several hours).

8.2. **Sensitivity of the cancellations to different parameters.** The graph on the right in Figure 6 shows that the contracted NG is not enough to satisfy maximal demands. Hence, we expect the model not to cancel all of the LNG loads. In order to see how the choice of different parameters affects the number of cancellations, in Figure 7 we report its sensitivity to variations of some of the data in the third column of Table 3, for three values of risk aversion parameter θ.

More precisely, to induce larger penalty fees, we multiply the coefficients $\beta^t_{n\ell}$ from (2) by a factor varying in the set $\{0.25, 0.5, 0.75, 1, 2+4i, i = 0, \ldots, 87\}$. The corresponding results are reported in the left graph in Figure 7.

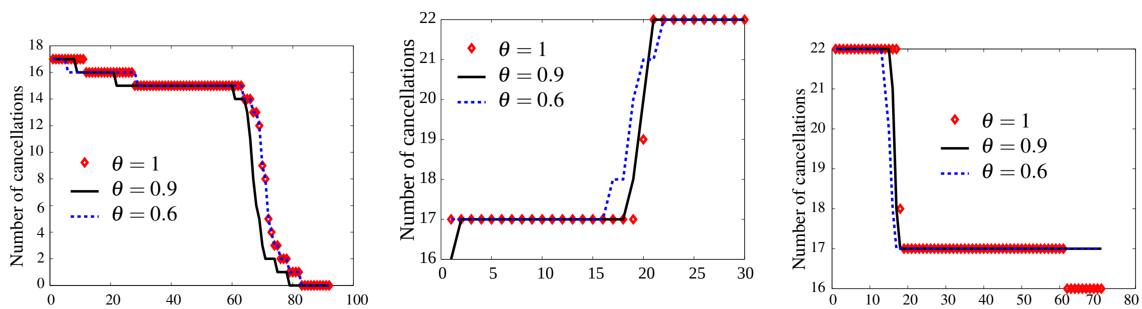

FIGURE 7. Cancellation sensitivity to the fees (left), NG buying prices (middle), and penalty levels (right).

We see in Figure 7 that in all cases the sensitivity is alike for the three different values of risk aversion. We also observe a natural behaviour in the left graph: the higher the fees, the lesser the cancellations. When fees are low, in particular for the initial values, only 5 loads are shipped (at the first time step) and 17 are cancelled. The shipped loads are enough to satisfy the maximal demand from time step 6 on. When fees are very large, it is more interesting to buy all the available loads, without any cancellation, even if these loads will not be used immediatly (such large fees are considered only to test the coherence of the cancellation policy, they would never be used in practice).

The middle graph in Figure 7 shows the number of cancellations is a non-decreasing function of the buying price. The NG purchasing price variation corresponds to taking K_n in (2) in the range $\{1.05+0.05i, i=0,\ldots,29\}$. As expected, when prices get high enough, all loads are cancelled. Finally, the graph on the right in Figure 7 shows a non-increasing relation of the number of cancellations on variations of the penalty levels $\delta^{i\,t} \in \{0.05(i-1)\mathscr{S}^t, i=1,\ldots,71\}$. Naturally, when penalties fall below a certain threshold, all loads are cancelled. Beyond this threshold, once the necessary loads (booked early) for satisfying the maximal demand have arrived, increasing the penalty no longer forces the system to ship more loads.

Figure 8 reports the sensitivity of cancellations to demand volatility (of customers in the first three segments) and to variations in the volume of contracted NG. The left graph in Figure 8 shows a high

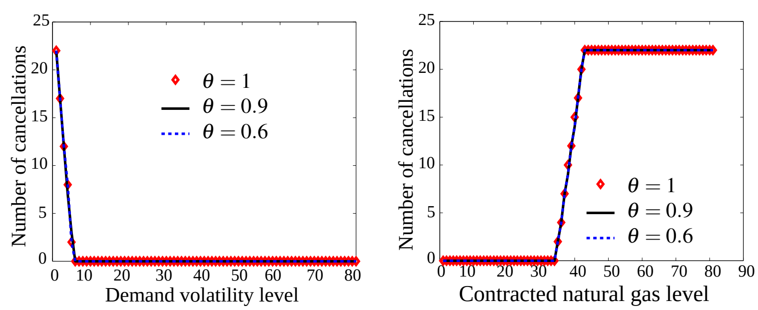

FIGURE 8. Cancellation sensitivity to demand volatility (left) and to contracted NG (right).

sensitivity to variations on the noise standard deviation σ_i in Table 1, chosen so that $\frac{\sigma_i}{\mathscr{D}^i_{init}} \in [0.005, 0.4]$. The fact that to higher demand volatility corresponds less cancellations is consistent with our model, designed to ensure a reliable supply of gas even in the worst case. In particular, when the volatility is very large (making higher demand more likely), not a single load is cancelled. As before, the behaviour is similar for the three tested values of risk aversion.

The sensitivity on NG volume variation is reported in the graph on the right in Figure 8, for contracted NG volumes ranging in $30.44 \times 60 \times 10^6 \times (0.8+0.005(i-1)), i=1,\ldots,81$. We see that the number of cancelled loads depends strongly on this parameter. When the NG contribution is sufficiently large, all of the 22 LNG loads are cancelled. We note that now the risk perception of the manager impacts the output. When $\theta = 1$ (risk neutral case), cancellation of a load always occurs as far as possible from the delivery date. When $\theta = 0.6$ and $\theta = 0.9$ (decreasing risk aversion), a few loads (almost none) are cancelled just two months (one month) before the delivery date.

Remark 8.1. *In the terminology of* [21]*, Figures 7 and 8 provide* willingness-to-buy *curves: from the left graph in Figure 7, for instance, it is possible to relate fees to amounts of LNG to buy. Since each load has the same volume, the multiplication of such a volume by the difference between the total number of loads and the number of cancellations gives the total volume of LNG the company is willing to buy for each given fee. A similar interpretation can be done for all figures in this section.*

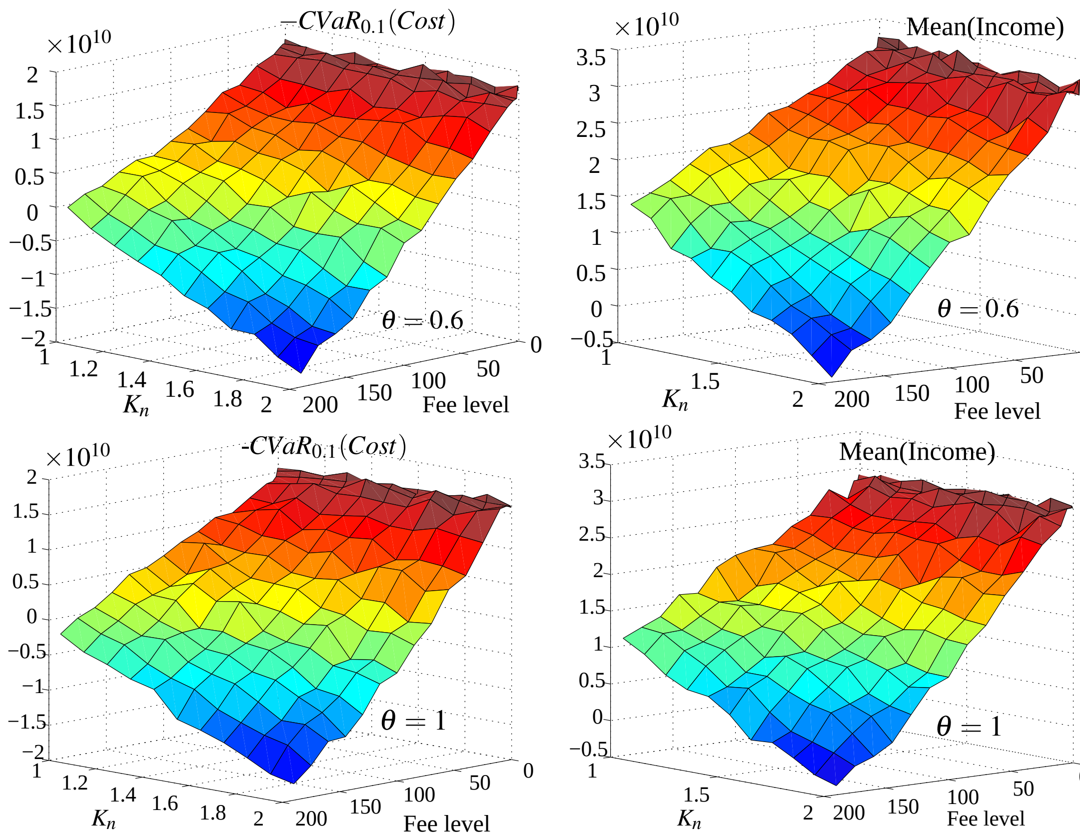

FIGURE 9. Sensitivity of the empirical mean and CVaR of the cost with respect to the fees and the gas buying price.

8.3. **Cost sensitivity and pricing.** In order to determine the impact of different contract configurations, we took different combinations of parameters K_n and $\beta^t_{n\ell}$ in (2). For these runs, the CVaR estimation (19) uses $NP = 200$ scenarios. We consider all combinations (buying price, fee) with $K_n \in$

$\{1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2\}$ and the fees $\beta_{n\ell}$ from Table 3, multiplied by $(0.5, 1, 2, 5)$ and $10 + 15(i-1)$ for $i = 0, \ldots, 13$.

Figure 9 reports the corresponding computed mean income and $-CVaR_{0.1}(Cost)$ (which is the empirical mean income for the 20 scenarios of lowest income). For a fixed fee $\beta_{n\ell}^t$, the income values naturally tend to decrease when K_n increases. Similarly, for a fixed price K_n, the incomes also tend to decrease when the fees increase. The graph also confirms the observation that the lowest admissible values will be chosen if the seller only fixes lower bounds for the set Δ_n, defining admissible prices. Otherwise, some of the values in the grid from Figure 9 may not be feasible, in which case the fees and prices providing the lowest mean income and CVaR, weighted by the risk aversion parameter θ, should be chosen.

The manager's perception of risk enters the problem by means of the parameter θ. Figure 10 shows the cost distribution, obtained with Variant C of Algorithm 6.2 on a set of 200 scenarios of demand and gas spot prices for $\theta \in \{1, 0.9, 0.6\}$, corresponding to a risk-neutral, moderately risk-averse and risk-averse manager.

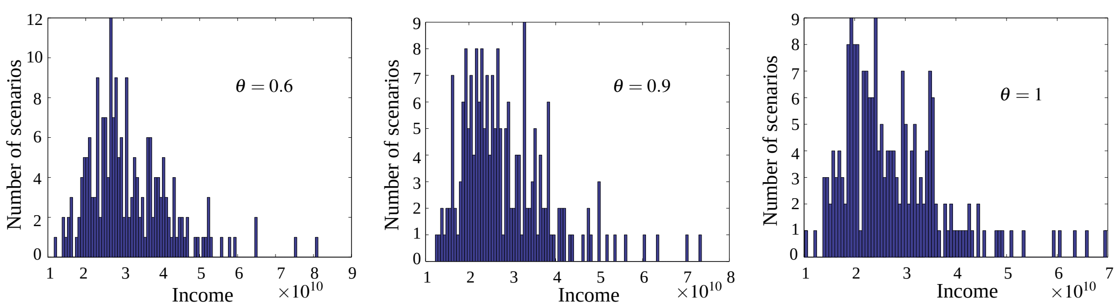

FIGURE 10. Empirical distribution of the cost for different values of θ.

We observe that all the distributions have a similar shape, in particular their empirical mean and $CVaR_{0.1}$ of the income are close. However, as expected, as θ gets lower and risk aversion increases, the estimate $-CVaR_{0.1}(Cost)$ gets higher. This is confirmed by the graph on the left in Figure 11, reporting the empirical estimation of the mean of the 20 scenarios of lowest income for $\theta \in [0.3, 1]$. Naturally, lower values of θ tend to limit the number of scenarios of very low income. An additional analysis from Figure 9 is reported in the graph on the right in Figure 11. More precisely, assume that the seller already proposed a contract configuration, for instance corresponding to the values in Table 3.

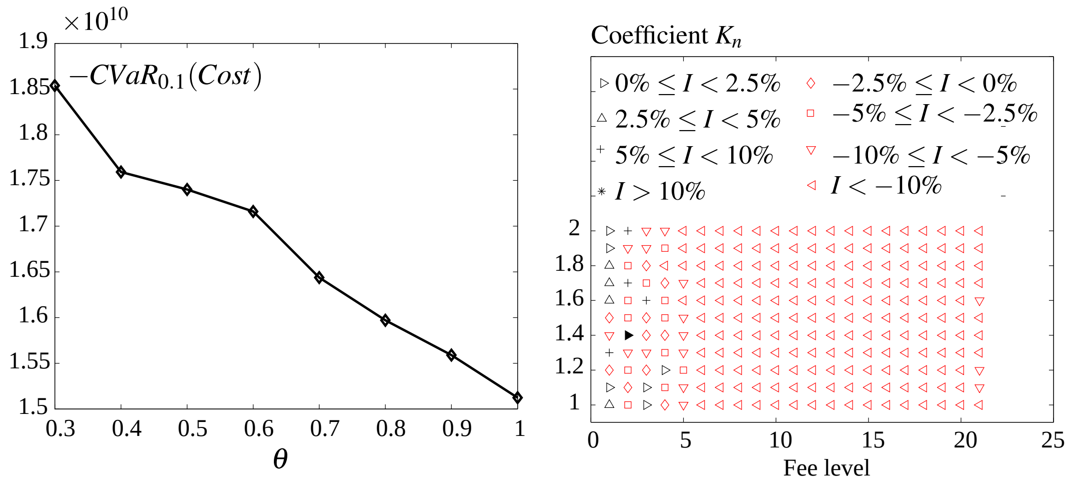

FIGURE 11. Sensitivity of the empirical CVaR of the cost to parameter θ (on the left) and to the fees and gas buying price for $\theta = 0.6$ (on the right).

Then, by running Algorithm 6.2 on a set of scenarios with different contract configurations (different combinations of K_n and $\beta^t_{n\ell}$, including the seller values), the manager can compute the corresponding empirical mean and $CVaR_{0.1}$ of the cost. With this information, the manager can negotiate with the seller a different configuration, proposing a combination with mean income and/or $-CVaR_{0.1}$ of the cost higher than the initial ones. For illustration, the graph on the right in Figure 11 represents the percentual increase I of $-CVaR_{0.1}$ of the cost, with respect to variations in the parameters from Table 3, using the grid used in Figure 9. In Figure 11 a filled triangle corresponds to the initial values of the parameters, while a black (respectively red) symbol corresponds to an increase in $-CVaR_{0.1}$ of the cost (respectively, decrease). Any configuration with a black symbol could be counter-proposed to the seller. To help the negotiation, black points that increase at least one of the seller coefficients $(K_n, \beta^t_{n\ell})$ should be proposed.

9. CONCLUDING REMARKS AND CONTRIBUTIONS

We have presented a computational model that is both realistic and tractable for pricing LNG contracts from a buyers' perspective. From the NG asset valuation viewpoint, our work permits to large NG companies dealing with storage to simultaneously hedge and valuate LNG contracts with cancellation options. For the company manager, an interesting feature of our pricing proposal is that it explicitly incorporates in the pricing process the actual strategy that will be eventually applied by the company to manage the selected LNG contracts. The computational tool developed in this work not only provides the manager with additional insight of how a specific LNG contract configuration will benefit the company business, but also with different contract configurations to counter-propose to the seller for negotiation.

From the Stochastic Programming point of view, our model builds risk-averse feasible policies for a multi-stage mixed-integer stochastic problem that is linear, but with uncertainty that is not stage-wise independent. This is a challenging issue in the area. The rolling-horizon approach, which is not new, provides a mechanism to hedge risk in a tractable manner. The numerical results on a simplified yet realistic case-study, illustrate the potential of the methodology. In particular, its relatively light computational effort gives the corporate manager a versatile pricing tool that takes explicitly into account the strategy the company will apply on the portfolio of contracts eventually selected.

An interesting extension would be to consider uncertain contracted NG volumes, Q^t. Indeed, if important reductions in the contracted volumes are forecast, it would be appealing to increase the number of loads. Also, since LNG markets are not liquid and the NG spot price tends to increase, it is important to book loads with enough anticipation. But this involves taking a large time horizon T, which in turn makes it unrealistic considering that the delivered NG is deterministic. Our methodology is applicable in this setting, working with uncertainty sets for the corresponding NG flows and applying the robust optimization paradigm.

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Appendix with notation and data

		Group 1	Group 2	Group 3	
Monthly demand $\mathscr{D}_{init}$ (30.44 × 10^7 million of NG m^3/day)		10	2	30	
Tendency μ_i		$0.001\mathscr{D}^i_{init}$			
Seasonal component $\mathscr{D}^i_S$	$t = 1:6$	$0.1, 0.05, 0, -0.1, -0.1, 0$			$\mathscr{D}^i_{init}$
	$t = 7:12$	$0.05, 0.1, 0.05, -0.05, 0.05, 0.1$			
Noise $\varepsilon^{i\ t} \sim \mathscr{N}(0, \sigma_i^2)$		$\sigma_i = 0.01\mathscr{D}^i_{init}$			

TABLE 1. Groups 1, 2, and 3 demand.

Mean of process ξ	$\mu_\xi = 0.15$
Standard deviation for ξ	$\sigma_\xi = 0.16$
Standard deviation for χ	$\sigma_\chi = 0.65$
Linear correlation coefficient	$\rho_{\chi\xi} = -0.000413$
Mean reverting parameter	$\kappa = 1.28$
Seasonal component	$se^t = 0.7 + 0.07 \times \cos(2 \times \pi \times (22t + 7.45)/250)$
Starting values for ξ and χ	$(\tilde{\xi}^0, \tilde{\chi}^0) = (2, 0)$

TABLE 2. Parameters of the S&S model.

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Number of periods	T	24
Number of LNG contracts	N	2
Number of NG contracts	1	1
Number of storage facilities	N_S	0
Number of ships in the port at $t=1$	N_{LG}	0
Number of loads for contract n	ℓ_n	22
Contracted NG at t	Q^t	$30.44 \times 60 \times 10^6 \mathrm{m}^3$
Quantity of gas for load (n,ℓ)	$Q_{n\ell}$	$83 \times 10^6\ m^3$
Fraction of evaporated gas at t in the ships	Evap^t	0
Delivery date for load (n,ℓ)	$t_{n\ell}$	$\begin{cases} 2\ell & \text{for } n=1 \\ 2\ell+1 & \text{for } n=2 \end{cases}$
Time step after which loads from contract n can be cancelled	t_{n0}	0
Demand for time step t, group of client i	$\mathscr{D}^{ti}$	Buys-Ballot model
Maximal/minimal entrance flow at time step t	$\overline{\mathrm{F}}^t_+/\underline{\mathrm{F}}^t_+$	$+\infty/0$
Maximal/minimal exit flow at time step t	$\overline{\mathrm{F}}^t_-/\underline{\mathrm{F}}^t_-$	$+\infty/0$
NG spot price at time t	$\mathscr{S}^t$	Two-factor SS model
Gas selling price for clients in group i at t	π^{it}	$1.4\mathscr{S}^t$
Alternative energy unit price at t	$p_A(\mathscr{S}^t)$	$2\mathscr{S}^t$
Penalties paid for clients in group i at t	δ^{it}	$3\mathscr{S}^t$ for i=3,4
Coefficients in (2)	(K_n, k_n^t)	$(1.05, 0)$
	$\beta^t_{n\ell}$	$(0.01, 0.025, 0.05)$
Unit price of each load in contract n	$p_n(\mathscr{S}^t)$	$1.05\ \mathscr{S}^t$
Fee paid at t for load (n,ℓ)	$f_{n\ell}(\mathscr{S}^t)$	$\begin{cases} 0.01 p_n(\mathscr{S}^t) Q_{n\ell} & \text{if } t < t_{n\ell}-2 \\ 0.025 p_n(\mathscr{S}^t) Q_{n\ell} & \text{if } t = t_{n\ell}-2 \\ 0.05 p_n(\mathscr{S}^t) Q_{n\ell} & \text{if } t = t_{n\ell}-1 \end{cases}$
Storage cost (in $US\$$ per m^3) for future load (n,ℓ)	$\alpha_{n\ell}$	20
Cost induced at time t for load (n,ℓ)	$\mathsf{C}^t_{n\ell}$	

TABLE 3. Main parameters for the gas portfolio problem. The third column contains the values taken in Section 8.