Global dominated splittings and the C^1 Newhouse phenomenon

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Abstract

We prove that given a compact n-dimensional boundaryless manifold M, $n \geq 2$, there exists a residual subset $\mathcal{R}$ of the space of C^1 diffeomorphisms $\mathrm{Diff}^1(M)$ such that given any chain-transitive set K of $f \in \mathcal{R}$ then either K admits a dominated splitting or else K is contained in the closure of an infinite number of periodic sinks/sources. This result generalizes the generic dichotomy for homoclinic classes in [BDP].

It follows from the above result that given a C^1-generic diffeomorphism f then either the nonwandering set $\Omega(f)$ may be decomposed into a finite number of pairwise disjoint compact sets each of which admits a dominated splitting, or else f exhibits infinitely many periodic sinks/sources (*the "C^1 Newhouse phenomenon"*). This result answers a question in [BDP] and generalizes the generic dichotomy for surface diffeomorphisms in [M].

1 Context and notation

Throughout this paper, M denotes a compact boundaryless manifold of dimension $n \geq 2$ and $\mathrm{Diff}^1(M)$ is the space of C^1-diffeomorphisms on M with the usual topology.

Given an open subset $\mathcal{U}$ of $\mathrm{Diff}^1(M)$, a subset $\mathcal{R}$ of $\mathcal{U}$ is *residual in* $\mathcal{U}$ if $\mathcal{R}$ contains the intersection of a countable family of open and dense subsets of $\mathcal{U}$; in this case $\mathcal{R}$ is dense in $\mathcal{U}$. A set $\mathcal{R} \subset \mathrm{Diff}^1(M)$ is said to be *residual* if it is residual in all of $\mathrm{Diff}^1(M)$. A property (P) is *locally generic* if there is some nonempty open set $\mathcal{U} \subset \mathrm{Diff}^1(M)$ such that (P) holds for all diffeomorphisms which belong to some residual subset of $\mathcal{U}$; in this case property (P) is said to be *generic in* $\mathcal{U}$. Property (P) is said to be *generic* if it is generic in all of $\mathrm{Diff}^1(M)$.

Given $f \in \mathrm{Diff}^1(M)$, an f-invariant compact set Λ is *transitive* if there is some $x \in \Lambda$ such that the ω-limit set $\omega_f(x)$ of x coincides with Λ.

A compact f-invariant set Λ is *aperiodic* if it contains no periodic orbits.

Given $f \in \mathrm{Diff}^1(M)$ and $x \in M$, the *f-orbit* of x is given by $\mathcal{O}_f(x) \equiv \{f^k(x) | k \in \mathbb{Z}\}$. In the absence of ambiguity we may write $\mathcal{O}(x)$ for $\mathcal{O}_f(x)$. If p is a periodic point we denote its period by $\Pi(p)$.

Given a compact subset $K \subset M$ and $\varepsilon > 0$, we set $B_\varepsilon(K) \equiv \{x \in M : d(x, K) < \varepsilon\}$. Two compact subsets C and D of a metric space X are *ε-Hausdorff-close* if $B_\varepsilon(C) \supset D$ and $B_\varepsilon(D) \supset C$. In this case we write $d_H(C, D) < \varepsilon$.

2 Introduction

In the hyperbolic (i.e., Axiom A) context, the non-trivial dynamics may always be decomposed into a finite number of pairwise disjoint transitive sets (the *spectral decomposition*), each of which contains a dense subset of periodic points. Such sets are called *basic sets*. A lot of recent literature is devoted to finding and studying nonhyperbolic substitutes for the concept of hyperbolic basic sets.

Homoclinic classes – defined as the closure of the transverse homoclinic points of a given hyperbolic periodic orbit[1] – are natural candidates for such a substitute. For one thing, they are always transitive. Furthermore, they are "robust" in that they always admit continuations. The basic sets of Axiom A diffeomorphisms are actually homoclinic classes. Finally, the definition of homoclinic class does not require any hyperbolicity other than that of periodic points.

The theory of "nonhyperbolic basic sets" is best approached from a *generic* point of view, that is, an approach that deals with properties of diffeomorphisms belonging to residual subsets of $\mathrm{Diff}^1(M)$ or of open regions of $\mathrm{Diff}^1(M)$. The convenience of this approach owes much to the genericity of Kupka-Smale diffeomorphisms, to Pugh's classical General Density Theorem [Pu], and to Hayashi's more recent Connecting Lemma [H].

Mañé [M] has shown that generic surface diffeomorphisms either are Axiom A (and hence admit spectral decompositions) or else exhibit infinitely many periodic sinks/sources. Partly motivated by this result, Bonatti-Diaz-Pujals [BDP] showed that, in any dimension, homoclinic classes of generic diffeomorphisms either admit dominated splittings (a weak form of hyperbolicity, see the definition in Section 2 below) or else are contained in the closure of infinitely many periodic sinks/sources. They then asked whether a global generalization of Mañé's theorem might hold: specifically, they asked whether the nonwandering sets of generic diffeomorphisms either exhibit infinitely many periodic sinks/sources or else admit finite partitions into compact invariant sets each of which admits a dominated splitting.

In a well-known result Newhouse [N] showed, on surfaces, the existence of C^2-locally residual subsets of $\mathrm{Diff}^2(M^2)$ consisting of diffeomorphisms which exhibit infinitely many periodic sinks/sources. Later Bonatti and Diaz [BD1] used a different mechanism to obtain the C^1-locally generic coexistence of infinitely many periodic sinks/sources, for manifolds of dimensions 3 and higher. We will henceforth refer to the C^1-generic coexistence of infinitely many periodic sinks/sources as the *C^1 Newhouse phenomenon*[2].

We can thus loosely rephrase the question in [BDP] as follows: is it true that generic diffeomorphisms either admit "nonhyperbolic spectral decompositions" or else exhibit the C^1 Newhouse phenomenon?

The main obstacle that [BDP] faced in using their dichotomy to solve this question was that at the time it was unknown whether generically the nonwandering set is a (not necessarily finite) union of homoclinic classes. In fact, Bonatti and Diaz [BD2] later constructed a C^1-locally generic region of diffeomorphisms exhibiting uncountably many transitive sets each of which is not contained in (and actually does not even intersect) any homoclinic class. So it turns out that even in the generic context the homoclinic classes do not always constitute basic dynamically indivisible "building blocks" of the dynamics. This fact suggests that an affirmative answer to the question above requires that the dichotomy in [BDP] be extended to a more general class of objects than that of homoclinic classes. This is the thrust of our first result. Before we state it, let us define what this "more general class of objects" is.

Given $\varepsilon > 0$ and $f \in \mathrm{Diff}^1(M)$, an *ε-pseudo-orbit of f* is a (finite or infinite) sequence $\{p_k\}$ of points in M such that $d(f(p_k), p_{k+1}) < \varepsilon$ for every k. Given any pair (x, y) of points in M we write $x \dashv y$ if for every $\varepsilon > 0$ there is an ε-pseudo-orbit $\{p_k\}_{0 \le k \le m}$ such that $p_0 = x$, $p_m = y$ and $m \ge 1$. A point $x \in M$ is *chain-recurrent* if $x \dashv x$. The set $R(f)$ of chain-recurrent points is compact and f-invariant, and contains the nonwandering set $\Omega(f)$. We define an equivalence relation on $R(f)$ by setting

$$x \vdash\dashv y \Leftrightarrow x \dashv y \text{ and } y \dashv x.$$

[1] To be precise: given $f \in \mathrm{Diff}^1(M)$ and p a periodic hyperbolic point of f, the *homoclinic class* of f relative to p is given by $H(p, f) \equiv \overline{W^s(\mathcal{O}(p)) \pitchfork W^u(\mathcal{O}(p))}$, where $\pitchfork$ denotes points of transverse intersection of the invariant manifolds.

[2] It is easy to show (see for instance [Ab]) that if a C^1-generic diffeomorphism f exhibits infinitely many periodic sinks/sources, then there is a C^1-neighborhood of f where residually there also are infinitely many sinks/sources.

Given a chain-recurrent point $x \in M$, its equivalence class $C(x)$ under this relation is called the *chain-recurrence class of* x. Clearly $C(x)$ is f-invariant and compact. More generally, a compact invariant set K is *chain-transitive* if given any two points x, y in K and $\varepsilon > 0$, there is an ε-pseudo-orbit $\{p_k\}_{0 \le k \le m}$ contained in K such that $p_0 = x$, $p_m = y$ and $m \ge 1$.

Bonatti and Crovisier [BC] have shown that for generic diffeomorphisms the chain-recurrent set $R(f)$ and the nonwandering set $\Omega(f)$ coincide. Thus, for generic diffeomorphisms, the nonwandering set is a pairwise disjoint union of the chain-recurrence classes $C(x)$. Furthermore, generically each chain-recurrence class $C(x)$ is either a homoclinic class (and in particular has a dense subset of periodic points) or else an aperiodic set. Hence chain-recurrence classes generalize, for generic diffeomorphisms, the concept of homoclinic class.

We can now state our first result in terms of these objects:

Theorem A. *There exists a residual subset $\mathcal{R} \subset \mathrm{Diff}^1(M)$ of diffeomorphisms f such that given any chain-transitive set K of f, then either (a) or (b) holds:*

a) there is a dominated splitting over K[3];

b) the set K is contained is the Hausdorff limit of a sequence of periodic sinks/sources of f.

Theorem A essentially follows from combining two recent results: a theorem by Crovisier [C] and a generalization by Bonatti-Gourmelon-Vivier [BGV] of a result of [BDP]. The theorem by [BGV] essentially states that an infinite set of periodic orbits either admits a partition into a finite number of subsets each of which admits a dominated splitting or else there is a small perturbation of one of the periodic orbits such that the orbit becomes a periodic sink or source. The theorem in [C] states that generically any chain-transitive set is the Hausdorff limit of some sequence of periodic points. The proof of Theorem A, roughly speaking, goes as follows: given a chain-transitive set K of a generic diffeomorphism which does not admit any dominated splitting, then by [C] it follows that K is Hausdorff-approached by a sequence of periodic orbits. The absence of a dominated splitting over K guarantees that the union of the periodic orbits of the sequence does not admit any dominated splitting either. We then apply [BGV] to this sequence in order to create sinks/sources after small perturbations. When combined with some generic arguments this ensures that K is contained in the Hausdorff limit of some sequence of periodic sinks/sources.

We remark that Theorem A may also be obtained – albeit with greater effort – by combining [C] with [BDP] and a result of Wen [W]. Wen's result relates "global" dominated splittings with the creation of homoclinic tangencies. From the generic viewpoint, he has shown that the closure of the set of periodic orbits of a given index admits a dominated splitting if no homoclinic tangencies can be created via small C^1-perturbations. One could extract a "local" version of this result from Wen's proof; this local result would say that given any set of periodic saddles with some common index, then either (i) the closure of this set admits a dominated splitting, or else (ii) one may create, via arbitrarily small C^1 perturbations, a homoclinic tangency associated to a periodic saddle orbit which is Hausdorff-close to one of the saddles in the set.

The alternative proof of Theorem A would then go as follows. As in the first proof outlined above, given a chain-transitive set K of a generic diffeomorphism which does not admit any dominated splitting, it follows by [C] that K is Hausdorff-approached by a sequence of periodic orbits. Again as above, there can be no dominated splitting over the union of the periodic orbits of the sequence. Applying [W] to this sequence one obtains homoclinic tangencies associated to orbits in the sequence after small perturbations. Once a tangency is created a further small perturbation creates a non-trivial homoclinic class associated to the periodic orbit. Thus (allowing for some generic arguments) one proves that the set K is (Hausdorff)-approached by non-trivial homoclinic classes.

[3]A dominated splitting is a weak form of hyperbolicity, see section 3.

One then applies the quantitative version of the generic dichotomy from [BDP], which not only provides a dichotomy between dominated splittings and Newhouse's phenomenon for homoclinic classes, but it also provides a uniform estimate for the "strength" of the dominated splittings. This quantitative version implies that either the union of these homoclinic classes admits a dominated splitting (and as before it then follows that K admits a dominated splitting) or else these homoclinic classes (and hence K) are contained in the closure of infinitely many periodic sinks/sources.

Once we have Theorem A, it is relatively easy to obtain an affirmative answer to the question in [BDP], and hence a generalization of Mañé's dichotomy to higher dimensions. This is the content of our next result:

Theorem B. *There exists a residual subset $\mathcal{R} \subset \mathrm{Diff}^1(M)$ of diffeomorphisms f such that either (a) or (b) holds:*

a) the nonwandering set of f admits a decomposition

$$\Omega(f) = \Lambda_1 \cup \ldots \cup \Lambda_{k_f}$$

where the Λ_i's are pairwise disjoint compact f-invariant sets each of which admits some dominated splitting;

b) there are infinitely many periodic sinks/sources of f.

We note that, unlike hyperbolic spectral decomposition, the decomposition of item (a) is in general not unique.

Remark. The same proof gives the following statement: for any diffeomorphism $f \in \mathcal{R}$, there exists an invariant compact subset K of $R(f)$, which is a union of chain-recurrence classes, and a increasing sequence of invariant compact subsets of $R(f)$.

The existence of a global dominated splitting appears – unlike hyperbolicity – to be a rather weak condition. In general a global dominated splitting (e.g., a dominated splitting over the nonwandering set, or over all of M) does not by itself yield much information on the underlying dynamics of the diffeomorphism (except in dimension 2, see [PS]). So far, most results in this direction also impose other, usually quite restrictive, hypotheses. What the above result suggests, however, is that – generically at least – the *absence* of a global dominated splitting is enough to guarantee pathological dynamics.

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3 Technical Preliminaries

In this section we list several definitions and results that will be used in the next sections.

Definition 1. Given $f \in \mathrm{Diff}^1(M)$, a compact f-invariant set $\Lambda \subset M$ *admits an dominated splitting of strength $\ell \in \mathbb{N}$ and dimension $i \in \{1, \ldots, d-1\}$* if there is a Df-invariant splitting $T_\Lambda M = E \oplus F$ such that the fibers E_x of E have constant dimension i and such that

$$||Df^\ell(x)|_E|| \cdot ||Df^{-\ell}((f^\ell(x))|_F|| \leq 1/2$$

for every $x \in \Lambda$.

In the absence of ambiguity we simply say that Λ *admits a dominated splitting.*

We now list two important properties of dominated splittings (see [BDP] section 1.3 or [V] section 6):

Proposition 1. *Let K be an f-invariant compact set, $f \in \mathrm{Diff}^1(M)$. Assume that there is a sequence of compact f-invariant sets $K_j \subset K$ such that:*

i) $K_j \subset K_{j+1}$ *for every* $j \in \mathbb{N}$*;*

ii) for every $j \in \mathbb{N}$, K_j *admits an dominated splitting* $T_{K_j}M = E_j \oplus F_j$ *of strength* ℓ *and dimension* i*;*

iii) $\overline{\cup_{j\in\mathbb{N}} K_j} = K$.

Then K admits a dominated splitting of strength ℓ and dimension i.

Proposition 2. *Let $f \in \mathrm{Diff}^1(M)$ and let Λ be a compact f-invariant set with an dominated splitting $T_\Lambda M = E \oplus F$ of strength ℓ and of dimension i. Then there is an open neighborhood U of Λ in M such that every f-invariant compact set $K \subset U$ admits an dominated splitting $T_K M = E \oplus F$ of strength ℓ and of dimension i which is a (unique) continuation of the dominated splitting over Λ.*

Let us recall a result from Conley's theory. A *filtrating neighborhood* of a compact set K is a neighborhood of K which is the intersection of two open sets U and V such that $f(\bar{U}) \subset U$ and $f^{-1}(\bar{V}) \subset V$.

Proposition 3. *Let $C(x)$ be a chain-recurrence class of some diffeomorphism f of a compact manifold M. Then, there are arbitrarily small filtrating neighborhoods of $C(x)$.*

Proof. Given $\varepsilon > 0$, let $U_\varepsilon(x) \equiv \{y \in M :$ there is an ε-pseudo-orbit from x to $y\}$, $V_\varepsilon(x) \equiv \{y \in M :$ there is an ε-pseudo-orbit from y to $x\}$, and $C_\varepsilon(x) \equiv U_\varepsilon(x) \cap V_\varepsilon(x)$.

Then both $U_\varepsilon(x)$ and $V_\varepsilon(x)$ are open neighborhoods of $C(x)$, and therefore so is $C_\varepsilon(x)$. Moreover, clearly we have that $C(x) = \bigcap_{n\in\mathbb{N}} C_{\frac{1}{n}}(x)$.

We claim that $f(\overline{U_\varepsilon(x)}) \subset U_\varepsilon(x)$ and that $f^{-1}(\overline{V_\varepsilon(x)}) \subset V_\varepsilon(x)$. From the definition of ε-pseudo-orbit, the ε-neighborhood of $f(U_\varepsilon(x))$ is contained in $U_\varepsilon(x)$ so that $U_\varepsilon(x)$ is an attractor. The proof for $V_\varepsilon(x)$ follows by applying the same argument to f^{-1}.

Hence, for n large, $C_{\frac{1}{n}}(x) = U_{\frac{1}{n}}(x) \cap V_{\frac{1}{n}}(x)$ is an arbitrarily small filtrating neighborhood of $C(X)$. ∎

Crovisier has obtained the following result concerning chain-transitive sets of generic diffeomorphisms:

Theorem 1. *[C, Corollary 1.9] There is a residual set $\mathcal{R} \subset \mathrm{Diff}^1(M)$ of diffeomorphisms f such that given any chain-transitive set K of f there is a sequence $\{\gamma_n\}_{n\in\mathbb{N}}$ of periodic orbits of f which converge to K in the Hausdorff topology.*

The key to the proof of Theorem A is the following generalization by Bonatti-Gourmelon-Vivier of a theorem from [BDP]:

Theorem 2. *[BGV] Let $f \colon M \to M$ be a diffeomorphism of compact manifold M and $\mathcal{U}$ be a C^1-neighborhood of f, then there exist positive integers ℓ and n such that any periodic point x of f with period $\Pi(x) \geq n$ satisfies one of the two following properties:*

- *there is a dominated splitting of strength ℓ over the orbit of x,*

- *for any neighborhood $U \subset M$ of the orbit $\mathcal{O}_f(x)$ of x there is $g \in \mathcal{U}$ coinciding with f on $M \setminus U$ and on $\mathcal{O}_f(x)$ such that the differential $Dg^{\Pi(x)}(x)$ whose eigenvalues are all real and have the same modulus, which is different from 1.*

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4 Proof of Theorem A

We start with a generic consequence of Theorem 2

Proposition 4. *There a residual subset $\mathcal{R} \subset \mathrm{Diff}^1(M)$ of diffeomorphisms f verifying the following property: for any $\eta > 0$ there is $\ell > 0$ such that given any periodic point x of f*

a) either there is a dominated splitting of strength ℓ over the orbit of x,

b) or there exists some periodic sink or source y whose orbit is η-Hausdorff close to the orbit of x.

Proof. Let $\mathcal{K}(M)$ be the set of (maybe empty) compact subsets of M, endowed with the Hausdorff distance[4]. Then $\mathcal{K}(M)$ is a compact metric space. Consider now the metric space $\mathcal{K}(\mathcal{K}(M))$ defined similarly.

To any diffeomorphism f in $\mathrm{Diff}^1(M)$, one associates the set $\mathcal{S}_f \in \mathcal{K}(\mathcal{K}(M))$ which is the set of compact sets of M which are Hausdorff limits of a sequence of sinks or sources. One notes that the map $\mathcal{S}\colon f \mapsto \mathcal{S}_f$ is lower semi-continuous. By a well-known result (see [K], page 71), there is a residual $\mathcal{R}_1 \subset \mathrm{Diff}^1(M)$ such that every $f \in \mathcal{R}_1$ is a point of continuity of $\mathcal{S}$.

We denote by $\mathcal{R}_2$ the Kupka-Smale residual set and by $\mathcal{R}$ the intersection $\mathcal{R}_1 \cap \mathcal{R}_2$.

Let us consider $f \in \mathcal{R}$ and $\eta > 0$. We work by contradiction and assume that the conclusion of the proposition does not hold. Hence, for any ℓ, there exists a periodic point x_ℓ whose orbit has no dominated splitting of strength ℓ and which is not η-Hausdorff close to any sink or source. Since f is Kupka-Smale, the lack of dominated splitting implies that the period of x_ℓ goes to infinity with ℓ.

To any neighborhood $\mathcal{U}$ of f Theorem 2 associates two integers ℓ_0 and n. For ℓ large enough, we have $\ell \geq \ell_0$ and $\Pi(x_\ell) \geq n$. Since the orbit of x_ℓ by f has no dominated splitting of strength ℓ, there is $g \in \mathcal{U}$ such that $\mathcal{O}_f(x_\ell) = \mathcal{O}_g(x_\ell)$ and x_ℓ is a hyperbolic sink or source of g. In particular the Hausdorff distance between $\mathcal{S}_f$ and $\mathcal{S}_g$ is larger than η.

The neighborhood $\mathcal{U}$ of f can be chosen arbitrarily small, so we conclude that f is not a continuity point of $\mathcal{S}$, thereby obtaining a contradiction. □

End of the proof of Theorem A. Let $\mathcal{R}$ be the residual set obtained by intersecting the residual sets given by Proposition 4 and Theorem 1. Let $f \in \mathcal{R}$ and let K be a chain-transitive set of f which does not admit any dominated splitting. Given $\eta > 0$ we will show that there is a periodic sink or source whose Hausdorff distance to K is less than 2η. Then taking η arbitrarily small this will imply that K is Hausdorff-accumulated by a sequence of sinks or sources, as desired.

By Theorem 1 there is a sequence of periodic orbits $\{p_n\}$ which Hausdorff-accumulates on K. Let $\ell > 0$ be the number given by Proposition 4. Then either there is some subsequence of $\{p_n\}$ which consists of periodic points whose orbits admit dominated splittings of strength ℓ and (taking another subsequence if necessary) of the same dimension, or else for arbitrarily large n there are periodic sinks or sources whose orbits are η-Hausdorff-close to the orbit of p_n. In the

[4]We previously defined the Hausdorff distance between two non-empty compact sets of M. The Hausdorff distance between the empty set and any non-empty compact set of M will be taken equal to twice the diameter of M.

first case K is Hausdorff-accumulated by a sequence of sets with uniform dominated splittings, so by Proposition 1 K admits a dominated splitting of the same type, a contradiction. Hence we are in the second case, where K is 2η-close to some periodic sink or source, as announced.

$\square$

5 Proof of Theorem B

Let $\mathcal{R}$ be the residual set given by Theorem A and let $f \in \mathcal{R}$ have a finite number of sinks and sources. By Theorem A, each chain-recurrence class $C(x)$ has a dominated splitting. By Proposition 3, $C(x)$ has an arbitrarily small filtrating neighborhood U. Notice that for any such filtrating neighborhood U, the intersection $U \cap R(f)$ is a union of chain-recurrence classes which furthermore is open and closed in $R(f)$. By Proposition 2, by choosing this U small enough we guarantee that any invariant compact set in U has a dominated splitting.

Let us cover the chain-recurrent set by a finite collection $U_1, \dots, U_n$ of such open sets. We define :

$$\Lambda_1 = U_1 \cap R(f),\ \Lambda_2 = (U_2 \setminus U_1) \cap R(f),\ \dots,\ \Lambda_i = \left(U_i \setminus (\bigcup_{j=1}^{i-1} U_j) \right) \cap R(f).$$

The Λ_i are pairwise disjoint by construction. Let us show that they are invariant compact sets: note that all of the sets $U_j \cap R(f))$ are compact and open in $R(f)$; moreover they are unions of whole recurrence classes (and so invariant). Hence we obtain the same properties for $\Lambda_i = (U_i \cap R(f) \setminus (\bigcup_{j=1}^{i-1}(U_j \cap R(f))$, and we are done.

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