Some Recollections of René Thom

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I thank the organizers of this publication and in particular Alain Chenciner for the opportunity they gave me of adding a few words to this volume honoring a great mathematician who was also a close friend of mine for several decades. René Thom made fundamental contributions to the two mathematical disciplines of

(i) Differential topology (including some algebraic topology) and

(ii) Singularities and stratification of mappings.

Both subjects were introduced by H. Whitney. In what concerns (ii) the work of Thom was completed and extended by J. Mather.

The contributions of Thom to (i) and (ii) were analyzed respectively by A Haefliger [1] and B. Teissier [2] in a volume honoring Thom for his 65th birthday. For that occasion Thom [3] himself made several comments about his own mathematical work.

In (i) is included the work of Thom on cobordism [4] for which he was awarded the Fields medal during the International Congress of Mathematicians at Edinburgh in 1958. At the award ceremony H. Hopf made a glowing report on that work [5] stressing the grand simplicity *(grossartiger Einfachheit)* of Thom's fundamental ideas.

Thom entered the mathematical world in 1949 with a short seminal paper [6] on the decomposition of a Riemannian manifold M into cells, associated to a Morse function $f : M \to R$ on it. This was followed for some ten years more by a string of papers full of ideas and in many directions which constitutes the great contribution of Thom to topology. It signaled the power and beauty of geometric methods in topology, as in the old days. Among these papers the most important and by far the most elaborate is the one on cobordism mentioned above [4]. An important role in this paper is played by Thom's transversality lemma which is introduced and proved there.

The decomposition of M proposed by Thom in [6] is obtained by considering the gradient flow

$$(1) \qquad \frac{dx}{dt} = -\,grad\, f(x) \qquad x \in M$$

and taking the union of all stable manifolds W^s associated to the singularities of the above flow.

Now to this picture Smale introduced the decomposition of M into unstable manifolds W^u of the singular points of (1) and the requirement that at every point in M the corresponding W^s and W^u be transversal. From this point, from the hands of Smale there sprang two of his major contributions to mathematics : a) his topological work that culminated in the h−cobordism theorem, containing among other things the proof of the high dimension Poincaré conjecture ; b) his work on dynamical systems theory. This is one of the reasons why the paper [6] of Thom was so important.

Around 1970 Thom lost interest in mathematical research, devoting all his energies to Catastrophe Theory. He became quite famous, well beyond the universe of mathematicians.

Here I will relate a number of episodes concerning my relationship with Thom over the years. All my conversations with Thom mentioned below were held in French.

First meeting

It was in 1958 in Edinburgh, after the above mentioned Fields medal ceremony. I was introduced to Thom by G. Reeb. Reeb had spent in 1957 some two months in Rio de Janeiro at IMPA. From Reeb I first heard that in Strasbourg there was a very good mathematician named René Thom. Reeb knew him well and told me that very likely Thom would be interested in what I was doing in the area of structural stability of differential equations.

But in our meeting in Edinburgh there were too many people and too much confusion around Thom. Later Thom told me that he forgot completely that he shook hands with me in Edinburgh. His recollection was that the first time he heard my name was after Edinburgh, from Lefschetz.

First letter

My first mathematical contact with Thom was through Smale while Smale was spending the first semester of 1960 at IMPA. In June Smale made a quick trip to Zürich to attend a conference and speak about his work on high dimensional Poincaré conjecture that he was then finishing. When he came back I told him that I was in trouble with what is called nowadays the "closing lemma". To which he said that at the conference he met Thom who had run into the same question in n dimensions, while dealing with some other problem.

This was the origin of an extensive correspondence and discussions I initiated with Thom on this and related matters. The expression closing lemma was coined by Thom at about that time.

I recall here the C^r closing lemma. Let M be a differentiable manifold, $p \in M$, X be a vector field on M and γ be its trajectory through p which is assumed to be recurrent i.e. γ meets any neighborhood of p at arbitrarily large times. Then we can find ΔX C^r small such $X + \Delta X$ has a closed orbit through p. If $r = 0$ it is fairly simple to prove the closing lemma ; if $r = 1$, Pugh [7] proved it, a very difficult proof ; if $r > 1$ the problem is considered to be wide open. See [8].

Talking mathematics with Thom

My first conversation with Thom was in March, 1961, at Berkeley where we both went to visit Smale. Later he spent the month of September, 1961, at IMPA. He was very much interested in structural stability and so he was quite helpful to me with my work on M^2. Several years later, we published two joint notes at the Comptes Rendus on some other subject. Reeb once told me : "*I like very much to* *discuss mathematics with Thom. I am always left with the impression that perhaps, I too, I am also a great mathematician.*"

Hassler Whitney

I was well aware that Thom had great admiration for the originality and depth of the mathematical work of Whitney. He had many opportunities to discuss mathematics at length with him whom he found very singular in one respect. Namely, he said Whitney was totally indifferent as to whether the outcome of a mathematical discussion would add to or subtract from his work. And this only increased his admiration for Whitney. In any case it was a surprise for me that Thom was preoccupied with such arcane psychological problems.

Before the publication of Pugh's paper I once asked Whitney about the C^r closing lemma. In about ten minutes he said : "*If* $r = 0$ *the lemma is true and it is fairly simple to get your closed orbit. Otherwise I think it is a very difficult problem*". I was amazed by the speed and confidence shown by Whitney. When I mentioned this fact to Thom he said : "*Of all the mathematicians that I know Whitney is the one who has most acute the sense of the differentiable. Altogether he is the American Riemann*".

An interrupted dialog on poetry

I once told Thom : "*I think that English poetry is better than French poetry. What do you think ?*" He answered : "*I agree with you and it is easy to see why. The point is that the French language is a language too explicit, too precise, too algebraic. Poetry requires something more ambiguous, more reticent, more shadowy and the English language is more congenial to that than is the French language.*" I then advanced my own quip : "*I think that poetry is something like a covering space.*" To which Thom said : "*Of course, of course.*" I found the insight of Thom a deep one and occasionally in our conversations we came to this point. Then I came across the book [9], an anthology of Chinese poetry. At the beginning of which there is, as some kind of introduction to the book, the following comment on the nature of poetry by Wei T'ai, an eleventh century scholar :

"*Poetry presents the thing in order to convey the feeling. It should be precise about the thing and reticent about the feeling, for as soon as the mind responds and connects with the thing the feeling shows in the words ; this is how poetry enters deeply into us. If the poet presents directly feelings which overwhelm him, and keeps nothing back to linger as an aftertaste, he stirs us superficially . . .* "

These words written some one thousand years ago contrast vividly with Thom's remark. Thom refers to something global, about languages which are more or less adapted to poetry. The comment of Wei T'ai on the contrary is something local, about the poem itself in any language. But then it goes very deep, it is one of these things that once you see it, you never forget. I was sure that Thom would be delighted to talk about Wei T'ai. In 1998 and afterwards I visited him several times but the occasion never presented itself for that talk.

Silence at Chartres

Over the years we visited the cathedral of Chartres some two or three times. Our last visit on a clear winter day was a memorable one. At a certain moment we both stopped and looked up in silence to one of those glorious stained glass windows. After several minutes René said : "*The blue.*" Awhile afterwards I answered : "*Yes.*" And we moved on.

So was my friend René Frédéric Thom, who died in his sleep on the 25th of October 2002 at Bures Sur Yvette, France.

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