Quandles at Finite Temperatures III

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Abstract

We resume work on telling embeddings of codimension two apart by counting colorings of the corresponding diagrams by given quandles. Previously, we illustrated the efficiency of this approach on classical knots. In the present paper we apply it to knotted surfaces. We recover work of Kamada in telling ribbon knots apart and we distinguish all elements of a class of twist-spun torus knots.

1 Introduction

Independently introduced by Joyce and Matveev, the quandle is an algebraic structure intimately related to knots (cf. [9], [17], also [15], and [5]). This relation arises as follows. Consider an arbitrary knot K i.e., an embedding of S^1 in $\mathbb{R}^3$. Its knot quandle, X_K, is presented by realizing, in any of its knot diagrams, arcs as generators and reading relations of the sort $a * b = c$ at crossings (where b is the over-arc, and a and c are the under-arcs meeting at the crossing). This quandle is an invariant of the knot under study (modulo deformation) for the defining axioms of this algebraic structure are such as to make it insensitive to the Reidemeister moves, in the above correspondence.

As a consequence, note that $Hom(X_K, X)$ (the set of homomorphisms from the knot quandle, X_K, to a fixed labelling quandle, X) is an invariant for knot K and so is its cardinality, $\#Hom(X_K, X)$. The elements of $Hom(X_K, X)$ are the so-called "colorings" and "counting colorings" is the evaluation of $\#Hom(X_K, X)$.

Despite the fact that the knot quandle is a classifying invariant (modulo mirror symmetry), the difficulties in handling presentations do not render it a good tool in distinguishing knots. Quandle (co)homology was one of the subsequent developments of quandle theory, which eventually led to the beautiful invariant now known as the CJKLS invariant (cf. [3], see also [6], [7]). The CJKLS invariant is, in fact, a collection of invariants, one for each choice of labelling quandle and 2-cocycle (in the cohomology of the labelling quandle). In particular, the CJKLS invariant yields the number of colorings when using the trivial cocyle to evaluate it. The invariance of the number of colorings was proved in yet another way

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in previous work by the second author (cf. [16]). The profusion of this fact led us into investigating the efficiency of counting colorings as an invariant. In [4] its effectiveness was illustrated by telling apart all but less than 3% of the (classical) knots of up to and including ten crossings. We will now describe the method used there since it is essentially the same that we will be using in this paper. We choose a class of knots - in the case in point it was the class of all prime knots of up to and including ten crossings. We fix a finite sequence of (labelling) quandles. Then for each (unordered) pair of (distinct) knots from the class we look for the first quandle in the finite sequence that yields different numbers of colorings for these two knots. Ideally, we would fix the sequence of all finite quandles and for each knot we would write the sequence whose terms would be the number of colorings by the corresponding quandle; this latter sequence would then be regarded as the "color spectrum" of the knot under study.

In this paper we look into the efficiency of this method in distinguishing knotted surfaces. Knotted surfaces (cf. [1]) are the one dimension higher analogs of (classical) knots. More precisely, they are embeddings of closed, orientable, locally flat 2-dimensional manifolds in $\mathbb{R}^4$. These embeddings can then be projected onto a 3-dimensional hyperplane of $\mathbb{R}^4$. The projection of the embeddding will in general self intersect. In order to make sense of what goes over and what goes under, the lower sheets are broken along the lines of self-intersection, splitting the projection into regions. This network of regions is the so-called *broken surface diagram* - the analog of the knot diagram (cf. [1]). Analogous to the Reidemeister moves, the Roseman moves relate broken surface diagrams corresponding to knotted surfaces which are deformable into each other (and vice versa, cf. [19]). The similarity to (classical) knot theory in the previous statement and the ones to follow should be noted. If we associate generators to the regions of a broken surface diagram, and read relations of the sort $a * b = c$ at the lines of self-intersection we obtain a presentation of a quandle - the knot quandle of the corresponding knotted surface. This knot quandle is also an invariant of the knotted surface under study modulo deformation (see below for a proof). The same list of objects and facts goes over from the set up of (classical) knots to the set up of knotted surfaces. In particular, we remark that, again, the number of colorings by a given quandle is an invariant for knotted surfaces. This is the invariant which we will lean on in this work. The method employed here is to fix a (finite) sequence of (labelling) quandles and to consider a class of knotted surfaces whose elements we would like to distinguish. Then for each (unordered) pair of knotted surfaces from this class, we look for the first quandle in the sequence that yields different numbers of colorings for the knots in the pair.

We work on two classes of knotted surfaces. One is a class of ribbon knots studied by Kamada in [12]. We recover his results (modulo three inconclusive cases) and improve on his conjecture (again modulo an extremely small percentage of inconclusive cases). The other is a class of twist-spun knots considered in [2]. The listing and counting of colorings was done with the help of computer programs implemented in C and Assembler.

The organization of this paper is as follows. In section 2 we lay out the basic material. We present the definition of quandles and briefly discuss knotted surfaces, knotted surface diagrams and Roseman moves. We prove that the knot quandle of a knotted surface is indeed an invariant. We also sketch ways of representing knotted surfaces, like 2-dimensional braids and braid charts. The latter are the ones we use for computer programs. We then describe how to compute the knot quandle from a braid chart of a knotted surface. In section 3 we detail the classes of knotted surfaces we consider, and work out a few examples to make the computations clearer.

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2 Preliminaries

2.1 The Quandles

Def. 2.1 (Quandle) *A quandle is an algebraic structure $(X, *)$ such that,*

- *(Idempotency) for any a in X, $a * a = a$;*
- *(Right-invertibility) for any a and b in X, there exists a unique x in X such that $x * b = a$;*

- *(Self-distributivity) for any a, b, and c in X,* $(a * b) * c = (a * c) * (b * c)$.

Note that the Right-invertibility axiom gives rise to a second operation on a quandle, namely $a \bar{*} b = c$, where c is the unique element such that $c * b = a$.

Prop. 2.1 *Suppose* $(X, *)$ *is a quandle. Then* $(X, \bar{*})$ *is also a quandle.*

Proof: It is a routine matter to check the axioms so we leave it to the reader. ■

Def. 2.2 *Keeping the notation above,* $(X, \bar{*})$ *is called the dual quandle of* $(X, *)$.

Prop. 2.2 *Consider a quandle* $(X, *)$ *along with its dual structure* $\bar{*}$ *. The operations* $*$ *and* $\bar{*}$ *are distributive with respect to each other.*

Proof: Again, this is a routine check so we leave it to the reader. ■

Quandles have been studied previouly in [9], [17], [15], and [5], for instance. The basic example is any group with group conjugation as the quandle operation $a * b := b^{-1}ab$. An important class of quandles is formed by those whose underlying set is $\mathbb{Z}_n$ and quandle operation is $a * b := 2b - a \pmod{n}$; they are denoted R_n. Another important class of quandles stems from the fact that the set of Laurent polynomials $\Lambda = \mathbb{Z}[T, T^{-1}]$ endowed with the operation $a * b = Ta + (1 - T)b$ is a quandle. It follows that any module over Λ is a quandle. We will be dealing with the so-called finite Alexander quandles which have the form $\mathbb{Z}_n[T, T^{-1}]/(h)$, where n is an integer, and h is a monic polynomial. In particular, we will be dealing with linear Alexander quandles i.e., Alexander quandles of the form $\mathbb{Z}_n[T, T^{-1}]/(T - a)$ where $gcd(n, a) = 1$ (cf. [18]). We remark that the R_n quandles can be regarded as finite linear Alexander quandles, namely $\mathbb{Z}_n[T, T^{-1}]/(T + 1)$.

2.2 Knotted Surfaces, Their Diagrams, and Roseman Moves

Knotted surfaces are the one-dimension higher counterparts of (classical) knots. By definition they are embeddings of closed, orientable, locally flat 2-dimensional manifolds in $\mathbb{R}^4$. In what follows we assume an orientation, along with the specification of a normal direction, has been attributed to the knotted surfaces under consideration. After a possible deformation of the knotted surface, its projection onto a 3-dimensional hyperplane of $\mathbb{R}^4$ (not containing the image of the embedding) will yield a generic surface i.e., locally the projected surface will look like the intersection of i coordinate planes ($i = 1, 2, 3$) or the cone on a figure eight (cf. [1]). By choosing a height function along the direction of projection from the hyperplane to the image of the embedding we can ascertain which is the lower and which is the higher sheet meeting along the lines of self-intersection. Cutting the lower sheet along these lines on the projection gives rise to the so-called broken surface diagrams. If a broken surface diagram can be obtained from another one by performing a finite number of local changes (called the Roseman moves, see figures 3 through 10) then the corresponding knotted surfaces are deformable into each other (and vice versa, [19]). According to the number of 2-dimensional manifolds they involve, the Roseman moves can be grouped into three types. Those that only involve one 2-dimensional manifold (type I moves); those that only involve two 2-dimensional manifolds (type II moves); and those that involve three or more 2-dimensional manifolds (type III moves). Also, these statements are local; globally the manifolds referred to may turn out to be "pieces" of the same manifold. The similarity to the Reidemeister moves should be noted. Furthermore, there are the bubble and the saddle type I moves; there are the bubble, the saddle and the "branch point through a plane" type II moves; and there are the "finger" and the "tetrahedral" type III moves (see figures 2 through 9).

Each broken surface diagram gives rise to a presentation of a quandle by regarding the regions of the diagram as generators and reading relations of the sort $a * b = c$ along each of the lines of self-intersection as follows (see figure 1). For the situation depicted in the figure (where the letters denote the quandle generators assigned to the corresponding regions), the following two statements are equivalent: $x * y = z$ and $z \bar{*} y = x$; it will be convenient to use one or the other, depending on the situation. The small arrow denotes the orientation of the normal to the surface. Note that in both equations, the over-sheet is the second factor of the $*$ (or $\bar{*}$) binary operation and that the orientation of the regions labelled x and z is irrelevant. Also note that when using the $*$ operation the normal to the over-sheet points to the under-sheet that receives the product, whereas, when using the $\bar{*}$ operation, the normal to the over-sheet

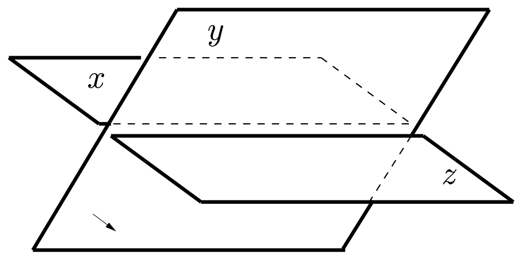

Figure 1: Regions and generators at (erased) line of self-intersection

points to the under-sheet which stands for the first factor. In the sequel, when the orientation of the normal to the surface is not explicitly shown, we will write $a *^{\epsilon} b = c$, with $\epsilon = \pm 1$. $*^{+1}$ means $*$, whereas $*^{-1}$ means $\bar{*}$.

Prop. 2.3 *Consider two broken surface diagrams D and D'. Assign quandle presentations to each of them in the manner discussed above i.e., regions stand for generators and each line of self-intersection gives rise to one relation of the type $a *^{\epsilon} b = c$. If D and D' are related by a finite number of Roseman moves, then the corresponding quandle presentations are isomorphic.*

Proof: We first make a few remarks in order to render the proof clearer. In this way note that:

- It suffices to prove the result assuming D and D' are related by one Roseman move and then check it for all Roseman moves;
- When performing a Roseman move, diagrams remain the same outside a neighborhood (dotted circles in the figures). As a consequence, generators corresponding to regions which extend from the outside of the neighborhood to the inside remain the same as the move develops. So it is enough to make sure that other generators and/or relations that might appear or disappear inside this neighborhood, as the move is performed, are expressed in terms of the remaining ones, and do not conflict with them.
- The figures depicting the moves only address special instances of them, corresponding to particular choices of over and under sheets. In the first four moves this is irrelevant since the other posibilities are obtained by reversing the move and/or by deformation of the local configuration (cf. [16]). For the last three moves, we leave it to the reader to check the remaining instances noting that the calculations are similar to the ones we do here.

So consider two knotted surface diagrams, D and D'.

- Assume D and D' are related by a type I Roseman move;
  - Suppose it is a bubble move (see figure 2).

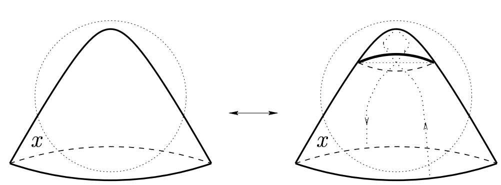

Figure 2: Type I Roseman move: bubble move

On the left hand side only one generator, say x, can be assigned to the region depicted and no relations are considered for there is no line of self-intersection. On the right-hand side, there is only one region and there is one line of self-intersection. The relation corresponding to this line of self-intersection is $x *^{\epsilon} x = x$ which is a trivial relation.

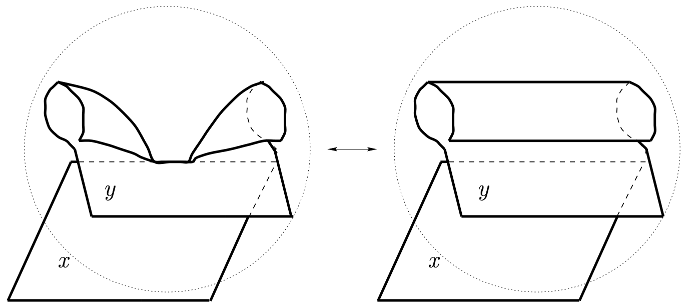

Figure 3: Type I Roseman move: saddle move

- Suppose it is a saddle move (see figure 3).

  By inspection of the left-hand side of figure 3, we see there is only one region, thus $x = y$. At the self-intersecting lines we have $y *^{\epsilon} y = y$ which are trivial relations. On the right-hand side, there are two regions with generators x and y; the self-intersecting line implies $x = y *^{\epsilon} y = y$.

- Assume D and D' are related by a type II Roseman move;

  - Suppose it is a bubble move (see figure 4).

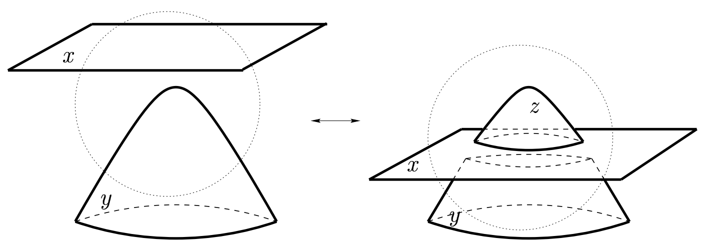

Figure 4: Type II Roseman move: bubble move

On the left-hand side of figure 4 there are two regions to which the generators x and y are associated, and there are no relations to be considered. On the right-hand side, there appears a new region, denoted z, and there is a relation which reads $y *^{\epsilon} x = z$. Hence z is expressed in terms of x and y.

- Suppose it is a saddle move (see figure 5).

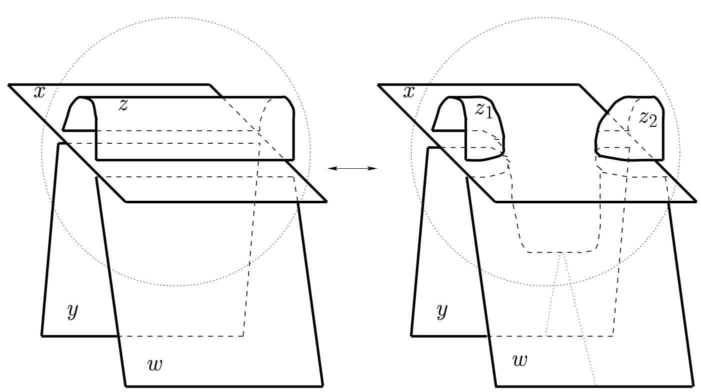

Figure 5: Type II Roseman move: saddle move

In the figure on the left-hand side there are four regions. Of these, three extend to the outside of the neighborhood where the move takes place, x, y, and w. z belongs to this neighborhood. The upper line of self-intersection (with respect to the page) yields $y *^{\epsilon} x = z$

whereas the other line of self-intersection yields $z *^{-\epsilon} x = w$ and using the previous equation $w = z *^{-\epsilon} x = (y *^{\epsilon} x) *^{-\epsilon} x = y$, i.e., $w = y$. Now consider the right-hand side. Again, x, y, and w are the generators associated to the outside and z_1, and z_2 are associated to the inside of the neighborhood. We have $z_1 = y *^{\epsilon} x = z_2$ and $z_1 *^{-\epsilon} x = w = z_2 *^{-\epsilon} x$ which again yields $y = w$. So, on both hand sides every generator is expressed in terms of x and y.

- Suppose it is a "branch point through a plane" move. (see figure 6).

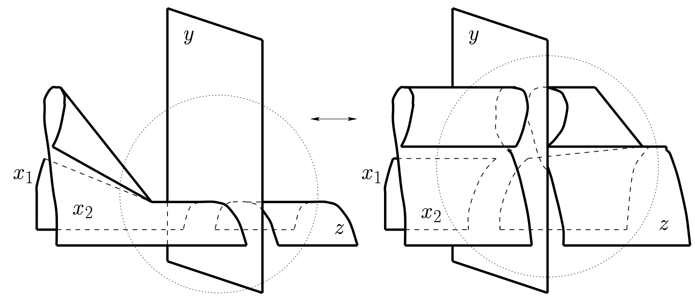

Figure 6: Type II Roseman move: "branch point through a plane" move

In the figure on the left we see that $x_1 = x_2$ and that $x_1 *^{\epsilon_Y} y = z$. In the figure on the right we have $x_1 *^{\epsilon_X} x_2 = x_2$ ($\Leftrightarrow x_1 = x_2$) and $x_1 *^{\epsilon_Y} y = z$ in this way every generator is expressed in terms of x_1 and y.

- Assume D and D' are related by a type III Roseman move;
  - Suppose it is a finger move (see figure 7).

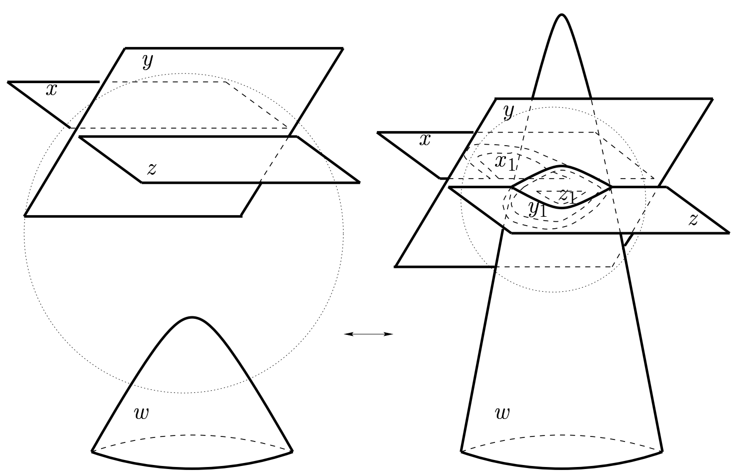

Figure 7: Type III Roseman move: finger

In the figure on the left we have $x *^{\epsilon_Y} y = z$. In the figure on the right $x *^{\epsilon_Y} y = z$, $x_1 *^{\epsilon_Y} y_1 = z_1$, $x *^{\epsilon_W} w = x_1$, $z *^{\epsilon_W} w = z_1$, $y *^{\epsilon_W} w = y_1$. Note that $x_1 *^{\epsilon_Y} y_1 = (x *^{\epsilon_W} w) *^{\epsilon_Y} (y *^{\epsilon_W} w) = (x *^{\epsilon_Y} y) *^{\epsilon_W} w = z_1$, so on either sides of the move all the generators are expressed in terms of x, y, and w.

  - Suppose it is a tetrahedral move (see figures 8 and 9).

    On the left-hand side of the move (figure 8) we have

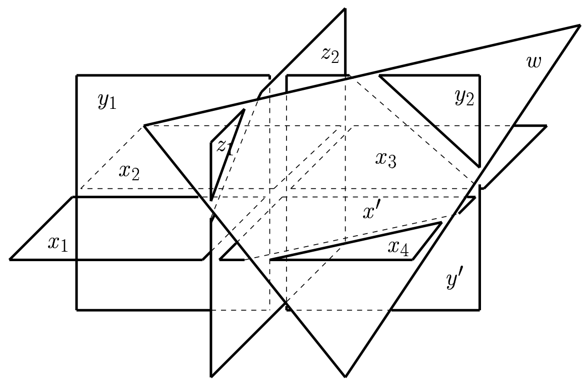

Figure 8: Type III Roseman move: tetrahedral move (LHS)

$$\begin{cases} x_1 *^{\epsilon_Y} y_1 = x_2 \\ x_4 *^{\epsilon_W} w = x' \\ x' *^{\epsilon_Y} y' = x_3 \\ x_1 *^{\epsilon_Z} z_2 = x' \\ x_2 *^{\epsilon_Z} z_2 = x_3 \\ y_1 *^{\epsilon_Z} z_2 = y' \\ y_2 *^{\epsilon_W} w = y' \\ z_1 *^{\epsilon_W} w = z_2 \end{cases} \qquad \Longleftrightarrow \qquad \begin{cases} x_2 & = x_1 *^{\epsilon_Y} y_1 \\ x_3 & = (x_1 *^{\epsilon_Y} y_1) *^{\epsilon_Z} (z_1 *^{\epsilon_W} w) \\ x_4 & = \Big(x_1 *^{\epsilon_Z} (z_1 *^{\epsilon_W} w)\Big) *^{-\epsilon_W} w \\ y' & = y_1 *^{\epsilon_Z} (z_1 *^{\epsilon_W} w) \\ z_2 & = z_1 *^{\epsilon_W} w \\ y_2 & = \Big(y_1 *^{\epsilon_Z} (z_1 *^{\epsilon_W} w)\Big) *^{-\epsilon_W} w \\ x_3 & = \Big(x_1 *^{\epsilon_Z} (z_1 *^{\epsilon_W} w)\Big) *^{\epsilon_Y} \Big(y_1 *^{\epsilon_Z} (z_1 *^{\epsilon_W} w)\Big) \\ x' & = x_1 *^{\epsilon_Z} (z_1 *^{\epsilon_W} w) \end{cases}$$

Hence, each generator is expressed in terms of x_1, y_1, z_1, and w. Note that the two expressions yielding x_3 are equivalent thanks to the self-distributivity axiom.

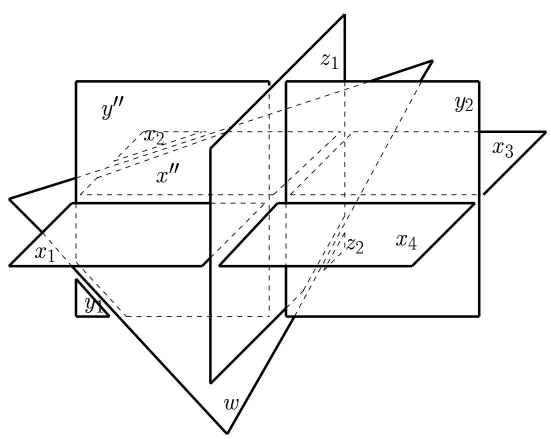

Figure 9: Type III Roseman move: tetrahedral move (RHS)

On the right-hand side of the move (figure 9) we have

$$
\begin{cases}
x_1 *^{\epsilon_Y} y'' = x'' \\
x_4 *^{\epsilon_Y} y_2 = x_3 \\
x'' *^{\epsilon_W} w = x_2 \\
x_1 *^{\epsilon_Z} z_1 = x_4 \\
x'' *^{\epsilon_Z} z_1 = x_3 \\
y_1 *^{-\epsilon_W} w = y'' \\
y'' *^{\epsilon_Z} z_1 = y_2 \\
z_1 *^{\epsilon_W} w = z_2
\end{cases}
\iff
\begin{cases}
x'' &= x_1 *^{\epsilon_Y} (y_1 *^{-\epsilon_W} w) \\
x_3 &= (x_1 *^{\epsilon_Z} z_1) *^{\epsilon_Y} \Big((y_1 *^{-\epsilon_W} w) *^{\epsilon_Z} z_1 \Big) \\
x_2 &= \Big(x_1 *^{\epsilon_Y} (y_1 *^{-\epsilon_W} w) \Big) *^{\epsilon_W} w \\
x_4 &= x_1 *^{\epsilon_Z} z_1 \\
x_3 &= \Big(x_1 *^{\epsilon_Y} (y_1 *^{-\epsilon_W} w) \Big) *^{\epsilon_Z} z_1 \\
y'' &= y_1 *^{-\epsilon_W} w \\
y_2 &= (y_1 *^{-\epsilon_W} w) *^{\epsilon_Z} z_1 \\
z_2 &= z_1 *^{\epsilon_W} w
\end{cases}
$$

Again, each generator is expressed in terms of x_1, y_1, z_1, and w, and note that the two expressions yielding x_3 are equivalent thanks to the self-distributivity axiom.

This concludes the proof. ■

Def. 2.3 *The knot quandle of a knotted surface is the quandle presented by assigning generators to regions in one of its knotted surface diagrams, and reading relations at each (erased) line of self-intersection as in Prop. 2.3.*

Thm. 2.1 *The knot quandle is an invariant modulo deformation of the corresponding knotted surface.* ■

Def. 2.4 *A coloring of a knotted surface by a given quandle X, is a homomorphism from the knot quandle of the knotted surface to X.*

Cor. 2.1 *The number of colorings of a knotted surface is an invariant of that knotted surface.* ■

Cor. 2.1 will be used in the sequel in the following form. If two knotted surfaces are such that there is a quandle yielding n_1 colorings for one of the knotted surfaces and n_2 for the other, with $n_1 \neq n_2$, then these knotted surfaces are **not** deformable into each other.

2.3 2-dimensional braids and braid charts

There are different ways of representing knotted surfaces (cf. [1] and [13]). We will concentrate on the so-called 2-dimensional braids (cf. [11], also known as surface braids cf. [13]) and their braid charts (cf. [10], [13]) for they give rise to encodings of the knotted surfaces appropriate for implementation of computer programs (cf. [3] and [2]).

Consider a knotted surface, S, in $\mathbb{R}^4 \cong \bigcup_{t\in\mathbb{R}}(\mathbb{R}^3 \times \{t\})$. For each $t \in \mathbb{R}$, either $\mathbb{R}^3 \times \{t\}$ meets the embedding or not. We may assume that the intersection is non-empty only for $-1 \leq t \leq 1$. In this way

$$
\left(\mathbb{R}^3 \times \{t\}, \quad S \cap \left(\mathbb{R}^3 \times \{t\}\right)\right)_{-1\leq t\leq 1}
$$

encompasses all the embedding. Without loss of generality, we may further regard $\mathbb{R}^3 \cong \bigcup_{u\in\mathbb{R}}(\mathbb{R}^2 \times \{u\})$ and by deletion of the u-coordinate and the insertion of the usual over/under information at crossings,

$$
\left(\mathbb{R}^2 \times \{t\}, \quad S \cap \left(\mathbb{R}^2 \times \{t\}\right)\right)_{-1\leq t\leq 1}
$$

can be regarded as a sequence of stills of a motion picture depicting the embedding. This is what became known as Fox's motion picture method. In the first applications of these ideas (cf. [8], for instance) knotted spheres were depicted with minima lying in $\mathbb{R}^3 \times \{-1\}$, maxima in $\mathbb{R}^3 \times \{1\}$ and saddle points in

$\bigcup_{i=1}^{n}(\mathbb{R}^3 \times \{t_i\})$, with $-1 < t_1 \leq t_2 \leq .... \leq t_n < 1$. For $t \in]-1,1[\setminus\{t_1,t_2,...,t_n\}$, $(\mathbb{R}^3 \times \{t\}, S \cap (\mathbb{R}^3 \times \{t\}))$ is a (classical) link. The changes in the stills

$$\left(\mathbb{R}^2 \times \{t\}, \quad S \cap \left(\mathbb{R}^2 \times \{t\}\right)\right)$$

from $t = t_i$ to t_{i+1}, are ambient isotopies of links ($i = 0,1,2,...,n$; $t_0 = -1$, and $t_n = 1$). In this way, for each $t_i < t < t_{i+1}$, $S \cap \left(\mathbb{R}^3 \times \{t\}\right) = h_{s(t)}(L_i)$ where $s(t) = \frac{t-t_i}{t_{i+1}-t_i}$, L_i is the link in the still "immediately after" $S \cap \left(\mathbb{R}^3 \times \{t_i\}\right)$ and $\{h_t\}_{i \leq t \leq i+1}$ is an ambient isotopy in $\mathbb{R}^3$. This is an operation that can be applied to a link in a process of (re)constructing an embedding. The other operation on a link we will be needing is the so-called *hyperbolic transformation* (see figure 10). That is, we attach a band, B, along

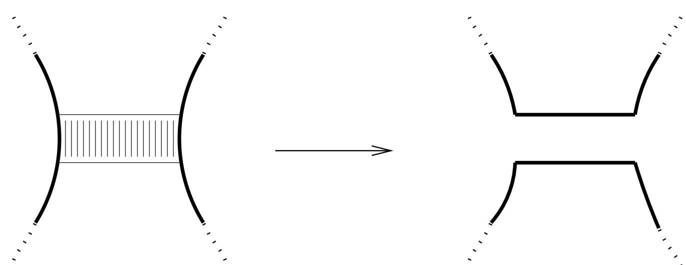

Figure 10: Hyperbolic Transformation

arcs of a link, L, then perform the transformation by removing the inside of the band and the sides of the band previously attached to the link, and leaving behind the other sides of the band. The result of this transformation is the link L'. The hyperbolic transformations are used for modelling the saddle points in the following way (where $\epsilon > 0$ is small enough).

$$S \cap \left(\mathbb{R}^3 \times \{t\}\right) = \begin{cases} L \times \{t\} & \text{for } t_i - \epsilon < t < t_i; \\ L \cup B & \text{for } t = t_i; \\ L' \times \{t\} & \text{for } t_i < t < t_i + \epsilon \end{cases}$$

In the piecewise linear category minima become minimum discs, maxima become maximum discs and saddle points become saddle bands. Working in this category, Kawauchi et al (cf. [14]) proved that any knotted surface can be deformed into the so-called *normal form.* That is, all minimum discs are standardly embedded in $\mathbb{R}^3 \times \{-1\}$, all saddle bands are standardly embedded in $\mathbb{R}^3 \times \{0\}$, and all maximum discs are standardly embedded in $\mathbb{R}^3 \times \{1\}$. Moreover,

$$\left(\mathbb{R}^3 \times \{t\}, \quad S \cap \left(\mathbb{R}^3 \times \{t\}\right)\right)_{-1<t<0}$$

corresponds to an ambient isotopy of trivial links from the boundary of the minimum discs, and

$$\left(\mathbb{R}^3 \times \{t\}, \quad S \cap \left(\mathbb{R}^3 \times \{t\}\right)\right)_{0<t<1}$$

corresponds to an ambient isotopy of trivial links to the boundary of the maximum discs. At $t = 0$ hyperbolic transformations occur.

In [11], the following improvements in the normal form were introduced. Deform the saddle bands so that the hyperbolic transformations look like figures 11 and/or 12. Then deform the trivial links right before and right after the hyperbolic transformation so that they look like closed trivial braids. It should be noted that this last step may involve hyperbolic transformations but they are all shaped as in figure 11 or 12. In short Kamada proved that any knotted surface is deformable so as to look like an ambient

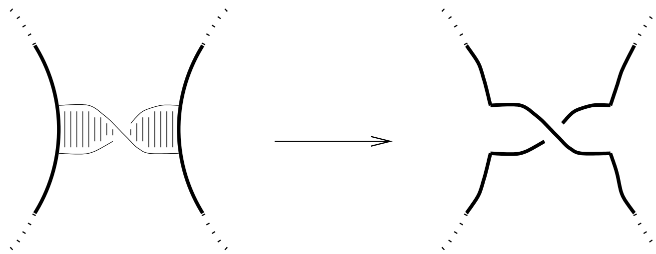

Figure 11: Simple Hyperbolic Transformation 1

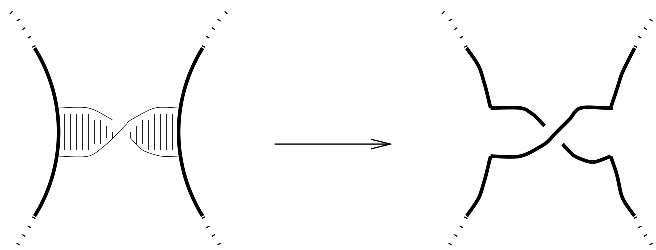

Figure 12: Simple Hyperbolic Transformation 2

isotopy from a closed trivial braid, followed by simple hyperbolic transformations as in figure 11 or 12 giving rise to a closed braid, that is a trivial link, which is then the starting point for a second ambient isotopy to a closed trivial braid. Finally, the closed trivial braids are capped off with discs. A remarkable fact here is that all braids involved have the same braid index, say m. A knotted surface in such a form is a closed 2-dimensional m-braid. By conveniently shrinking the saddle bands to a point, we have singular braids at the hyperbolic transformations (see figures 13 and 14).

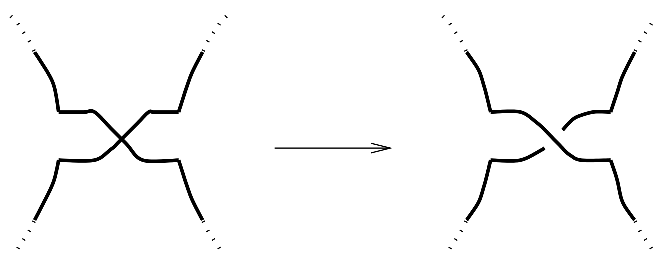

Figure 13: Simple Hyperbolic Transformation 1 (from a singular braid)

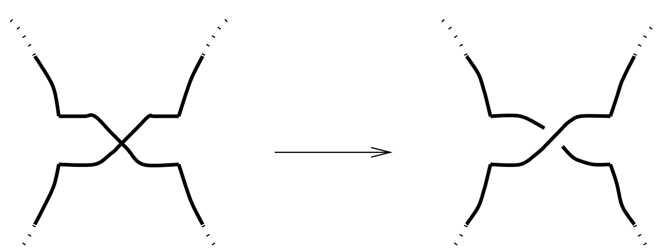

Figure 14: Simple Hyperbolic Transformation 2 (from a singular braid)

In this way the formal definition of a 2-dimensional m-braid is

Def. 2.5 *(cf. [13]) A 2-dimensional m-braid (or surface braid) is an oriented 2-dimensional manifold S embedded in $D_1^2 \times D_2^2$ such that the restriction map $pr_2|_S : S \to D_2^2$ of the second factor projection*

$pr_2 : D_1^2 \times D_2^2 \to D_2^2$ is a simple branched covering of degree m and $\partial S = X_m \times \partial D_2^2$ (where X_m is a set of m interior points of D_1^2).

There is also a closed 2-dimensional m-braid:

Def. 2.6 *(cf. [13]) A closed 2-dimensional m-braid (or surface braid) is an oriented 2-dimensional closed manifold S embedded in $D_1^2 \times S^2$ such that the restriction map $pr_2|_S : S \to S^2$ of the second factor projection $pr_2 : D_1^2 \times S^2 \to S^2$ is a simple branched covering of degree m.*

Loosely speaking, a closed 2-dimensional m-braid is obtained by "capping off" the boundary of a 2-dimensional m-braid with m standardly embedded discs.

The relevant stills (modulo closure of the braids) of the motion pictures of two 2-dimensional braids are depicted in figures 15 and 16. Also note that saddle points do not have to be all concentrated in $\mathbb{R}^3 \times \{0\}$.

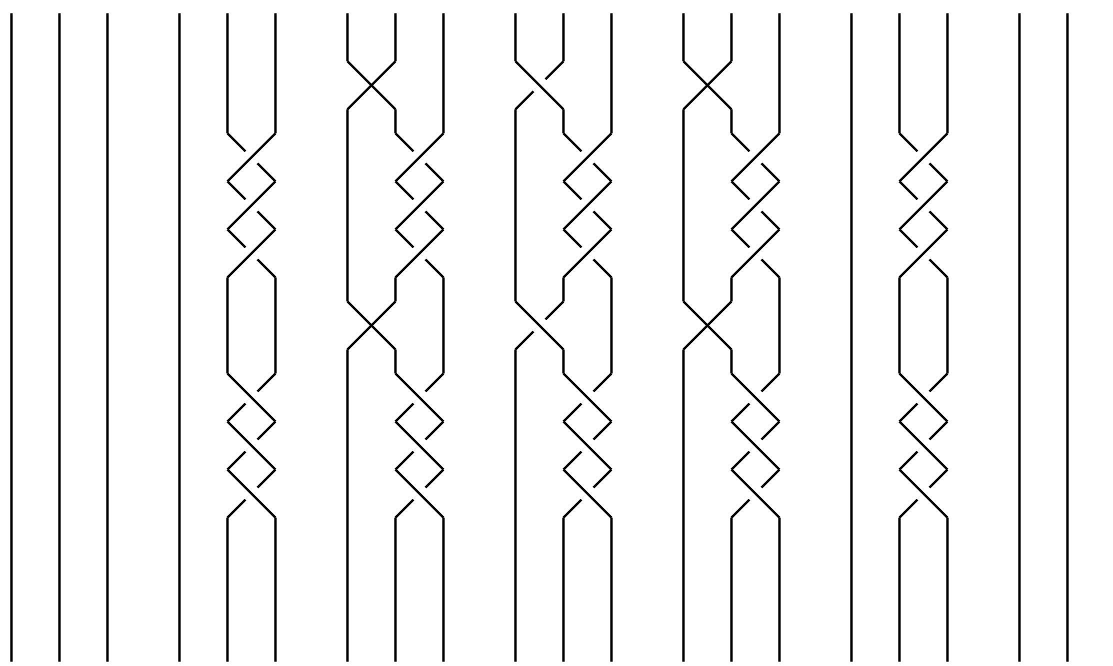

Figure 15: Movie for 2-dimensional 3-braid

If we draw the projection on the second disc factor, D_2^2, of the lines of self-intersection of the projection of the embedding then we obtain what is called a braid chart.

Def. 2.7 *(cf. [13]) A surface braid chart of degree m is a finite graph Γ in D_2^2 whose edges are oriented and labelled satisfying the following conditions:*

- *The graph is disjoint from the boundary of D_2^2;*
- *Every vertex has degree one, four or six;*
- *The labels of edges are integers in $\{1, 2, ..., m-1\}$;*
- *For each degree six vertex, three consecutive edges are oriented inward and the others are oriented outward, and the six edges are labelled i and $i+1$ alternately for some i;*

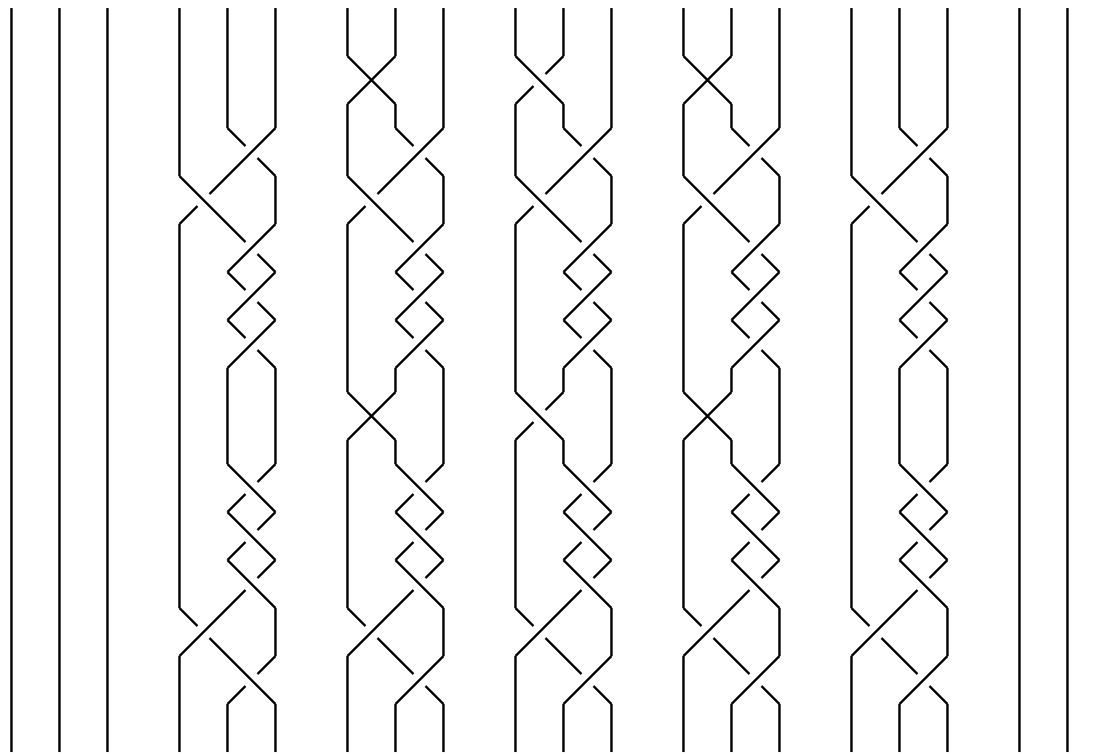

Figure 16: Movie for 2-dimensional 3-braid

- *For each degree four vertex, diagonal edges have the same label and are oriented coherently, and the labels i and j satisfy $|i-j|>1$.*

We call a degree one vertex a black vertex and a degree six vertex a white vertex. In [10] (see also [13]) it is proved that any 2-dimensional braid is described by a chart. Figures 17 and 18 are braid charts (ignore the dotted lines for now) corresponding to the 2-dimensional braids in figures 15 and 16 (repect.). A note on terminology. A *free edge* is an edge connecting two black vertices. An *oval nest* is a free edge with concentric loops around it (cf. [13]).

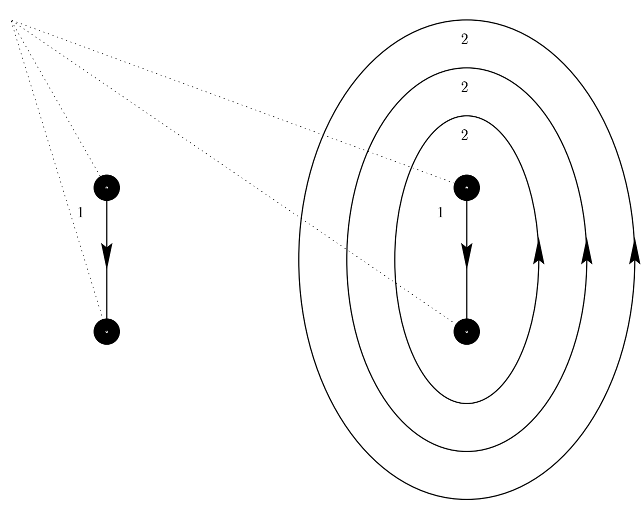

Figure 17: Chart for 2-dimensional 3-braid

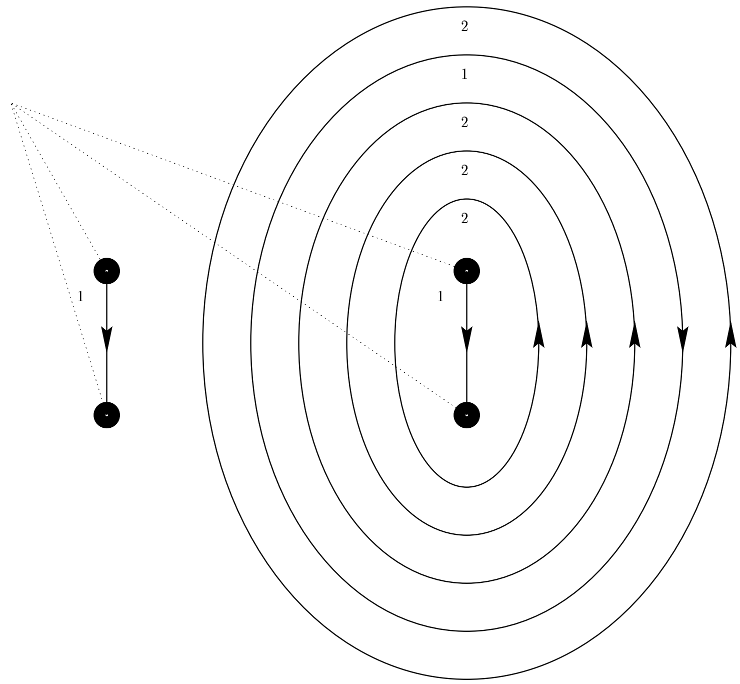

Figure 18: Chart for 2-dimensional 3-braid

2.4 The presentation of the knot quandle from braid charts

The purpose of this subsection is to describe a way of retrieving the knot quandle of a knotted surface from a corresponding chart - see [3] for the full development. Following this reference, we assign to each braid $\beta \in B_m$ an automorphism, $Q(\beta)$, of the free quandle on m generators, $FQ < x_1, ..., x_m >$ such that:

$$\begin{aligned} Q(\sigma_i)(x_i) &= x_{i+1} \bar{*} x_i \\ Q(\sigma_i)(x_{i+1}) &= x_i \\ Q(\sigma_i)(x_j) &= x_j \qquad \text{for } j \notin \{i, i+1\} \end{aligned} \qquad \text{and} \qquad \begin{aligned} Q(\sigma_i^{-1})(x_i) &= x_{i+1} \\ Q(\sigma_i^{-1})(x_{i+1}) &= x_i * x_{i+1} \\ Q(\sigma_i^{-1})(x_j) &= x_j \qquad \text{for } j \notin \{i, i+1\} \end{aligned}$$

Also note that this a right action i.e., $Q(\beta\beta')(x_k) = Q(\beta')\Big(Q(\beta)(x_k)\Big)$. Also we will let $Q(\beta)(w_1, ..., w_m) := (Q(\beta)(w_1), ..., Q(\beta)(w_m))$, where the w_i are words in the x_j.

Consider a 2-dimensional m-braid S along with one of its braid charts, Γ with n black vertices. Pick a point $q_0 \in \partial D_2^2$ where D_2^2 is the disc containing Γ. From each black vertex in Γ draw an arc to q_0 with the following properties. Each arc avoids the vertices of Γ (except the one it emanates from), each arc crosses edges transversally, and any two arcs meet only at q_0. Such a collection of arcs is called a Hurwitz arc system for the chart in point (note the dotted lines in figures 15 and 16). For each arc i write a braid word w_i such that its k-th syllable is $\sigma_{i_k}^{\epsilon_{i_k}}$ if the k-th edge crossed by arc i is labelled with i_k and $\epsilon_{i_k} = +1$ if the edge being crossed is oriented from right to left and $\epsilon_{i_k} = -1$ otherwise. In this way the following is a presentation of the knot quandle of S:

$$\langle\ x_1, ..., x_m \ \mid\ Q(w_i)(x_{k_i}) = Q(w_i)(x_{k_i+1}),\ \ i = 1, ..., n\ \rangle$$

where k_i is the label of the edge connecting to the i-th black vertex (cf. [3]).

We will now describe a simple way of retrieving this presentation.

The so-called *slide* is a left action of the braid group, B_m, on m copies of the same quandle, say Q, defined as follows.

$$slide(\sigma_i)(..., \overset{i^{th} entry}{q}, q', ...) = (..., \overset{i^{th} entry}{q' \bar{*} q}, q, ...)$$
$$slide(\sigma_i^{-1})(..., \overset{i^{th} entry}{q}, q', ...) = (..., \overset{i^{th} entry}{q'}, q * q', ...)$$

Prop. 2.4 *Consider the free quandle on m elements, $FQ\langle x_1, ..., x_m\rangle$, and pick any $\beta \in B_m$. Then, $Q(\beta)(x_1, ..., x_m) = slide(\beta)(x_1, ..., x_m)$.*

Proof: By induction on the length of β. If $\beta = \sigma_i$, then

$$Q(\sigma_i)(x_1, ..., x_i, x_{i+1}, ..., x_m) = (x_1, ..., x_{i+1} \bar{*} x_i, x_i, ..., x_m) = slide(\sigma_i)(x_1, ..., x_m)$$

and analogously for $\beta = \sigma_i^{-1}$. Now for the induction step. Suppose $\beta \in B_m$ is a word of length n.

$$\begin{aligned} slide(\sigma_i\beta)(x_1, ..., x_i, x_{i+1}, ..., x_m) &= slide(\sigma_i)slide(\beta)(x_1, ..., x_i, x_{i+1}, ..., x_m) = \\ &= slide(\sigma_i)(Q(\beta)(x_1), ..., Q(\beta)(x_i), Q(\beta)(x_{i+1}), ..., Q(\beta)(x_m)) = \\ &= ((Q(\beta)(x_1), ..., Q(\beta)(x_{i+1}) \bar{*} Q(\beta)(x_i), Q(\beta)(x_i), ..., Q(\beta)(x_m)) = \\ &= Q(\sigma_i\beta)(x_1), ..., Q(\sigma_i\beta)(x_i), Q(\sigma_i\beta)(x_{i+1}), ..., Q(\beta)(x_m)) = Q(\sigma_i\beta)(x_1, ..., x_m) \end{aligned}$$

and similarly, had we considered $\sigma_i^{-1}\beta$ instead of $\sigma_i\beta$, which concludes the proof. ■

We can now give a graphical interpretation to the calculation of $Q(\beta)$ which simplifies it. The slide action of a braid word of B_m, say $\sigma_{i_1}^{\epsilon_1}...\sigma_{i_n}^{\epsilon_n}$ on $FQ\langle x_1, ..., x_m\rangle$ tells us first to consider x_1,...,x_m at the bottom of the braid with the strands emerging from each of them. Then we compute the action of $\sigma_{i_n}^{\epsilon_n}$ on the x_1,...,x_m, in the usual quandle context of arcs meeting at a crossing. Then we compute the action of $\sigma_{i_{n-1}}^{\epsilon_{n-1}}$ on the result of the previous action, and so on, and so forth until we reach the top. There, from left to right we find $Q(\sigma_{i_1}^{\epsilon_1}...\sigma_{i_n}^{\epsilon_n})(x_1)$, $Q(\sigma_{i_1}^{\epsilon_1}...\sigma_{i_n}^{\epsilon_n})(x_2)$,...,$Q(\sigma_{i_1}^{\epsilon_1}...\sigma_{i_n}^{\epsilon_n})(x_m)$. We believe figure 19 will now be self-explanatory.

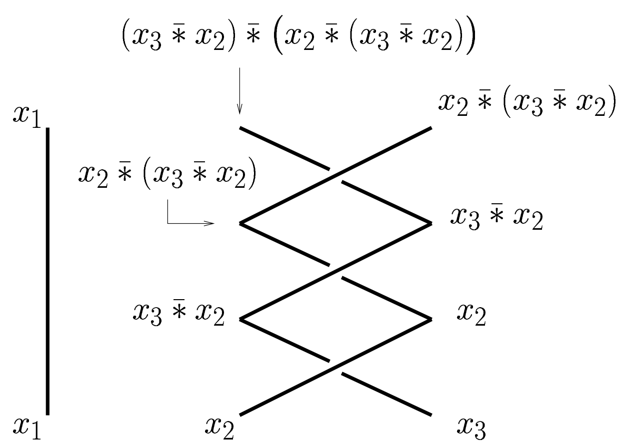

Figure 19: Calculating $Q(\sigma_2\sigma_2\sigma_2)$

We will work out the calculations associated with the charts depicted in figures 17 and 18. As remarked, the dotted lines oriented from the black vertices to a point in the upper left part of figure 17 stand for a Hurwitz arc system of this chart (analogously for 18). Numbering the arcs from left to right we have the corresponding braid words, $w_1 = 1$, $w_2 = 1$, $w_3 = \sigma_2^3$, $w_4 = \sigma_2^3$. The relations they give rise to are $x_1 = x_2$ and $x_1 = (x_3 \bar{*} x_2) \bar{*} (x_2 \bar{*} (x_3 \bar{*} x_2))$. Hence a presentation of the associated knot quandle is

$$\langle\ x_1, x_3 \ \mid\ x_1 = (x_3 \bar{*} x_1) \bar{*} (x_1 \bar{*} (x_3 \bar{*} x_1))\ \rangle$$

Analogously, in figure 18 consider the dotted lines from the black vertices to the point in the upper left part of the figure and note that they stand for a Hurwitz arc system for this chart. The associated braid words are $w_1 = 1$, $w_2 = 1$, $w_3 = \sigma_2^3\sigma_1^{-1}\sigma_2$, $w_4 = \sigma_2^3\sigma_1^{-1}\sigma_2$. The relations they give rise to are $x_1 = x_2$ and $x_3 \bar{*} x_2 = \left(x_2 \bar{*} \left(x_1 * (x_3 \bar{*} x_2)\right)\right) \bar{*} \left(\left(x_1 * (x_3 \bar{*} x_2)\right) \bar{*} \left(x_2 \bar{*} \left(x_1 * (x_3 \bar{*} x_2)\right)\right)\right)$. A presentation of the associated knot quandle is

$$\left\langle \; x_1, x_3 \;\middle|\; x_3 \bar{*} x_1 = \left(x_1 \bar{*} \left(x_1 * (x_3 \bar{*} x_1)\right)\right) \bar{*} \left(\left(x_1 * (x_3 \bar{*} x_1)\right) \bar{*} \left(x_1 \bar{*} \left(x_1 * (x_3 \bar{*} x_1)\right)\right)\right) \; \right\rangle$$

3 Distinguishing Elements From Classes of Knotted Surfaces

In this section we describe the classes of knotted surfaces whose elements we propose to distinguish via comparing numbers of colorings by given quandles and we describe how the counting of colorings was implemented. We also remark on the results we obtained.

3.1 A Class Of Ribbon Knots

The first class of knotted surfaces we consider was studied by Kamada in [12]. This is a class of closed 2-dimensional braids of braid index less than or equal to 3 and Euler characteristic 2. Since the braid index is less than or equal to 3 we are regarding ribbon knots i.e., knotted surfaces which are obtained from a disjoint union of standardly embedded 2-spheres by surgery along 1-handles attached to them (cf. [10]). We will now give a short description of Kamada's work in [12]. There it is shown that 2-dimensional 3-braids of Euler characteristic 2 can be described by braid charts with two free edges (one of which together with an oval nest) and label 1 on the free edges. The loops in the oval nests are labelled with 1's and 2's; the loops labelled with 1's are oriented clockwise and the loops labelled with 2's are oriented counterclockwise. The innermost and the outermost loops are always labelled with 2 for otherwise the free edges would absorb these loops (see figures 17 and 18). A first classification of these charts is done according to the number of loops in the oval nest, say α. For each α, the $\alpha-2$ loops between the innermost and the outermost loops may each be labelled either with a 1 or a 2, so there are as many possibilities as there are sets in G_α, the power set of $\{1, 2, ..., \alpha - 2\}$. For each $g \in G_\alpha$, the oval nest (and hence the braid chart) is recovered as follows. If $1 \in g$ then the second loop (counting from the outermost loop) is labelled with one, otherwise two. If $2 \in g$ then the third loop is labelled with one, otherwise two. And so on and so forth. The following operations on each $g \in G_\alpha$ are considered. $g^{co} = \{1, 2, ..., \alpha - 2\} \setminus g$, $g^{op} = \{\alpha - 1 - j | j \in g\}$, and $g^{coop} = g^{opco}$. It is then proved that knotted surfaces corresponding to g^{co} and g^{op} are deformable into mirror images of those corresponding to g (cf. lemma 3.1 in [12]). In this way, an equivalence relation, $\sim$, is defined on G_α such that $g \sim g^{co} \sim g^{op} \sim g^{coop} = g^{opco}$ and a surjective map from $\{G_\alpha/\sim\}_{\alpha\in\mathbb{N}_2}$ to the class of deformations of closed 2-dimensional 3-braids of Euler characteristic 2 (modulo mirror image) is set up in the following way. For each $[g] \in G_\alpha/\sim$, we pick a representative $g \in [g]$ and construct the oval nest as above. Is this map injective? For $\alpha \leq 10$, Kamada proves it is by looking at the Alexander polynomials of the corresponding elements (modulo three pairs which he also distinguishes using another method). He also tells which of these knots are invertible and which are not. The questions remain open for $\alpha > 10$. Note that for $\alpha \leq 10$ there are 151 knotted surfaces.

We recover the results above (modulo the three pairs just referred to and modulo telling apart reverses) by counting colorings. We now describe the methods and algorithms for listing these knotted surfaces and retrieving their knot quandles. Instead of considering, for each α, representatives of $G_\alpha/\sim$ we consider sequences of $\alpha - 2$ binary digits, 0's and 1's, and we recover the oval nest in a similar way. For each sequence of $\alpha - 2$ 0's and 1's, if the first digit is 1 then the second loop (counting from the innermost loop) is labelled with 1, otherwise 2. If the second digit is 1 then the the third loop is labelled with 1, otherwise 2. And so on, and so forth. The co- operation corresponds to binary negation ($0 \to 1$, and $1 \to 0$) and the op- operation to reversing the sequence. In this way, we consider, for each α, sequences of $\alpha - 2$ binary digits as numbers ordered by magnitude, and arrange them into $\sim$-equivalence classes in the following way. For a given α, the first one is 00...0 ($\alpha - 2$ digits). So its equivalence class is made up of itself, 00...0 = 00...0^{op}, and 00...0^{co} = 00...0^{coop} = 11...1; we pick the lowest i.e., 00...0, for its representative. We then consider the next one in the ordering, 00...1 ($\alpha - 2$ digits), calculate 00...1^{co}, 00...1^{op}, 00...1^{coop} and pick the lowest i.e., 00...1, for its representative. We proceed in this way. If for a given $i_1 i_2 ... i_{\alpha-2}$ with $i_j \in \{0,1\}$, there is ${i_1 i_2 ... i_{\alpha-2}}^{co}$, ${i_1 i_2 ... i_{\alpha-2}}^{op}$, or ${i_1 i_2 ... i_{\alpha-2}}^{coop}$ which is strictly less than $i_1 i_2 ... i_{\alpha-2}$ we know we are looking at a class which was already calculated and do not consider it again.

If $\alpha - 2 = 2N$, for some integer $N \geq 1$, there are $2^{2N-2} + 2^{N-1}$ equivalence classes. If $\alpha - 2 = 2N + 1$, for some integer $N \geq 1$, there are $2^{2N-1} + 2^{N-1}$ equivalence classes. In [12], Kamada distinguished all equivalence classes from $\alpha = 2$ to $\alpha = 10$ which amounts to 11325 comparisons. We distinguished all but three pairs: 000010000 and 001111100 ($\alpha = 9$); 0000101000 and 0000110100 ($\alpha = 10$); and 0001011100 and 0010111100 ($\alpha = 10$). In a second experience we let α range from 2 to 18, which amounts to $549,477,825$ comparisons. We distinguished all but 628 pairs.

Let us now see how to retrieve the knot quandle. Consider the braid charts of figures 17 and 18. They are particular cases of the ribbon knots we have been discussing. On both cases, the contribution of the free edge on the left to the knot quandle presentation is the relation $x_1 = x_2$. Now for the contribution of the oval nests. The oval nest of figure 17 is encoded 000, whereas that of figure 18 is encoded 00010. We let the digits 0 and 1 act on the left on (a, b, c) as follows. $0(a, b, c) = (a, c \bar{*} b, b)$ and $1(a, b, c) = (b, a * b, c)$ (see figure 20 for a graphical interpretation). In the case of figure 17 we have:

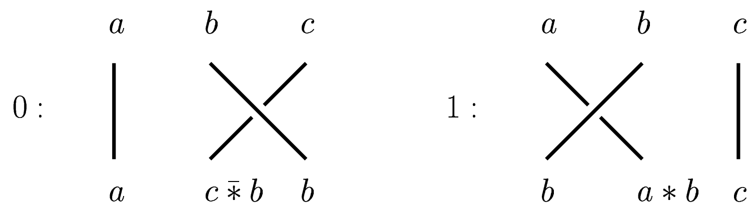

Figure 20: 0 and 1 acting on (a, b, c)

$$
\begin{aligned}
000(x_1, x_2, x_3) &= 00(x_1, x_3 \bar{*} x_2, x_2) = 0\big(x_1, x_2 \bar{*} (x_3 \bar{*} x_2), x_3 \bar{*} x_2\big) = \\
&= \big(x_1, (x_3 \bar{*} x_2) \bar{*} \big((x_2 \bar{*} (x_3 \bar{*} x_2)\big), x_3 \bar{*} (x_3 \bar{*} x_2)\big)
\end{aligned}
$$

So the relation obtained here is $x_1 = (x_3 \bar{*} x_2) \bar{*} \big((x_2 \bar{*} (x_3 \bar{*} x_2)\big)$ and the presentation of the knot quandle corresponding to the braid chart of figure 17 is

$$
\langle \; x_1, x_3 \;\mid\; x_1 = (x_3 \bar{*} x_1) \bar{*} \big(x_1 \bar{*} (x_3 \bar{*} x_1)\big) \; \rangle
$$

As for the oval nest in figure 18, we have:

$$
\begin{aligned}
&00010(x_1, x_2, x_3) = 0001(x_1, x_3 \bar{*} x_2, x_2) = 000(x_3 \bar{*} x_2, x_1\big(x_3 \bar{*} x_2), x_2\big) = \\
&\quad = 00\big(x_3 \bar{*} x_2, x_2 \bar{*} (x_1 * (x_3 \bar{*} x_2)\big), x_1 * (x_3 \bar{*} x_2)\big) = \\
&\quad = 0\Bigg(x_3 \bar{*} x_2, (x_1 * (x_3 \bar{*} x_2)) \bar{*} \Big(x_2 \bar{*} (x_1 * (x_3 \bar{*} x_2))\Big), x_2 \bar{*} (x_1 * (x_3 \bar{*} x_2))\Bigg) = \\
&\quad = \Big(x_3 \bar{*} x_2, \Big(x_2 \bar{*} (x_1 * (x_3 \bar{*} x_2))\Big) \bar{*} \Big(\big(x_1 * (x_3 \bar{*} x_2)\big) \bar{*} \Big(x_2 \bar{*} (x_1 * (x_3 \bar{*} x_2))\Big)\Big), \\
&\quad (x_1 * (x_3 \bar{*} x_2)) \bar{*} \Big(x_2 \bar{*} (x_1 * (x_3 \bar{*} x_2))\Big)\Big)
\end{aligned}
$$

whence a presentation for the knot quandle corresponding to the braid chart of figure 18 is

$$
\Big\langle \; x_1, x_3 \;\mid\; x_3 \bar{*} x_1 = \Big(x_1 \bar{*} (x_1 * (x_3 \bar{*} x_1))\Big) \bar{*} \Big(\big(x_1 * (x_3 \bar{*} x_1)\big) \bar{*} \Big(x_1 \bar{*} (x_1 * (x_3 \bar{*} x_1))\Big)\Big) \; \Big\rangle
$$

Note that we arrived at the same expressions as in 2.4, as expected. Once the presentation of a knot quandle is known it is a simple matter to list the colorings by a labelling quandle and subsequently count them. We first map the generators of the knot quandle to elements of the labelling quandle. Then, if upon replacement of the generators of the knot quandle by their images, the relations are still satisfied in the labelling quandle, we know the map is a coloring. Otherwise it is not a coloring. We will next exemplify this using R_3 as a labelling quandle to color the two ribbon knots just considered. R_3 is significantly powerful in telling knots apart (via counting colorings). Also, it has only three elements, it coincides with its dual, and has a pretty simple multiplication table. The "product" of two distinct elements yields the third element - the remaining case is covered by the first axiom. In this way, it is easy to see that, in the case of the braid chart of figure 17, any assignment of x_1 and x_3 to two elements of R_3 yields a coloring. The number of colorings here is then 9. On the other hand, for the braid chart of figure 18, x_1 and x_3 have to be mapped to the same element in R_3 in order to have a coloring (i.e., it only admits trivial colorings). The number of colorings here is then 3. Since $9 \neq 3$, these braid charts correspond to knotted surfaces which are not deformable into each other.

Our choice of labelling quandles relied heavily on linear Alexander quandles for they are simpler and less memory consuming. We used all linear Alexander quandles from $\mathbb{Z}_3[T, T^{-1}]/(T+1)$, through $\mathbb{Z}_{245}[T, T^{-1}]/(T-179)$, beyond which we started to have processing and memory problems. The other Alexander quandles in the list allowed us to distinguish more cases. We raise the question whether linear quandles alone would suffice should we be able to increase their list.

For each $\alpha \geq 2$ and for each $g \in G_\alpha$ if $g = g^{op}$ then the corresponding ribbon knots are amphicheiral (hence invertible). Kamada proves the converse is also true (for $2 \leq \alpha \leq 10$) by looking at Alexander polynomials (cf. [12]). We could not reproduce these results. Specifically, for all $2 \leq \alpha \leq 18$, for all $g \in G_\alpha$ such that $g \neq g^{op}$, there was **no** quandle in the list mentioned above producing different numbers of colorings for g and g^{op}.

The results concerning the above mentioned class of knotted surfaces from $\alpha = 2$ through $\alpha = 10$ are shown in appendix A. We split the list of labelling quandles into sublists of 16 quandles. In table A.1 we display the number of pairs distinguished by the first 16 quandles and in the last row we display the number of pairs these quandles were not able to distinguish. In tables A.2 through A.4 we show the numbers of colorings each of these quandles produced on all knotted surfaces in point. In table A.5 we show the number of pairs distinguished by the next 16 quandles from the pairs not distinguished by the first 16 quandles, and also the number of pairs not distinguished so far. In tables A.6 through A.8 we show the numbers of colorings the second 16 quandles produced on the knotted surfaces not distinguished by the first 16 quandles. And so on and so forth until we reached quandle $\mathbb{Z}_{245}[T, T^{-1}]/(T-179)$. Note that we reached the final result (three inconclusive cases) by quandle 198, $\mathbb{Z}_{27}[T, T^{-1}]/(T-5)$. Note that quandle 33, $\mathbb{Z}_{11}[T, T^{-1}]/(T-3)$, had already distinguished two of those pairs - see table 3.1. The pairs of equivalence classes not told apart by counting colorings are exactly the same pairs that are not told apart by their Alexander polynomials (in Kamada's work) - see table 3.2.

	$\mathbb{Z}_{11}[T, T^{-1}]/(T-3)$
0000101000	121
0000110100	11
0001011100	11
0010111100	121

Table 3.1: Number of Colorings by $\mathbb{Z}_{11}[T, T^{-1}]/(T-3)$ on the Indicated Equivalence Classes ($\alpha = 10$)

$\alpha = 9$	000010000	-	001111100
$\alpha = 10$	0000101000	-	0010111100
$\alpha = 10$	0000110100	-	0001011100

Table 3.2: Pairs of Equivalence Classes Which Were not Told Apart Either by Counting Colorings or by the Alexander Polynomial

We repeated this procedure for the equivalence classes from $\alpha = 2$ through $\alpha = 18$ which amounts to comparing $549,477,825$ pairs of equivalence classes. We distinguished them all but 628 pairs. We used the same list of labelling quandles as above. The huge amount of data this time prevents us from displaying it.

3.2 A Class Of Twist-spun Knots

In this subsection we consider a class of twist-spun knots. Following [13], a d-twist-spun knot is defined as follows. Let (B, K_0) be a pair of a 3-ball B^3 and a properly embedded arc K_0. Let A be an unknotted arc in B with $\partial A = \partial K_0$. Let $R_\theta : B \mapsto B$ be rotation through the angle θ about A in the positive meridian direction. Define:

$$\Sigma = B \times S^1/(x,\theta) \sim (x,\theta') \text{ for } x \in \partial B, \text{ and } \theta, \theta' \in S^1;$$

$$F = \cup_\theta (R_{d\theta}(K_0) \times \{e^{i\theta}\})/\sim .$$

Then Σ is a 4-sphere and F is a 2-sphere. Removing a point from Σ, we have a 2-knot F in $\mathbb{R}^4$. This is a d-twist-spun knot.

Let $\tau^d T(n,m)$ denote the d-twist-spin of $T(n,m)$. In [2], we found presentations of the knot quandle of the $\tau^2 T(2,m)$ for integer $m \geq 3$:

$$\langle x_2, x_3 \quad | \quad x_3 * (x_2 x_3)^{(m-1)/2} = x_2, \quad x_2 * x_3^2 = x_2 \rangle \quad \text{for odd } m; \text{ and}$$

$$\langle x_2, x_3 \quad | \quad x_2 * (x_3 x_2)^{m/2} = x_2, \quad x_2 * x_3^2 = x_2 \rangle \quad \text{for even } m.$$

Using these expressions we implemented a program to count colorings by specific labelling quandles in order to try to tell apart the corresponding $\tau^2 T(2,m)$ for different m's. (Note that, in the previous subsection we first had to find a way of encoding the knotted surfaces, then to retrieve the knot quandle of each of them before we could count colorings.) From computations in [2], we know that, for any integer $m \geq 3$, the knot quandle of the $\tau^2 T(2,m)$ is isomorphic with the knot quandle of its mirror image, of its reverse, and of the mirror image of its reverse.

In order to fix a set of these knotted surfaces, we let $3 \leq m \leq 1000$ which gave rise to 523,776 comparisons. We first used the same list of labelling quandles we used in the previous subsection. With this list of quandles we ended up with a little bit less that 2% of inconclusive cases. On the other hand, since it was basically the dihedral quandles that were doing the job, we set up a new list of labelling quandles, this time with R_n quandles only ($3 \leq n \leq 512$). With these R_n's for labelling quandles we got 3014 inconclusive cases (which is less than 1%). In appendix B we display the number of pairs of distinct $\tau^2 T(2,m)$'s each R_n distinguished. In appendix C we exhibit the numbers of colorings of each of the R_n quandles for $3 \leq n \leq 26$. Inspection of the first few tables of appendix C suggests that the number of colorings each R_n produces on the $\tau^2 T(2,m)$'s as function of m has periodicity n. This is, in fact, true.

Prop. 3.1 *Consider the presentations above for the knot quandle of $\tau^2 T(2,m)$ for $m \geq 3$. If R_n is a labelling quandle for the $\tau^2 T(2,m)$ then the assignment $x_2 \mapsto a$, and $x_3 \mapsto b$, (for a, and $b \in R_n$) stands for a coloring if and only if $ma \equiv mb \pmod n$.*

Proof: We will use induction on m for odd m; it is analogous for even m.

For $m = 3$ we have:

$$\begin{cases} b * (ab)^{(3-1)/2} = a \\ a * b^2 = a \end{cases} \iff \begin{cases} (b * a) * b = a \\ (a * b) * b = a \end{cases} \iff \begin{cases} (2a - b) * b = a \\ (2b - a) * b = a \end{cases} \iff \begin{cases} 2b - (2a - b) = a \\ 2b - (2b - a) = a \end{cases} \iff$$

$$\begin{cases} 3b - 2a = a \\ a = a \end{cases}$$

In particular, $3a \equiv 3b \pmod{n}$. Now assume that, for some odd m

$$b * (ab)^{(m-1)/2} = a \Longleftrightarrow mb - (m-1)a = a$$

Then

$$a = b * (ab)^{((m+2)-1)/2} = \Big(b * (ab)^{(m-1)/2}\Big) * (ab) = \Big(mb - (m-1)a\Big) * (ab) = \Big(2a - mb + (m-1)a\Big) * b$$
$$= 2b - 2a + mb - (m-1)a = (m+2)b - (m+1)a$$

and the result now follows. ■

Cor. 3.1 *Fix an integer $n \geq 3$. If R_n is the labelling quandle for the $\tau^2 T(2,m)$'s, the number of colorings has periodicity n as a function of m.*

Proof: From the previous proposition it is clear that the periodicity is equal to n, at most. Suppose n is prime. Then $\mathbb{Z}_n$ is a field and

$$m(a-b) \equiv 0 \pmod{n} \qquad \Longleftrightarrow \qquad m \equiv 0 \pmod{n} \quad \text{or} \quad (a-b) \equiv 0 \pmod{n}$$

If it is not true that $m \equiv 0 \pmod{n}$ then $a \equiv b \pmod{n}$, which amounts to n solutions. If $m \equiv 0 \pmod{n}$ then any choice of a, and b is a solution which amounts to n^2 solutions. The result follows for prime n.

Suppose n is not prime. If $n \mid m$ then $m(a-b) \equiv 0 \pmod{n}$ is satisfied by any choice of a and b (n^2 solutions). If $n \nmid m$ then let $m' \equiv m \pmod{n}$ such that $0 < m' < n$. If $m' \nmid n$ then $m(a-b) \equiv 0 \pmod{n} \Longleftrightarrow a = b \pmod{n}$ (n solutions). If $m' \mid n$ then pick $0 < k < n$ such that $n = km'$. Then any a and b such that $a - b \equiv k \pmod{n}$ or $a \equiv b \pmod{n}$ is a solution. In particular, $(a,b) = (k,0)$ is a solution, and $(a,b) = (k,1)$ is **not** a solution. Thus, the number of solutions here is greater than n but less than n^2. The result follows for non-prime n which concludes the proof. ■

We will now show, for each choice of distinct integers m, $m' \geq 3$, how to produce a labelling R_n yielding different numbers of colorings for $\tau^2 T(2,m)$ and $\tau^2 T(2,m')$.

Thm. 3.1 *Let m, and m' be distinct integers strictly greater than 2.*

- *If there is a prime p such that $p \mid m$ and $p \nmid m'$, then R_p yields p^2 colorings for the $\tau^2 T(2,m)$ and p colorings for the $\tau^2 T(2,m')$;*
- *Otherwise pick a prime, p, such that, for some integer $k > 1$, $p^{k-1} \mid m$, $p^k \nmid m$, and $p^k \mid m'$. Then R_{p^k} yields p^{2k} colorings for the $\tau^2 T(2,m')$ while for the $\tau^2 T(2,m)$ it yields strictly less than p^{2k} colorings.*

Proof: Follows easily from the proof of corollary above. ■

In this way, we have shown that it is always possible to tell apart any two distinst elements of this class of knotted surfaces by counting colorings by dihedral quandles of orders p^k, where p is a prime and k is a positive integer.

As a concluding remark, we feel we have given convincing evidence that counting of colorings is an invariant of knotted surfaces that should be further studied and applied.

References

[1] J. S. Carter, M. Saito, *Knotted Surfaces and Their Diagrams*, Mathematical Surveys and Monographs **55**, American Mathematical Society (1998)

[2] J. S. Carter, D. Jelsovsky, S. Kamada, M. Saito, *Computations of Quandle Cocycle Invariants of Knotted Curves and Surfaces*, Adv. Math. **157** (2001), no. 1, 36-94

[3] J. S. Carter, D. Jelsovsky, S. Kamada, L. Langford, M. Saito, *Quandle Cohomology and State-sum Invariants of Knotted Curves and Surfaces*, Trans. Amer. Math. Soc., **355** (2003), no. 10, 3947- 3989

[4] F. M. Dionísio, P. Lopes, *Quandles at Finite Temperatures II*, to appear in J. Knot Theory Ramifications, available at math.GT/0205053

[5] R. Fenn, C. Rourke, *Racks and Links in Codimension Two*, J. of Knot Theory Ramifications, **1** (1992), no. 4, 343-406

[6] R. Fenn, C. Rourke, B. Sanderson, *Trunks and Classifying Spaces*, Appl. Categ. Structures, **3** (1995), no. 4, 321-356

[7] R. Fenn, C. Rourke, B. Sanderson, *James Bundles and Applications*, preprint found at http://www.maths.warwick.ac.uk/~bjs/

[8] R. H. Fox, *A Quick Trip Through Knot Theory*, in "Topology of 3-Manifolds and Related Topics" (Georgia, 1961), Prentice-Hall (1962) 120-167

[9] D. Joyce, *A Classifying Invariant of Knots, The Knot Quandle*, J. Pure Appl. Alg., **23** (1982), 37-65

[10] S. Kamada, *Surfaces in $\mathbb{R}^4$ of Braid Index Three Are Ribbon*, J. Knot Theory Ramifications, **1** (1992), no. 2, 137-160

[11] S. Kamada, *A Characterization of Groups of Closed Orientable Surfaces in 4-Space*, Topology, **33** (1994), no. 1, 113-122

[12] S. Kamada, *Standard Forms of 3-Braid 2-Knots and their Alexander Polynomials*, Michigan Math. J., **45** (1998), no. 1, 189-205

[13] S. Kamada, *Braid and Knot Theory in Dimension Four*, Mathematical Surveys and Monographs **95**, American Mathematical Society (2002)

[14] A. Kawauchi, T. Shibuya, S. Suzuki, *Descriptions on Surfaces in Four-Space, I: Normal Forms*, Mathematics Seminar Notes Kobe University, **10** (1982), no. 1, 75-125

[15] L. H. Kauffman, *Knots and Physics*, 3rd edition, Series on Knots and Everything **1**, World Scientific Publishing Co., River Edge, NJ (2001)

[16] P. Lopes, *Quandles at Finite Temperatures I*, J. Knot Theory Ramifications, **12** (2003), no. 2, 159-186

[17] S. V. Matveev, *Distributive Groupoids in Knot Theory*, Math. USSR Sbornik, **47** (1984), no. 1, 73-83

[18] S. Nelson, *Classification of Finite Alexander Quandles*, preprint at http://xxx.lanl.gov/abs/math.GT/0202281

[19] D. Roseman, *Reidemeister-Type Moves for Surfaces in Four-Dimensional Space*, in "Knot Theory" (Warsaw, 1995), Banach Center Publ., 42, Polish Academy of Sciences, Warsaw (1998) 347-380

A Distinguishing Ribbon Knots ($2 \leq \alpha \leq 10$)

Quandle	Quandle Number	Number of Pairs Distinguished
R3	1	4578
$\mathbb{Z}_3[T, T^{-1}]/(T^2 + 1)$	2	2250
$\mathbb{Z}_2[T, T^{-1}]/(T^2 + T + 1)$	3	0
$\mathbb{Z}_3[T, T^{-1}]/(T^2 + T - 1)$	4	965
$\mathbb{Z}_2[T, T^{-1}]/(T^2 + 1)$	5	1119
$\mathbb{Z}_2[T, T^{-1}]/(T^3 + T^2 + 1)$	6	598
$\mathbb{Z}_2[T, T^{-1}]/(T^3 + T + 1)$	7	139
$\mathbb{Z}_3[T, T^{-1}]/(T^2 - 1)$	8	0
$\mathbb{Z}_3[T, T^{-1}]/(T^2 - T + 1)$	9	0
$\mathbb{Z}_2[T, T^{-1}]/(T^3 + 1)$	10	0
$\mathbb{Z}_4[T, T^{-1}]/(T - 3)$	11	0
$\mathbb{Z}_5[T, T^{-1}]/(T - 2)$	12	0
$\mathbb{Z}_5[T, T^{-1}]/(T - 3)$	13	0
$\mathbb{Z}_5[T, T^{-1}]/(T - 4)$	14	516
$\mathbb{Z}_6[T, T^{-1}]/(T - 5)$	15	0
$\mathbb{Z}_7[T, T^{-1}]/(T - 2)$	16	297

Undistinguished Pairs So Far:	863

Table A.1: Quandles 1-16 and Their Performances

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
00	3	9	4	9	8	8	8	27	9	16	8	5	5	5	12	7
000	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0000	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	7
0010	3	9	4	9	4	64	8	9	9	8	4	5	5	25	6	49
00000	3	9	4	9	4	8	8	9	9	8	4	5	5	25	6	7
00010	3	9	4	81	4	8	8	9	9	8	4	5	5	5	6	7
00100	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	7
000000	9	9	16	9	8	8	8	81	81	64	8	5	5	5	36	49
000010	9	9	16	9	4	8	64	27	81	32	4	5	5	5	18	7
000100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	49
000110	3	9	4	9	8	8	8	27	9	16	8	5	5	25	12	7
001010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
001100	9	81	16	9	16	8	8	81	81	64	16	25	25	5	36	7
0000000	3	9	4	9	4	64	64	9	9	8	4	5	5	5	6	7
0000010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0000100	3	9	4	9	8	8	8	27	9	16	8	5	5	5	12	7
0000110	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	49
0001000	9	9	16	9	4	8	8	27	81	32	4	5	5	25	18	7
0001010	9	9	16	9	8	8	64	81	81	64	8	5	5	5	36	7
0001100	3	9	4	9	4	64	8	9	9	8	4	5	5	5	6	7
0010010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0010100	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0011100	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	7
00000000	3	81	4	81	16	8	8	27	9	16	16	25	25	5	12	7
00000010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
00000100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
00000110	3	81	4	9	16	64	8	27	9	16	16	25	25	5	12	7
00001000	3	9	4	9	4	64	8	9	9	8	4	5	5	5	6	7
00001010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
00001100	3	9	4	81	8	8	8	27	9	16	8	5	5	5	12	7
00001110	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
00010010	3	9	4	9	8	8	8	27	9	16	8	5	5	5	12	7
00010100	3	9	4	9	4	64	8	9	9	8	4	5	5	5	6	7
00010110	3	9	4	9	4	8	8	9	9	8	4	5	5	25	6	7
00011000	9	81	16	9	16	8	8	81	81	64	16	25	25	5	36	7
00011010	9	9	16	9	4	8	64	27	81	32	4	5	5	5	18	7
00011100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7

Table A.2: Configurations vs. Quandles 1-16: Numbers of Colorings

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
00100100	9	9	16	9	8	8	8	81	81	64	8	5	5	25	36	7
00101010	3	9	4	9	8	8	8	27	9	16	8	5	5	5	12	7
00101100	3	9	4	81	4	8	8	9	9	8	4	5	5	5	6	7
00110010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
00111100	3	81	4	9	16	8	8	27	9	16	16	25	25	25	12	7
000000000	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
000000010	9	9	16	9	4	8	8	27	81	32	4	5	5	25	18	7
000000100	3	81	4	9	16	8	8	27	9	16	16	25	25	25	12	7
000000110	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
000001000	3	9	4	81	4	8	8	9	9	8	4	5	5	5	6	7
000001010	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	7
000001100	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
000001110	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
000010000	9	81	16	9	16	8	8	81	81	64	16	25	25	5	36	49
000010010	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
000010100	3	9	4	81	4	8	8	9	9	8	4	5	5	5	6	49
000010110	3	81	4	9	16	64	8	27	9	16	16	25	25	5	12	7
000011000	3	9	4	9	4	64	8	9	9	8	4	5	5	5	6	7
000011010	3	9	4	9	4	8	8	9	9	8	4	5	5	25	6	7
000011100	9	9	16	9	8	8	8	81	81	64	8	5	5	25	36	7
000100010	3	9	4	9	8	8	8	27	9	16	8	5	5	5	12	7
000100100	3	9	4	9	4	64	8	9	9	8	4	5	5	5	6	7
000100110	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
000101000	3	81	4	9	16	8	8	27	9	16	16	25	25	25	12	7
000101010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
000101100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
000110010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
000110100	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	49
000111000	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
000111010	9	81	16	9	16	8	64	81	81	64	16	25	25	5	36	7
000111100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
001000010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
001000100	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
001001010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
001001100	3	9	4	9	8	8	8	27	9	16	8	5	5	5	12	49
001010010	3	9	4	9	8	64	8	27	9	16	8	5	5	25	12	49
001010100	3	9	4	81	4	8	8	9	9	8	4	5	5	25	6	7
001011100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
001100010	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
001101100	9	9	16	9	4	8	8	27	81	32	4	5	5	25	18	7
001111100	9	81	16	9	16	8	8	81	81	64	16	25	25	5	36	49
0000000000	3	9	4	9	8	8	8	27	9	16	8	5	5	25	12	7
0000000010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	49
0000000100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0000000110	3	9	4	9	8	8	8	27	9	16	8	5	5	5	12	7
0000001000	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0000001010	9	9	16	81	4	8	8	27	81	32	4	5	5	5	18	7
0000001100	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	7
0000001110	3	9	4	81	4	8	8	9	9	8	4	5	5	25	6	7
0000010000	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0000010010	3	81	4	9	16	8	64	27	9	16	16	25	25	25	12	7
0000010100	9	9	16	9	4	8	8	27	81	32	4	5	5	25	18	7
0000010110	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0000011000	3	9	4	81	8	8	8	27	9	16	8	5	5	5	12	7
0000011010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	49
0000011100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0000011110	3	9	4	9	8	8	64	27	9	16	8	5	5	5	12	7
0000100010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0000100100	3	81	4	81	16	8	64	27	9	16	16	25	25	5	12	7
0000100110	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	49
0000101000	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0000101010	9	81	16	9	16	8	8	81	81	64	16	25	25	25	36	7
0000101100	3	9	4	9	4	8	8	9	9	8	4	5	5	25	6	7
0000101110	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	49
0000110000	3	81	4	9	16	8	8	27	9	16	16	25	25	25	12	7
0000110010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0000110100	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0000110110	9	81	16	9	16	64	8	81	81	64	16	25	25	5	36	7
0000111000	3	9	4	9	4	64	8	9	9	8	4	5	5	5	6	7
0000111010	3	9	4	81	4	8	8	9	9	8	4	5	5	5	6	7
0000111100	3	9	4	9	8	8	64	27	9	16	8	5	5	5	12	7
0001000010	9	9	16	81	8	8	8	81	81	64	8	5	5	5	36	49
0001000100	3	9	4	9	4	64	8	9	9	8	4	5	5	5	6	7
0001000110	3	9	4	81	4	8	8	9	9	8	4	5	5	5	6	7
0001001000	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	7
0001001010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7

Table A.3: Configurations vs. Quandles 1-16: Numbers of Colorings (cont'd)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0001001100	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0001010010	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0001010100	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	7
0001010110	3	9	4	9	4	8	8	9	9	8	4	5	5	25	6	7
0001011000	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0001011010	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	49
0001011100	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0001100010	3	9	4	9	4	8	8	9	9	8	4	5	5	25	6	7
0001100100	3	9	4	9	4	8	8	9	9	8	4	5	5	25	6	7
0001100110	3	9	4	9	8	8	8	27	9	16	8	5	5	5	12	49
0001101010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0001101100	3	81	4	9	16	8	8	27	9	16	16	25	25	5	12	7
0001110010	3	81	4	81	16	8	8	27	9	16	16	25	25	5	12	7
0001110100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0001111000	9	9	16	9	8	8	8	81	81	64	8	5	5	5	36	7
0001111010	9	9	16	9	4	8	64	27	81	32	4	5	5	25	18	7
0001111100	3	9	4	9	4	8	8	9	9	8	4	5	5	25	6	7
0010000010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0010000100	9	81	16	9	16	8	8	81	81	64	16	25	25	5	36	49
0010001010	3	81	4	81	16	8	8	27	9	16	16	25	25	5	12	7
0010001100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0010010010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0010010100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0010011010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0010011100	3	9	4	9	8	64	8	27	9	16	8	5	5	5	12	7
0010100010	9	9	16	9	8	8	8	81	81	64	8	5	5	5	36	7
0010101010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0010101100	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	49
0010110010	9	9	16	9	4	64	8	27	81	32	4	5	5	25	18	49
0010110100	3	81	4	9	16	8	8	27	9	16	16	25	25	25	12	7
0010111100	9	9	16	9	4	8	8	27	81	32	4	5	5	5	18	7
0011000010	3	9	4	81	4	8	8	9	9	8	4	5	5	5	6	7
0011001100	3	9	4	9	8	8	8	27	9	16	8	5	5	25	12	7
0011010010	3	9	4	9	8	8	64	27	9	16	8	5	5	5	12	7
0011011100	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	49
0011100010	3	9	4	9	4	8	8	9	9	8	4	5	5	5	6	7
0011111100	3	81	4	9	16	64	64	27	9	16	16	25	25	5	12	7

Table A.4: Configurations vs. Quandles 1-16: Numbers of Colorings (end)

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_7[T, T^{-1}](T-3)$	17	0
$\mathbb{Z}_7[T, T^{-1}](T-4)$	18	183
$\mathbb{Z}_7[T, T^{-1}](T-5)$	19	0
$\mathbb{Z}_7[T, T^{-1}](T-6)$	20	97
$\mathbb{Z}_8[T, T^{-1}](T-3)$	21	12
$\mathbb{Z}_8[T, T^{-1}](T-5)$	22	0
$\mathbb{Z}_8[T, T^{-1}](T-7)$	23	0
$\mathbb{Z}_9[T, T^{-1}](T-2)$	24	39
$\mathbb{Z}_9[T, T^{-1}](T-4)$	25	13
$\mathbb{Z}_9[T, T^{-1}](T-5)$	26	0
$\mathbb{Z}_9[T, T^{-1}](T-7)$	27	0
$\mathbb{Z}_9[T, T^{-1}](T-8)$	28	0
$\mathbb{Z}_{10}[T, T^{-1}](T-3)$	29	0
$\mathbb{Z}_{10}[T, T^{-1}](T-7)$	30	0
$\mathbb{Z}_{10}[T, T^{-1}](T-9)$	31	0
$\mathbb{Z}_{11}[T, T^{-1}](T-2)$	32	15

Undistinguished Pairs So Far:	504

Table A.5: Quandles 17-32 and Their Performances

	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
00	7	7	7	7	16	32	16	9	27	9	27	9	20	20	20	11
000	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
0000	7	7	7	7	32	64	32	9	27	9	27	9	100	100	20	11
0010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	11
00000	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	121
00010	7	49	7	49	8	8	8	9	9	9	9	9	10	10	10	11
00100	7	7	7	7	64	64	64	9	81	9	81	9	100	100	20	11
000010	49	7	49	7	8	8	8	81	9	81	9	81	10	10	10	11
000100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000110	7	7	7	7	16	32	16	9	81	9	81	9	20	20	100	11
001010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
001100	49	7	49	7	32	64	32	27	27	27	27	27	100	100	20	11
0000010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000100	7	7	7	49	16	32	16	9	27	9	27	9	20	20	20	11
0000110	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0001000	49	7	49	7	8	8	8	27	9	27	9	27	10	10	50	121
0001100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0010010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0010100	49	7	49	49	8	8	8	27	9	27	9	27	10	10	10	11
0011100	7	7	7	7	64	64	64	9	81	9	81	9	100	100	20	11
00000000	7	7	7	7	64	64	64	9	27	9	27	9	100	100	20	11
00000010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00000100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00000110	7	7	7	7	64	64	64	9	27	9	27	9	100	100	20	11
00001000	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00001010	7	49	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00001100	7	7	7	7	16	32	16	9	81	9	81	9	20	20	20	11
00001110	7	49	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00010010	7	7	7	7	16	32	16	9	81	9	81	9	20	20	20	11
00010100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00010110	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	11
00011000	49	7	49	7	64	64	64	27	27	27	27	27	100	100	20	11
00011010	49	7	49	7	8	8	8	81	9	81	9	81	10	10	10	11
00011100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00100010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	121
00100100	49	7	49	7	16	32	16	27	27	27	27	27	20	20	100	121

Table A.6: Configurations vs. Quandles 17-32: Numbers of Colorings

	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
00101010	7	49	7	7	16	32	16	9	27	9	27	9	20	20	20	11
00101100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00110010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
00111100	7	7	7	7	32	64	32	9	27	9	27	9	100	100	100	121
000000000	49	7	49	7	8	8	8	81	9	81	9	81	10	10	10	11
000000010	49	7	49	7	8	8	8	27	9	27	9	27	10	10	50	11
000000100	7	7	7	7	32	64	32	9	27	9	27	9	100	100	100	11
000000110	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000001000	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000001010	7	7	7	49	32	64	32	9	27	9	27	9	100	100	20	11
000001100	49	7	49	7	8	8	8	81	9	81	9	81	10	10	10	11
000001110	49	7	49	49	8	8	8	27	9	27	9	27	10	10	10	11
000010000	49	49	49	7	64	64	64	27	81	27	81	27	100	100	20	11
000010010	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
000010110	7	7	7	7	64	64	64	9	81	9	81	9	100	100	20	11
000011000	7	49	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000011010	7	7	7	49	8	8	8	9	9	9	9	9	10	10	50	11
000011100	49	7	49	7	16	32	16	27	27	27	27	27	20	20	100	11
000100010	7	7	7	7	16	32	16	9	27	9	27	9	20	20	20	11
000100100	7	49	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000100110	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000101000	7	7	7	7	64	64	64	9	81	9	81	9	100	100	100	121
000101010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000101100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000110010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
000110100	7	7	7	7	32	64	32	9	27	9	27	9	100	100	20	11
000111000	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
000111100	7	49	7	7	8	8	8	9	9	9	9	9	10	10	10	11
001000010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
001000100	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
001001010	7	7	7	49	8	8	8	9	9	9	9	9	10	10	10	11
001001100	7	7	7	7	16	32	16	9	27	9	27	9	20	20	20	11
001010100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	121
001011100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
001100010	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
001101100	49	7	49	7	8	8	8	81	9	81	9	81	10	10	50	121
001111100	49	49	49	7	64	64	64	27	81	27	81	27	100	100	20	11
0000000000	7	7	7	7	16	32	16	9	27	9	27	9	20	20	100	121
0000000010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000000100	7	49	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000000110	7	7	7	7	16	32	16	9	27	9	27	9	20	20	20	11
0000001000	49	7	49	7	8	8	8	81	9	81	9	81	10	10	10	11
0000001100	7	7	7	7	64	64	64	9	27	9	27	9	100	100	20	11
0000001110	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	121
0000010000	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000010100	49	7	49	7	8	8	8	81	9	81	9	81	10	10	50	11
0000010110	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
0000011000	7	7	7	7	16	32	16	9	27	9	27	9	20	20	20	11
0000011010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000011100	7	49	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000011110	7	7	7	7	16	32	16	9	27	9	27	9	20	20	20	11
0000100010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000100110	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000101000	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
0000101100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	11
0000101110	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000110000	7	7	7	7	64	64	64	9	27	9	27	9	100	100	100	121
0000110010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000110100	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
0000111000	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000111010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0000111100	7	7	7	7	16	32	16	9	81	9	81	9	20	20	20	11
0001000100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0001000110	7	49	7	49	8	8	8	9	9	9	9	9	10	10	10	11
0001001000	7	7	7	49	64	64	64	9	27	9	27	9	100	100	20	11
0001001010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0001001100	49	7	49	49	8	8	8	81	9	81	9	81	10	10	10	11
0001010010	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
0001010100	7	49	7	7	32	64	32	9	27	9	27	9	100	100	20	11
0001010110	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	11
0001011000	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0001011010	7	7	7	7	32	64	32	9	27	9	27	9	100	100	20	11

Table A.7: Configurations vs. Quandles 17-32: Numbers of Colorings (cont'd)

	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
0001011100	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
0001100010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	121
0001100100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	50	121
0001100110	7	7	7	7	16	32	16	9	27	9	27	9	20	20	20	11
0001101010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0001101100	7	49	7	7	64	64	64	9	27	9	27	9	100	100	20	11
0001110010	7	49	7	49	64	64	64	9	27	9	27	9	100	100	20	11
0001110100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0001111000	49	7	49	49	16	32	16	27	27	27	27	27	20	20	20	11
0001111100	7	7	7	49	8	8	8	9	9	9	9	9	10	10	50	121
0010000010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0010000100	49	49	49	7	64	64	64	27	81	27	81	27	100	100	20	11
0010001010	7	7	7	7	64	64	64	9	81	9	81	9	100	100	20	11
0010001100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0010010010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0010010100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0010011010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0010100010	49	7	49	7	16	32	16	27	27	27	27	27	20	20	20	11
0010101010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0010110100	7	7	7	7	64	64	64	9	81	9	81	9	100	100	100	121
0010111100	49	7	49	7	8	8	8	27	9	27	9	27	10	10	10	11
0011000010	7	49	7	49	8	8	8	9	9	9	9	9	10	10	10	11
0011001100	7	7	7	49	16	32	16	9	81	9	81	9	20	20	100	121
0011010010	7	7	7	7	16	32	16	9	81	9	81	9	20	20	20	11
0011011100	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11
0011100010	7	7	7	7	8	8	8	9	9	9	9	9	10	10	10	11

Table A.8: Configurations vs. Quandles 17-32: Numbers of Colorings (end)

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{11}[T,T^{-1}]/(T-3)$	33	147
$\mathbb{Z}_{11}[T,T^{-1}]/(T-4)$	34	30
$\mathbb{Z}_{11}[T,T^{-1}]/(T-5)$	35	75
$\mathbb{Z}_{11}[T,T^{-1}]/(T-6)$	36	0
$\mathbb{Z}_{11}[T,T^{-1}]/(T-7)$	37	0
$\mathbb{Z}_{11}[T,T^{-1}]/(T-8)$	38	0
$\mathbb{Z}_{11}[T,T^{-1}]/(T-9)$	39	12
$\mathbb{Z}_{11}[T,T^{-1}]/(T-10)$	40	14
$\mathbb{Z}_{12}[T,T^{-1}]/(T-5)$	41	0
$\mathbb{Z}_{12}[T,T^{-1}]/(T-7)$	42	0
$\mathbb{Z}_{12}[T,T^{-1}]/(T-11)$	43	0
$\mathbb{Z}_{13}[T,T^{-1}]/(T-2)$	44	10
$\mathbb{Z}_{13}[T,T^{-1}]/(T-3)$	45	62
$\mathbb{Z}_{13}[T,T^{-1}]/(T-4)$	46	0
$\mathbb{Z}_{13}[T,T^{-1}]/(T-5)$	47	0
$\mathbb{Z}_{13}[T,T^{-1}]/(T-6)$	48	16

Undistinguished Pairs So Far:	138

Table A.9: Quandles 33-48 and Their Performances

	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
00	11	11	11	11	11	11	11	11	48	72	24	13	13	13	13	13
000	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
00000	11	11	11	121	121	121	11	11	12	12	12	13	13	13	13	13
00010	11	11	11	11	11	11	11	11	12	12	12	169	13	13	13	13
00100	11	11	11	11	11	11	11	11	48	144	48	13	13	13	169	13
000010	11	11	11	11	11	11	121	11	36	12	36	13	13	169	13	169
000100	11	11	11	11	11	11	11	121	12	12	12	13	13	13	13	13
001010	121	11	121	11	11	11	11	11	12	12	12	13	169	13	13	13
0000010	11	11	121	11	11	11	11	121	12	12	12	13	13	13	13	13
0000110	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0001100	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0010010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0010100	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
0011100	11	11	11	11	11	11	11	11	48	144	48	13	13	13	169	13
00000010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
00000100	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
00001000	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
00001010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
00001110	11	121	121	11	11	11	11	11	12	12	12	13	13	13	13	13
00010100	11	11	11	11	11	11	11	11	12	12	12	13	169	13	13	13
00010110	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
00011010	11	11	11	11	11	11	121	11	36	12	36	13	13	169	13	169
00011100	11	11	11	11	11	11	11	11	12	12	12	13	169	13	13	13
00101100	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
00110010	11	11	121	11	11	11	11	11	12	12	12	13	13	13	13	13
000000000	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
000000110	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
000001000	121	11	11	11	11	11	121	11	12	12	12	13	13	13	13	13
000001100	11	11	11	11	11	11	11	11	36	12	36	169	13	169	13	13
000001110	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
000010000	11	11	11	11	11	11	11	11	144	144	144	13	169	169	169	13
000010010	11	11	121	11	11	11	11	121	36	12	36	13	13	169	13	13

Table A.10: Configurations vs. Quandles 33-48: Numbers of Colorings

	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
000011000	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
000100010	11	11	11	11	11	11	11	11	48	72	24	13	13	13	13	13
000100100	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
000100110	11	11	11	11	11	11	11	11	12	12	12	13	169	13	13	13
000101000	11	11	11	121	121	121	11	11	48	144	48	13	13	13	169	13
000101010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
000101100	11	11	121	11	11	11	11	11	12	12	12	13	13	13	13	13
000110010	121	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
000110100	11	11	11	11	11	11	11	121	48	144	48	13	13	13	169	13
000111000	121	121	11	11	11	11	11	121	36	12	36	13	13	169	13	13
000111100	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
001000010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
001000100	11	11	11	11	11	11	11	11	36	12	36	169	13	169	13	13
001001100	11	11	121	11	11	11	11	11	48	72	24	169	13	13	13	13
001010100	11	11	11	121	121	121	11	121	12	12	12	13	13	13	13	13
001011100	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
001100010	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
001111100	11	11	11	11	11	11	11	11	144	144	144	13	169	169	169	13
0000000010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0000000100	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	169
0000000110	11	11	11	11	11	11	11	121	48	72	24	13	13	13	13	13
0000001000	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
0000001110	11	11	11	121	121	121	11	11	12	12	12	13	13	13	13	13
0000010000	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	169
0000010110	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
0000011010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0000011100	11	11	11	11	11	11	121	11	12	12	12	13	13	13	13	13
0000100010	121	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0000100110	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0000101000	121	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
0000101100	11	11	11	11	11	11	11	121	12	12	12	13	13	13	13	13
0000101110	11	11	121	11	11	11	11	11	12	12	12	13	13	13	13	13
0000110010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0000110100	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
0000111000	11	11	11	11	11	11	11	11	12	12	12	169	169	13	13	13
0000111010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0000111100	11	11	11	11	11	11	11	11	48	72	24	13	13	13	13	13
0001000100	11	11	11	11	11	11	121	11	12	12	12	13	13	13	13	13
0001000110	11	121	11	11	11	11	11	11	12	12	12	169	13	13	13	13
0001001010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0001010010	121	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
0001010110	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0001011000	121	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0001011010	11	11	11	11	11	11	11	11	48	144	48	13	13	13	169	13
0001011100	11	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
0001100010	11	11	11	121	121	121	11	121	12	12	12	13	169	13	13	13
0001100100	11	11	11	121	121	121	11	11	12	12	12	13	13	13	13	13
0001100110	11	11	11	11	11	11	11	11	48	72	24	13	13	13	13	13
0001101010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0001110100	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0010000010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0010000100	11	11	11	11	11	11	11	11	144	144	144	13	169	169	169	13
0010001100	11	11	11	11	11	11	11	11	12	12	12	13	169	13	13	13
0010010010	11	11	11	11	11	11	11	11	12	12	12	13	169	13	13	13
0010010100	11	121	121	11	11	11	11	11	12	12	12	13	13	13	13	13
0010011010	121	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0010101010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13
0010110100	11	11	11	121	121	121	11	11	48	144	48	13	13	13	169	13
0010111100	121	11	11	11	11	11	11	11	36	12	36	13	13	169	13	13
0011000010	121	11	11	11	11	11	11	11	12	12	12	169	13	13	13	13
0011010010	11	11	121	11	11	11	11	11	48	72	24	13	13	13	13	13
0011011100	11	11	11	11	11	11	121	11	12	12	12	13	13	13	13	13
0011100010	11	11	11	11	11	11	11	11	12	12	12	13	13	13	13	13

Table A.11: Configurations vs. Quandles 33-48: Numbers of Colorings (end)

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{13}[T, T^{-1}]/(T-7)$	49	36
$\mathbb{Z}_{13}[T, T^{-1}]/(T-8)$	50	0
$\mathbb{Z}_{13}[T, T^{-1}]/(T-9)$	51	29
$\mathbb{Z}_{13}[T, T^{-1}]/(T-10)$	52	0
$\mathbb{Z}_{13}[T, T^{-1}]/(T-11)$	53	13
$\mathbb{Z}_{13}[T, T^{-1}]/(T-12)$	54	13
$\mathbb{Z}_{14}[T, T^{-1}]/(T-3)$	55	0
$\mathbb{Z}_{14}[T, T^{-1}]/(T-5)$	56	0
$\mathbb{Z}_{14}[T, T^{-1}]/(T-9)$	57	0
$\mathbb{Z}_{14}[T, T^{-1}]/(T-11)$	58	0
$\mathbb{Z}_{14}[T, T^{-1}]/(T-13)$	59	0
$\mathbb{Z}_{15}[T, T^{-1}]/(T-2)$	60	0
$\mathbb{Z}_{15}[T, T^{-1}]/(T-4)$	61	0
$\mathbb{Z}_{15}[T, T^{-1}]/(T-7)$	62	0
$\mathbb{Z}_{15}[T, T^{-1}]/(T-8)$	63	0
$\mathbb{Z}_{15}[T, T^{-1}]/(T-11)$	64	0

Undistinguished Pairs So Far:	47

Table A.12: Quandles 49-64 and Their Performances

	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
00	13	13	13	13	13	13	28	28	28	28	28	15	45	45	15	75
000	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
00000	13	13	13	13	13	13	14	14	14	14	14	15	75	15	15	15
00100	13	169	13	13	13	13	28	28	28	28	28	75	45	225	75	75
000010	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
0000110	13	13	13	13	169	169	14	14	98	14	14	15	15	15	15	15
0001100	13	13	169	13	13	13	14	14	14	14	14	15	15	15	15	15
0010010	169	13	169	13	169	13	14	14	14	14	14	15	15	15	15	15
0010100	13	13	13	169	13	13	98	98	14	14	98	45	15	15	45	45
0011100	13	169	13	13	13	13	28	28	28	28	28	75	45	225	75	75
00000010	13	13	13	13	13	169	14	14	14	14	14	15	15	15	15	15
00000100	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
00001000	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
00001010	13	13	13	13	13	13	14	14	14	98	14	15	15	15	15	15
00010110	13	13	169	13	13	13	14	14	14	14	14	15	75	15	15	15
00011010	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
00011100	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
00101100	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
00110010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
000000000	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
000000110	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
000001110	13	13	13	169	13	13	98	98	14	14	98	45	15	15	45	45
000010000	13	169	169	169	13	13	196	196	196	196	28	225	45	225	225	225
000011000	13	13	13	13	13	13	14	14	14	98	14	15	15	15	15	15
000100010	13	13	13	13	169	13	28	28	28	28	28	15	45	45	15	75
000100100	13	13	13	13	13	13	14	14	14	98	14	15	15	15	15	15
000100110	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
000101000	13	169	13	13	13	13	28	28	28	28	28	75	225	225	75	75
000101010	13	13	13	13	169	13	14	14	14	14	14	15	15	15	15	15
000101100	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
000110010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
000111100	13	13	13	13	13	13	14	14	14	98	14	15	15	15	15	15
001000010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
001011100	13	13	169	13	13	13	14	14	14	14	14	15	15	15	15	15
001100010	13	13	13	169	13	169	98	98	14	14	14	45	15	15	45	45
001111100	13	169	169	169	13	13	196	196	196	196	28	225	45	225	225	225
0000000010	169	13	13	13	13	13	14	14	98	14	14	15	15	15	15	15
0000001000	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
0000010110	13	13	13	169	13	169	98	98	14	14	14	45	15	15	45	45
0000011010	13	13	13	13	13	13	14	14	98	14	14	15	15	15	15	15
0000100010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
0000100110	13	13	13	13	13	13	14	14	98	14	14	15	15	15	15	15
0000101000	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
0000110010	13	13	169	13	13	13	14	14	14	14	14	15	15	15	15	15
0000110100	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
0000111010	13	13	169	13	13	13	14	14	14	14	14	15	15	15	15	15
0001001010	13	13	169	13	13	13	14	14	14	14	14	15	15	15	15	15
0001010010	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
0001010110	13	13	13	13	13	169	14	14	14	14	14	15	75	15	15	15
0001011000	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
0001011100	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
0001100100	13	13	13	13	13	169	14	14	14	14	14	15	75	15	15	15
0001101010	169	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
0001110100	169	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
0010000010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
0010000100	13	169	169	169	13	13	196	196	196	196	28	225	45	225	225	225
0010001100	13	13	13	13	169	13	14	14	14	14	14	15	15	15	15	15
0010010010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
0010011010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
0010101010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
0010110100	13	169	13	13	13	13	28	28	28	28	28	75	225	225	75	75
0010111100	13	13	13	169	13	13	98	98	14	14	14	45	15	15	45	45
0011100010	13	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15

Table A.13: Configurations vs. quandles 49-64: Numbers of Colorings

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{15}[T, T^{-1}]/(T-13)$	65	0
$\mathbb{Z}_{15}[T, T^{-1}]/(T-14)$	66	0
$\mathbb{Z}_{16}[T, T^{-1}]/(T-3)$	67	4
$\mathbb{Z}_{16}[T, T^{-1}]/(T-5)$	68	0
$\mathbb{Z}_{16}[T, T^{-1}]/(T-7)$	69	0
$\mathbb{Z}_{16}[T, T^{-1}]/(T-9)$	70	0
$\mathbb{Z}_{16}[T, T^{-1}]/(T-11)$	71	0
$\mathbb{Z}_{16}[T, T^{-1}]/(T-13)$	72	0
$\mathbb{Z}_{16}[T, T^{-1}]/(T-15)$	73	0
$\mathbb{Z}_{17}[T, T^{-1}]/(T-2)$	74	0
$\mathbb{Z}_{17}[T, T^{-1}]/(T-3)$	75	10
$\mathbb{Z}_{17}[T, T^{-1}]/(T-4)$	76	0
$\mathbb{Z}_{17}[T, T^{-1}]/(T-5)$	77	3
$\mathbb{Z}_{17}[T, T^{-1}]/(T-6)$	78	0
$\mathbb{Z}_{17}[T, T^{-1}]/(T-7)$	79	3
$\mathbb{Z}_{17}[T, T^{-1}]/(T-8)$	80	4

Undistinguished Pairs So Far:	23

Table A.14: Quandles 65-80 and Their Performances

	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
000	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
00100	225	15	128	256	128	256	128	256	128	17	17	289	17	17	17	17
000010	15	45	16	16	16	16	16	16	16	17	17	17	17	289	17	17
0010100	15	45	16	16	16	16	16	16	16	17	17	17	289	17	289	17
0011100	225	15	256	256	256	256	256	256	256	17	17	289	17	17	17	17
00000100	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
00001010	15	15	16	16	16	16	16	16	16	17	289	17	17	17	17	17
00011010	15	45	16	16	16	16	16	16	16	17	17	17	17	289	17	17
00011100	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
00110010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
000000000	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
000000110	15	15	16	16	16	16	16	16	16	17	289	17	17	17	17	17
000001110	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
000010000	225	45	128	256	128	256	128	256	128	17	17	289	17	17	17	17
000011000	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
000100100	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
000100110	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
000101000	225	75	128	256	128	256	128	256	128	17	17	289	17	17	17	17
000101100	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
000110010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
000111100	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
001000010	15	15	16	16	16	16	16	16	16	17	289	17	289	17	17	17
001011100	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
001100010	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
001111100	225	45	128	256	128	256	128	256	128	17	17	289	17	17	17	17
0000001000	15	45	16	16	16	16	16	16	16	17	17	17	289	17	17	17
0000010110	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0000011010	15	15	16	16	16	16	16	16	16	17	289	17	17	17	17	17
0000100010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0000100110	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0000101000	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0000110010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0000110100	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0001001010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0001010010	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0001011000	15	15	16	16	16	16	16	16	16	17	17	17	17	17	289	17
0001011100	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0001101010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	289
0001110100	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0010000010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0010000100	225	45	256	256	256	256	256	256	256	17	17	289	17	17	17	17
0010010010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0010011010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0010101010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	289
0010110100	225	75	256	256	256	256	256	256	256	17	17	289	17	17	17	17
0010111100	15	45	16	16	16	16	16	16	16	17	17	17	17	17	17	17
0011100010	15	15	16	16	16	16	16	16	16	17	17	17	17	17	17	17

Table A.15: Configurations vs. Quandles 65-80

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{17}[T, T^{-1}]/(T-9)$	81	1
$\mathbb{Z}_{17}[T, T^{-1}]/(T-10)$	82	0
$\mathbb{Z}_{17}[T, T^{-1}]/(T-11)$	83	0
$\mathbb{Z}_{17}[T, T^{-1}]/(T-12)$	84	2
$\mathbb{Z}_{17}[T, T^{-1}]/(T-13)$	85	0
$\mathbb{Z}_{17}[T, T^{-1}]/(T-14)$	86	0
$\mathbb{Z}_{17}[T, T^{-1}]/(T-15)$	87	2
$\mathbb{Z}_{17}[T, T^{-1}]/(T-16)$	88	4
$\mathbb{Z}_{18}[T, T^{-1}]/(T-5)$	89	0
$\mathbb{Z}_{18}[T, T^{-1}]/(T-7)$	90	0
$\mathbb{Z}_{18}[T, T^{-1}]/(T-11)$	91	0
$\mathbb{Z}_{18}[T, T^{-1}]/(T-13)$	92	0
$\mathbb{Z}_{18}[T, T^{-1}]/(T-17)$	93	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-2)$	94	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-3)$	95	2
$\mathbb{Z}_{19}[T, T^{-1}]/(T-4)$	96	1

Undistinguished Pairs So Far:	11

Table A.16: Quandles 81-96 and Their Performances

	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96
000	17	17	17	17	17	17	17	17	54	18	54	18	54	19	19	19
000010	17	17	17	17	17	17	17	17	162	18	162	18	162	19	19	19
00000100	17	17	17	17	17	17	17	289	18	18	18	18	18	19	19	19
00011010	17	17	17	17	17	17	17	17	162	18	162	18	162	19	19	19
00011100	17	17	17	17	17	17	289	17	18	18	18	18	18	19	19	19
00110010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
000010000	17	17	17	17	289	17	17	17	108	324	108	324	108	19	19	19
000011000	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
000100100	289	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
000100110	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
000101100	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
000110010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
001011100	17	17	17	17	17	17	17	17	18	18	18	18	18	19	361	19
001100010	17	17	17	17	17	17	17	17	54	18	54	18	54	19	19	19
001111100	17	17	17	17	289	17	17	17	108	324	108	324	108	19	19	19
0000010110	17	17	17	17	17	17	17	17	54	18	54	18	54	19	19	19
0000100010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
0000101000	17	17	17	17	17	17	17	289	54	18	54	18	54	19	19	19
0000110010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
0000110100	17	17	17	289	17	17	17	17	54	18	54	18	54	19	19	19
0001001010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
0001010010	17	17	17	17	17	17	17	17	54	18	54	18	54	19	19	19
0001011100	17	17	17	289	17	17	17	17	54	18	54	18	54	19	19	19
0010000010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
0010010010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
0010011010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	19
0010111100	17	17	17	17	17	17	17	289	54	18	54	18	54	19	19	19
0011100010	17	17	17	17	17	17	17	17	18	18	18	18	18	19	19	361

Table A.17: Configurations vs. Quandles 81-96: Numbers of Colorings

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{19}[T, T^{-1}]/(T-5)$	97	2
$\mathbb{Z}_{19}[T, T^{-1}]/(T-6)$	98	3
$\mathbb{Z}_{19}[T, T^{-1}]/(T-7)$	99	1
$\mathbb{Z}_{19}[T, T^{-1}]/(T-8)$	100	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-9)$	101	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-10)$	102	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-11)$	103	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-12)$	104	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-13)$	105	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-14)$	106	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-15)$	107	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-16)$	108	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-17)$	109	0
$\mathbb{Z}_{19}[T, T^{-1}]/(T-18)$	110	0
$\mathbb{Z}_{20}[T, T^{-1}]/(T-3)$	111	0
$\mathbb{Z}_{20}[T, T^{-1}]/(T-7)$	112	0

Undistinguished Pairs So Far:	5

Table A.18: Quandles 97-112 and Their Performances

	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112
000010	19	19	19	361	19	19	19	361	19	19	19	19	19	19	20	20
00011010	19	19	19	361	19	19	19	361	19	19	19	19	19	19	20	20
00110010	19	361	19	19	19	19	361	19	19	19	19	19	19	19	20	20
000010000	19	19	361	361	19	19	361	361	19	19	19	19	19	19	400	400
000100110	19	361	19	19	19	19	19	19	19	19	19	19	361	19	20	20
000101100	19	19	361	19	19	19	19	19	19	19	19	19	19	19	20	20
000110010	19	19	361	19	19	19	19	19	19	19	19	19	19	19	20	20
001100010	361	19	19	361	19	361	19	361	19	19	19	19	19	19	20	20
001111100	19	19	361	361	19	19	361	361	19	19	19	19	19	19	400	400
0000010110	19	19	19	361	19	19	19	361	19	19	19	19	19	19	20	20
0000100010	19	19	19	19	19	19	19	19	19	19	19	19	19	19	20	20
0000101000	19	19	19	361	19	361	19	361	19	19	19	19	19	19	20	20
0000110010	19	19	19	19	19	19	19	19	19	19	19	19	19	19	20	20
0000110100	19	19	19	361	19	19	19	361	19	19	19	19	19	361	20	20
0001001010	19	19	19	19	19	19	19	19	19	19	19	19	19	19	20	20
0001011100	19	19	19	361	19	19	19	361	19	19	19	19	19	361	20	20
0010010010	361	19	19	19	19	19	19	19	19	19	19	19	19	19	20	20
0010011010	19	361	19	19	19	19	19	19	19	19	19	19	19	19	20	20
0010111100	19	19	19	361	19	361	19	361	19	19	19	19	19	19	20	20

Table A.19: Configurations vs. Quandles 97-112: Numbers of Colorings

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{20}[T, T^{-1}]/(T - 9)$	113	0
$\mathbb{Z}_{20}[T, T^{-1}]/(T - 11)$	114	0
$\mathbb{Z}_{20}[T, T^{-1}]/(T - 13)$	115	0
$\mathbb{Z}_{20}[T, T^{-1}]/(T - 17)$	116	0
$\mathbb{Z}_{20}[T, T^{-1}]/(T - 19)$	117	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 2)$	118	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 4)$	119	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 5)$	120	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 8)$	121	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 10)$	122	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 11)$	123	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 13)$	124	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 16)$	125	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 17)$	126	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 19)$	127	0
$\mathbb{Z}_{21}[T, T^{-1}]/(T - 20)$	128	0

Undistinguished Pairs So Far:	5

Table A.20: Quandles 113-128 and Their Performances

	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128
000010	20	20	20	20	20	63	21	441	63	147	63	21	21	441	147	63
00011010	20	20	20	20	20	63	21	441	63	147	63	21	21	441	147	63
000010000	80	400	400	400	80	441	441	441	441	441	441	63	441	441	441	63
001111100	80	400	400	400	80	441	441	441	441	441	441	63	441	441	441	63
0000101000	20	20	20	20	20	63	21	441	63	147	63	21	21	441	147	63
0000110010	20	20	20	20	20	21	21	21	21	21	21	21	21	21	21	21
0000110100	20	20	20	20	20	63	21	441	63	147	63	21	21	441	147	63
0001001010	20	20	20	20	20	21	21	21	21	21	21	21	21	21	21	21
0001011100	20	20	20	20	20	63	21	441	63	147	63	21	21	441	147	63
0010111100	20	20	20	20	20	63	21	441	63	147	63	21	21	441	147	63

Table A.21: Configurations vs. Quandles 113-128: Numbers of Colorings

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 3)$	129	0
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 5)$	130	0
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 7)$	131	0
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 9)$	132	0
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 13)$	133	0
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 15)$	134	0
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 17)$	135	0
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 19)$	136	0
$\mathbb{Z}_{22}[T, T^{-1}]/(T - 21)$	137	0
$\mathbb{Z}_{23}[T, T^{-1}]/(T - 2)$	138	0
$\mathbb{Z}_{23}[T, T^{-1}]/(T - 3)$	139	0
$\mathbb{Z}_{23}[T, T^{-1}]/(T - 4)$	140	0
$\mathbb{Z}_{23}[T, T^{-1}]/(T - 5)$	141	0
$\mathbb{Z}_{23}[T, T^{-1}]/(T - 6)$	142	0
$\mathbb{Z}_{23}[T, T^{-1}]/(T - 7)$	143	0
$\mathbb{Z}_{23}[T, T^{-1}]/(T - 8)$	144	0

Undistinguished Pairs So Far:	5

Table A.22: Quandles 129-144 and Their Performances

	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144
000010	22	22	22	242	22	22	22	22	22	23	23	23	23	23	23	23
00011010	22	22	22	242	22	22	22	22	22	23	23	23	23	23	23	23
000010000	44	44	44	44	44	44	44	44	44	23	23	23	23	23	23	23
001111100	44	44	44	44	44	44	44	44	44	23	23	23	23	23	23	23
0000101000	242	22	22	22	22	22	22	22	22	23	23	23	23	23	23	23
0000110010	22	22	22	22	22	22	22	22	22	23	23	23	23	23	23	23
0000110100	22	22	22	22	22	22	22	22	22	529	23	23	23	23	23	23
0001001010	22	22	22	22	22	22	22	22	22	23	23	23	23	23	23	23
0001011100	22	22	22	22	22	22	22	22	22	529	23	23	23	23	23	23
0010111100	242	22	22	22	22	22	22	22	22	23	23	23	23	23	23	23

Table A.23: Configurations vs. Quandles 129-144: Numbers of Colorings

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{23}[T,T^{-1}]/(T-9)$	145	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-10)$	146	1
$\mathbb{Z}_{23}[T,T^{-1}]/(T-11)$	147	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-12)$	148	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-13)$	149	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-14)$	150	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-15)$	151	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-16)$	152	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-17)$	153	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-18)$	154	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-19)$	155	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-20)$	156	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-21)$	157	0
$\mathbb{Z}_{23}[T,T^{-1}]/(T-22)$	158	0
$\mathbb{Z}_{24}[T,T^{-1}]/(T-5)$	159	0
$\mathbb{Z}_{24}[T,T^{-1}]/(T-7)$	160	0

Undistinguished Pairs So Far:	4

Table A.24: Quandles 145-160 and Their Performances

	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
000010	23	23	23	23	23	23	23	23	23	23	529	23	23	23	72	24
00011010	23	23	23	23	23	23	23	23	23	23	529	23	23	23	72	24
000010000	23	23	23	23	23	23	23	23	23	23	23	23	23	23	576	576
001111100	23	23	23	23	23	23	23	23	23	23	23	23	23	23	576	576
0000101000	23	23	23	23	23	23	23	23	23	23	23	23	23	23	72	24
0000110010	23	529	23	23	23	23	23	23	23	23	23	23	23	23	24	24
0000110100	23	23	23	23	23	23	23	23	23	23	23	23	23	23	72	24
0001001010	23	23	23	23	23	23	23	529	23	23	23	529	23	23	24	24
0001011100	23	23	23	23	23	23	23	23	23	23	23	23	23	23	72	24
0010111100	23	23	23	23	23	23	23	23	23	23	23	23	23	23	72	24

Table A.25: Configurations vs. Quandles 145-160: Numbers of Colorings

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{24}[T,T^{-1}]/(T-11)$	161	0
$\mathbb{Z}_{24}[T,T^{-1}]/(T-13)$	162	0
$\mathbb{Z}_{24}[T,T^{-1}]/(T-17)$	163	0
$\mathbb{Z}_{24}[T,T^{-1}]/(T-19)$	164	0
$\mathbb{Z}_{24}[T,T^{-1}]/(T-23)$	165	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-2)$	166	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-3)$	167	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-4)$	168	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-6)$	169	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-7)$	170	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-8)$	171	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-9)$	172	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-11)$	173	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-12)$	174	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-13)$	175	0
$\mathbb{Z}_{25}[T,T^{-1}]/(T-14)$	176	0

Undistinguished Pairs So Far:	4

Table A.26: Quandles 161-176 and Their Performances

	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176
000010	72	24	72	24	72	25	25	25	25	25	25	25	25	25	25	25
00011010	72	24	72	24	72	25	25	25	25	25	25	25	25	25	25	25
000010000	576	576	576	576	576	125	125	25	625	625	125	25	625	125	125	25
001111100	576	576	576	576	576	125	125	25	625	625	125	25	625	125	125	25
0000101000	72	24	72	24	72	25	25	25	25	25	25	25	25	25	25	25
0000110100	72	24	72	24	72	25	25	25	25	25	25	25	25	25	25	25
0001011100	72	24	72	24	72	25	25	25	25	25	25	25	25	25	25	25
0010111100	72	24	72	24	72	25	25	25	25	25	25	25	25	25	25	25

Table A.27: Configurations vs. Quandles 161-176: Numbers of Colorings

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{25}[T, T^{-1}]/(T - 16)$	177	0
$\mathbb{Z}_{25}[T, T^{-1}]/(T - 17)$	178	0
$\mathbb{Z}_{25}[T, T^{-1}]/(T - 18)$	179	0
$\mathbb{Z}_{25}[T, T^{-1}]/(T - 19)$	180	0
$\mathbb{Z}_{25}[T, T^{-1}]/(T - 21)$	181	0
$\mathbb{Z}_{25}[T, T^{-1}]/(T - 22)$	182	0
$\mathbb{Z}_{25}[T, T^{-1}]/(T - 23)$	183	0
$\mathbb{Z}_{25}[T, T^{-1}]/(T - 24)$	184	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 3)$	185	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 5)$	186	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 7)$	187	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 9)$	188	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 11)$	189	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 15)$	190	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 17)$	191	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 19)$	192	0

Undistinguished Pairs So Far:	4

Table A.28: Quandles 177-192 and Their Performances

	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192
000010	25	25	25	25	25	25	25	25	26	26	26	26	26	26	338	338
00011010	25	25	25	25	25	25	25	25	26	26	26	26	26	26	338	338
000010000	625	125	625	25	625	125	125	25	676	676	52	676	52	52	676	52
001111100	625	125	625	25	625	125	125	25	676	676	52	676	52	52	676	52
0000101000	25	25	25	25	25	25	25	25	26	26	26	26	26	26	338	26
0000110100	25	25	25	25	25	25	25	25	26	26	26	26	26	26	338	26
0001011100	25	25	25	25	25	25	25	25	26	26	26	26	26	26	338	26
0010111100	25	25	25	25	25	25	25	25	26	26	26	26	26	26	338	26

Table A.29: Configurations vs. Quandles 177-192: Numbers of Colorings

Quandle	Quandle Number	Number of Pairs Distinguished
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 21)$	193	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 23)$	194	0
$\mathbb{Z}_{26}[T, T^{-1}]/(T - 25)$	195	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 2)$	196	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 4)$	197	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 5)$	198	1
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 7)$	199	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 8)$	200	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 10)$	201	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 11)$	202	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 13)$	203	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 14)$	204	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 16)$	205	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 17)$	206	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 19)$	207	0
$\mathbb{Z}_{27}[T, T^{-1}]/(T - 20)$	208	0

Undistinguished Pairs So Far:	3

Table A.30: Quandles 193-208 and Their Performances

	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208
000010	26	338	26	729	27	243	27	243	27	729	27	243	27	243	27	729
00011010	26	338	26	729	27	729	27	729	27	729	27	729	27	729	27	729
000010000	676	676	52	81	729	81	729	81	729	81	729	81	729	81	729	81
001111100	676	676	52	81	729	81	729	81	729	81	729	81	729	81	729	81
0000101000	26	338	26	81	27	81	27	81	27	81	27	81	27	81	27	81
0000110100	26	338	26	81	27	81	27	81	27	81	27	81	27	81	27	81
0001011100	26	338	26	81	27	81	27	81	27	81	27	81	27	81	27	81
0010111100	26	338	26	81	27	81	27	81	27	81	27	81	27	81	27	81

Table A.31: Configurations vs. Quandles 193-208: Numbers of Colorings

B Distinguishing 2-Twist-spun Torus Knots: The Quandles' Performances

Quandle	Pairs Distinguished
R3	233244
R4	181902
R5	35037
R6	0
Undistinguished Pairs So Far: 73593	

Quandle	Pairs Distinguished
R7	18146
R8	4710
R9	4643
R10	0
Undistinguished Pairs So Far: 46094	

Quandle	Pairs Distinguished
R11	7718
R12	0
R13	5503
R14	0
Undistinguished Pairs So Far: 32873	

Quandle	Pairs Distinguished
R15	0
R16	769
R17	3491
R18	0
Undistinguished Pairs So Far: 28613	

Quandle	Pairs Distinguished
R19	2668
R20	0
R21	0
R22	0
Undistinguished Pairs So Far: 25945	

Quandle	Pairs Distinguished
R23	1816
R24	0
R25	392
R26	0
Undistinguished Pairs So Far: 23737	

Quandle	Pairs Distinguished
R27	213
R28	0
R29	1014
R30	0
Undistinguished Pairs So Far: 22510	

Quandle	Pairs Distinguished
R31	813
R32	77
R33	0
R34	0
Undistinguished Pairs So Far: 21620	

Quandle	Pairs Distinguished
R35	0
R36	0
R37	598
R38	0
Undistinguished Pairs So Far: 21022	

Quandle	Pairs Distinguished
R39	0
R40	0
R41	567
R42	0
Undistinguished Pairs So Far: 20455	

Quandle	Pairs Distinguished
R43	540
R44	0
R45	0
R46	0
Undistinguished Pairs So Far: 19915	

Quandle	Pairs Distinguished
R47	515
R48	0
R49	71
R50	0
Undistinguished Pairs So Far: 19329	

Quandle	Pairs Distinguished
R51	0
R52	0
R53	489
R54	0
Undistinguished Pairs So Far: 18840	

Quandle	Pairs Distinguished
R55	0
R56	0
R57	0
R58	0
Undistinguished Pairs So Far: 18840	

Quandle	Pairs Distinguished
R59	469
R60	0
R61	451
R62	0
Undistinguished Pairs So Far: 17920	

Quandle	Pairs Distinguished
R63	0
R64	16
R65	0
R66	0
Undistinguished Pairs So Far: 17904	

Quandle	Pairs Distinguished
R67	435
R68	0
R69	0
R70	0
Undistinguished Pairs So Far: 17469	

Quandle	Pairs Distinguished
R71	420
R72	0
R73	406
R74	0
Undistinguished Pairs So Far: 16643	

Quandle	Pairs Distinguished
R75	0
R76	0
R77	0
R78	0
Undistinguished Pairs So Far: 16643	

Quandle	Pairs Distinguished
R79	391
R80	0
R81	12
R82	0
Undistinguished Pairs So Far: 16240	

Quandle	Pairs Distinguished
R83	379
R84	0
R85	0
R86	0
Undistinguished Pairs So Far: 15861	

Quandle	Pairs Distinguished
R87	0
R88	0
R89	367
R90	0
Undistinguished Pairs So Far: 15494	

Quandle	Pairs Distinguished
R91	0
R92	0
R93	0
R94	0
Undistinguished Pairs So Far: 15494	

Quandle	Pairs Distinguished
R95	0
R96	0
R97	355
R98	0
Undistinguished Pairs So Far: 15139	

Quandle	Pairs Distinguished
R99	0
R100	0
R101	345
R102	0
Undistinguished Pairs So Far: 14794	

Quandle	Pairs Distinguished
R103	335
R104	0
R105	0
R106	0
Undistinguished Pairs So Far: 14459	

Quandle	Pairs Distinguished
R107	326
R108	0
R109	317
R110	0
Undistinguished Pairs So Far: 13816	

Quandle	Pairs Distinguished
R111	0
R112	0
R113	308
R114	0
Undistinguished Pairs So Far: 13508	

Quandle	Pairs Distinguished
R115	0
R116	0
R117	0
R118	0
Undistinguished Pairs So Far: 13508	

Quandle	Pairs Distinguished
R119	0
R120	0
R121	8
R122	0
Undistinguished Pairs So Far: 13500	

Quandle	Pairs Distinguished
R123	0
R124	0
R125	8
R126	0
Undistinguished Pairs So Far: 13492	

Quandle	Pairs Distinguished
R127	299
R128	8
R129	0
R130	0
Undistinguished Pairs So Far: 13185	

Quandle	Pairs Distinguished
R131	291
R132	0
R133	0
R134	0
Undistinguished Pairs So Far: 12894	

Quandle	Pairs Distinguished
R135	0
R136	0
R137	284
R138	0
Undistinguished Pairs So Far: 12610	

Quandle	Pairs Distinguished
R139	277
R140	0
R141	0
R142	0
Undistinguished Pairs So Far: 12333	

Quandle	Pairs Distinguished
R143	0
R144	0
R145	0
R146	0
Undistinguished Pairs So Far: 12333	

Quandle	Pairs Distinguished
R147	0
R148	0
R149	270
R150	0
Undistinguished Pairs So Far: 12063	

Quandle	Pairs Distinguished
R151	264
R152	0
R153	0
R154	0
Undistinguished Pairs So Far: 11799	

Quandle	Pairs Distinguished
R155	0
R156	0
R157	258
R158	0
Undistinguished Pairs So Far: 11541	

Quandle	Pairs Distinguished
R159	0
R160	0
R161	0
R162	0
Undistinguished Pairs So Far: 11541	

Quandle	Pairs Distinguished
R163	252
R164	0
R165	0
R166	0
Undistinguished Pairs So Far: 11289	

Quandle	Pairs Distinguished
R167	246
R168	0
R169	6
R170	0
Undistinguished Pairs So Far: 11037	

Quandle	Pairs Distinguished
R171	0
R172	0
R173	240
R174	0
Undistinguished Pairs So Far: 10797	

Quandle	Pairs Distinguished
R175	0
R176	0
R177	0
R178	0
Undistinguished Pairs So Far: 10797	

Quandle	Pairs Distinguished
R179	235
R180	0
R181	230
R182	0
Undistinguished Pairs So Far: 10332	

Quandle	Pairs Distinguished
R183	0
R184	0
R185	0
R186	0
Undistinguished Pairs So Far: 10332	

Quandle	Pairs Distinguished
R187	0
R188	0
R189	0
R190	0
Undistinguished Pairs So Far: 10332	

Quandle	Pairs Distinguished
R191	225
R192	0
R193	220
R194	0
Undistinguished Pairs So Far: 9887	

Quandle	Pairs Distinguished
R195	0
R196	0
R197	215
R198	0
Undistinguished Pairs So Far: 9672	

Quandle	Pairs Distinguished
R199	210
R200	0
R201	0
R202	0
Undistinguished Pairs So Far: 9462	

Quandle	Pairs Distinguished
R203	0
R204	0
R205	0
R206	0
Undistinguished Pairs So Far: 9462	

Quandle	Pairs Distinguished
R207	0
R208	0
R209	0
R210	0
Undistinguished Pairs So Far: 9462	

Quandle	Pairs Distinguished
R211	205
R212	0
R213	0
R214	0
Undistinguished Pairs So Far: 9257	

Quandle	Pairs Distinguished
R215	0
R216	0
R217	0
R218	0
Undistinguished Pairs So Far: 9257	

Quandle	Pairs Distinguished
R219	0
R220	0
R221	0
R222	0
Undistinguished Pairs So Far: 9257	

Quandle	Pairs Distinguished
R223	201
R224	0
R225	0
R226	0
Undistinguished Pairs So Far: 9056	

Quandle	Pairs Distinguished
R227	197
R228	0
R229	193
R230	0
Undistinguished Pairs So Far: 8666	

Quandle	Pairs Distinguished
R231	0
R232	0
R233	189
R234	0
Undistinguished Pairs So Far: 8477	

Quandle	Pairs Distinguished
R235	0
R236	0
R237	0
R238	0
Undistinguished Pairs So Far: 8477	

Quandle	Pairs Distinguished
R239	185
R240	0
R241	181
R242	0
Undistinguished Pairs So Far: 8111	

Quandle	Pairs Distinguished
R243	4
R244	0
R245	0
R246	0
Undistinguished Pairs So Far: 8107	

Quandle	Pairs Distinguished
R247	0
R248	0
R249	0
R250	0
Undistinguished Pairs So Far: 8107	

Quandle	Pairs Distinguished
R251	177
R252	0
R253	0
R254	0
Undistinguished Pairs So Far: 7930	

Quandle	Pairs Distinguished
R255	0
R256	4
R257	173
R258	0
Undistinguished Pairs So Far: 7753	

Quandle	Pairs Distinguished
R259	0
R260	0
R261	0
R262	0
Undistinguished Pairs So Far: 7753	

Quandle	Pairs Distinguished
R263	170
R264	0
R265	0
R266	0
Undistinguished Pairs So Far: 7583	

Quandle	Pairs Distinguished
R267	0
R268	0
R269	167
R270	0
Undistinguished Pairs So Far: 7416	

Quandle	Pairs Distinguished
R271	164
R272	0
R273	0
R274	0
Undistinguished Pairs So Far: 7252	

Quandle	Pairs Distinguished
R275	0
R276	0
R277	161
R278	0
Undistinguished Pairs So Far: 7091	

Quandle	Pairs Distinguished
R279	0
R280	0
R281	158
R282	0
Undistinguished Pairs So Far: 6933	

Quandle	Pairs Distinguished
R283	155
R284	0
R285	0
R286	0
Undistinguished Pairs So Far: 6778	

Quandle	Pairs Distinguished
R287	0
R288	0
R289	3
R290	0
Undistinguished Pairs So Far: 6775	

Quandle	Pairs Distinguished
R291	0
R292	0
R293	152
R294	0
Undistinguished Pairs So Far: 6623	

Quandle	Pairs Distinguished
R295	0
R296	0
R297	0
R298	0
Undistinguished Pairs So Far: 6623	

Quandle	Pairs Distinguished
R299	0
R300	0
R301	0
R302	0
Undistinguished Pairs So Far: 6623	

Quandle	Pairs Distinguished
R303	0
R304	0
R305	0
R306	0
Undistinguished Pairs So Far: 6623	

Quandle	Pairs Distinguished
R307	149
R308	0
R309	0
R310	0
Undistinguished Pairs So Far: 6474	

Quandle	Pairs Distinguished
R311	146
R312	0
R313	143
R314	0
Undistinguished Pairs So Far: 6185	

Quandle	Pairs Distinguished
R315	0
R316	0
R317	140
R318	0
Undistinguished Pairs So Far: 6045	

Quandle	Pairs Distinguished
R319	0
R320	0
R321	0
R322	0
Undistinguished Pairs So Far: 6045	

Quandle	Pairs Distinguished
R323	0
R324	0
R325	0
R326	0
Undistinguished Pairs So Far: 6045	

Quandle	Pairs Distinguished
R327	0
R328	0
R329	0
R330	0
Undistinguished Pairs So Far: 6045	

Quandle	Pairs Distinguished
R331	137
R332	0
R333	0
R334	0
Undistinguished Pairs So Far: 5908	

Quandle	Pairs Distinguished
R335	0
R336	0
R337	134
R338	0
Undistinguished Pairs So Far: 5774	

Quandle	Pairs Distinguished
R339	0
R340	0
R341	0
R342	0

Undistinguished Pairs So Far: 5774

Quandle	Pairs Distinguished
R343	2
R344	0
R345	0
R346	0

Undistinguished Pairs So Far: 5772

Quandle	Pairs Distinguished
R347	131
R348	0
R349	129
R350	0

Undistinguished Pairs So Far: 5512

Quandle	Pairs Distinguished
R351	0
R352	0
R353	127
R354	0

Undistinguished Pairs So Far: 5385

Quandle	Pairs Distinguished
R355	0
R356	0
R357	0
R358	0

Undistinguished Pairs So Far: 5385

Quandle	Pairs Distinguished
R359	125
R360	0
R361	2
R362	0

Undistinguished Pairs So Far: 5258

Quandle	Pairs Distinguished
R363	0
R364	0
R365	0
R366	0

Undistinguished Pairs So Far: 5258

Quandle	Pairs Distinguished
R367	123
R368	0
R369	0
R370	0

Undistinguished Pairs So Far: 5135

Quandle	Pairs Distinguished
R371	0
R372	0
R373	121
R374	0

Undistinguished Pairs So Far: 5014

Quandle	Pairs Distinguished
R375	0
R376	0
R377	0
R378	0

Undistinguished Pairs So Far: 5014

Quandle	Pairs Distinguished
R379	119
R380	0
R381	0
R382	0

Undistinguished Pairs So Far: 4895

Quandle	Pairs Distinguished
R383	117
R384	0
R385	0
R386	0

Undistinguished Pairs So Far: 4778

Quandle	Pairs Distinguished
R387	0
R388	0
R389	115
R390	0

Undistinguished Pairs So Far: 4663

Quandle	Pairs Distinguished
R391	0
R392	0
R393	0
R394	0

Undistinguished Pairs So Far: 4663

Quandle	Pairs Distinguished
R395	0
R396	0
R397	113
R398	0

Undistinguished Pairs So Far: 4550

Quandle	Pairs Distinguished
R399	0
R400	0
R401	111
R402	0

Undistinguished Pairs So Far: 4439

Quandle	Pairs Distinguished
R403	0
R404	0
R405	0
R406	0

Undistinguished Pairs So Far: 4439

Quandle	Pairs Distinguished
R407	0
R408	0
R409	109
R410	0

Undistinguished Pairs So Far: 4330

Quandle	Pairs Distinguished
R411	0
R412	0
R413	0
R414	0

Undistinguished Pairs So Far: 4330

Quandle	Pairs Distinguished
R415	0
R416	0
R417	0
R418	0

Undistinguished Pairs So Far: 4330

Quandle	Pairs Distinguished
R419	107
R420	0
R421	105
R422	0

Undistinguished Pairs So Far: 4118

Quandle	Pairs Distinguished
R423	0
R424	0
R425	0
R426	0
Undistinguished Pairs So Far: 4118	

Quandle	Pairs Distinguished
R427	0
R428	0
R429	0
R430	0
Undistinguished Pairs So Far: 4118	

Quandle	Pairs Distinguished
R431	103
R432	0
R433	101
R434	0
Undistinguished Pairs So Far: 3914	

Quandle	Pairs Distinguished
R435	0
R436	0
R437	0
R438	0
Undistinguished Pairs So Far: 3914	

Quandle	Pairs Distinguished
R439	99
R440	0
R441	0
R442	0
Undistinguished Pairs So Far: 3815	

Quandle	Pairs Distinguished
R443	97
R444	0
R445	0
R446	0
Undistinguished Pairs So Far: 3718	

Quandle	Pairs Distinguished
R447	0
R448	0
R449	95
R450	0
Undistinguished Pairs So Far: 3623	

Quandle	Pairs Distinguished
R451	0
R452	0
R453	0
R454	0
Undistinguished Pairs So Far: 3623	

Quandle	Pairs Distinguished
R455	0
R456	0
R457	93
R458	0
Undistinguished Pairs So Far: 3530	

Quandle	Pairs Distinguished
R459	0
R460	0
R461	91
R462	0
Undistinguished Pairs So Far: 3439	

Quandle	Pairs Distinguished
R463	89
R464	0
R465	0
R466	0
Undistinguished Pairs So Far: 3350	

Quandle	Pairs Distinguished
R467	87
R468	0
R469	0
R470	0
Undistinguished Pairs So Far: 3263	

Quandle	Pairs Distinguished
R471	0
R472	0
R473	0
R474	0
Undistinguished Pairs So Far: 3263	

Quandle	Pairs Distinguished
R475	0
R476	0
R477	0
R478	0
Undistinguished Pairs So Far: 3263	

Quandle	Pairs Distinguished
R479	85
R480	0
R481	0
R482	0
Undistinguished Pairs So Far: 3178	

Quandle	Pairs Distinguished
R483	0
R484	0
R485	0
R486	0
Undistinguished Pairs So Far: 3178	

Quandle	Pairs Distinguished
R487	83
R488	0
R489	0
R490	0
Undistinguished Pairs So Far: 3095	

Quandle	Pairs Distinguished
R491	81
R492	0
R493	0
R494	0
Undistinguished Pairs So Far: 3014	

Quandle	Pairs Distinguished
R495	0
R496	0
R497	0
R498	0
Undistinguished Pairs So Far: 3014	

C Distinguishing 2-Twist-spun Torus Knots: Some Numbers of Colorings

Quandle	Pairs Distinguished
R3	233244
R4	181902
R5	35037
R6	0
Undistinguished Pairs So Far: 73593	

Table C.1: Quandles R3-R6 and Their Performances

	R3	R4	R5	R6
3	9	4	5	18
4	3	16	5	12
5	3	4	25	6
6	9	8	5	36
7	3	4	5	6
8	3	16	5	12
9	9	4	5	18
10	3	8	25	12
11	3	4	5	6
12	9	16	5	36
13	3	4	5	6
14	3	8	5	12
15	9	4	25	18
16	3	16	5	12
17	3	4	5	6
18	9	8	5	36
19	3	4	5	6
20	3	16	25	12
21	9	4	5	18
22	3	8	5	12
23	3	4	5	6
24	9	16	5	36
25	3	4	25	6
26	3	8	5	12
27	9	4	5	18
28	3	16	5	12
29	3	4	5	6
30	9	8	25	36
31	3	4	5	6
32	3	16	5	12
33	9	4	5	18
34	3	8	5	12
35	3	4	25	6
36	9	16	5	36
37	3	4	5	6
38	3	8	5	12
39	9	4	5	18
40	3	16	25	12
41	3	4	5	6
42	9	8	5	36
43	3	4	5	6
44	3	16	5	12
45	9	4	25	18
46	3	8	5	12
47	3	4	5	6
48	9	16	5	36
49	3	4	5	6
50	3	8	25	12
51	9	4	5	18
52	3	16	5	12
53	3	4	5	6
54	9	8	5	36
55	3	4	25	6
56	3	16	5	12
57	9	4	5	18
58	3	8	5	12
59	3	4	5	6
60	9	16	25	36
61	3	4	5	6
62	3	8	5	12
63	9	4	5	18
64	3	16	5	12
65	3	4	25	6
66	9	8	5	36
67	3	4	5	6
68	3	16	5	12
69	9	4	5	18
70	3	8	25	12
71	3	4	5	6
72	9	16	5	36
73	3	4	5	6
74	3	8	5	12
75	9	4	25	18
76	3	16	5	12
77	3	4	5	6
78	9	8	5	36
79	3	4	5	6
80	3	16	25	12
81	9	4	5	18
82	3	8	5	12
83	3	4	5	6
84	9	16	5	36
85	3	4	25	6
86	3	8	5	12
87	9	4	5	18
88	3	16	5	12
89	3	4	5	6
90	9	8	25	36
91	3	4	5	6
92	3	16	5	12
93	9	4	5	18
94	3	8	5	12
95	3	4	25	6
96	9	16	5	36
97	3	4	5	6
98	3	8	5	12
99	9	4	5	18
100	3	16	25	12
101	3	4	5	6
102	9	8	5	36
103	3	4	5	6
104	3	16	5	12
105	9	4	25	18
106	3	8	5	12
107	3	4	5	6
108	9	16	5	36
109	3	4	5	6
110	3	8	25	12
111	9	4	5	18
112	3	16	5	12
113	3	4	5	6
114	9	8	5	36
115	3	4	25	6
116	3	16	5	12
117	9	4	5	18
118	3	8	5	12
119	3	4	5	6
120	9	16	25	36
121	3	4	5	6
122	3	8	5	12
123	9	4	5	18
124	3	16	5	12
125	3	4	25	6
126	9	8	5	36
127	3	4	5	6
128	3	16	5	12
129	9	4	5	18
130	3	8	25	12
131	3	4	5	6
132	9	16	5	36
133	3	4	5	6
134	3	8	5	12
135	9	4	25	18
136	3	16	5	12
137	3	4	5	6
138	9	8	5	36
139	3	4	5	6
140	3	16	25	12
141	9	4	5	18
142	3	8	5	12
143	3	4	5	6
144	9	16	5	36
145	3	4	25	6
146	3	8	5	12
147	9	4	5	18
148	3	16	5	12
149	3	4	5	6
150	9	8	25	36
151	3	4	5	6
152	3	16	5	12
153	9	4	5	18
154	3	8	5	12
155	3	4	25	6
156	9	16	5	36
157	3	4	5	6
158	3	8	5	12
159	9	4	5	18
160	3	16	25	12
161	3	4	5	6
162	9	8	5	36
163	3	4	5	6
164	3	16	5	12
165	9	4	25	18
166	3	8	5	12
167	3	4	5	6

Table C.2: 2-Twist-spun $T(2,m)$ $(3 \leq m \leq 167)$ vs. Quandles R3-R6: Numbers of Colorings

	R3	R4	R5	R6
168	9	16	5	36
169	3	4	5	6
170	3	8	25	12
171	9	4	5	18
172	3	16	5	12
173	3	4	5	6
174	9	8	5	36
175	3	4	25	6
176	3	16	5	12
177	9	4	5	18
178	3	8	5	12
179	3	4	5	6
180	9	16	25	36
181	3	4	5	6
182	3	8	5	12
183	9	4	5	18
184	3	16	5	12
185	3	4	25	6
186	9	8	5	36
187	3	4	5	6
188	3	16	5	12
189	9	4	5	18
190	3	8	25	12
191	3	4	5	6
192	9	16	5	36
193	3	4	5	6
194	3	8	5	12
195	9	4	25	18
196	3	16	5	12
197	3	4	5	6
198	9	8	5	36
199	3	4	5	6
200	3	16	25	12
201	9	4	5	18
202	3	8	5	12
203	3	4	5	6
204	9	16	5	36
205	3	4	25	6
206	3	8	5	12
207	9	4	5	18
208	3	16	5	12
209	3	4	5	6
210	9	8	25	36
211	3	4	5	6
212	3	16	5	12
213	9	4	5	18
214	3	8	5	12
215	3	4	25	6
216	9	16	5	36
217	3	4	5	6
218	3	8	5	12
219	9	4	5	18
220	3	16	25	12
221	3	4	5	6
222	9	8	5	36
223	3	4	5	6
224	3	16	5	12
225	9	4	25	18
226	3	8	5	12
227	3	4	5	6
228	9	16	5	36
229	3	4	5	6
230	3	8	25	12
231	9	4	5	18
232	3	16	5	12
233	3	4	5	6
234	9	8	5	36
235	3	4	25	6
236	3	16	5	12
237	9	4	5	18
238	3	8	5	12
239	3	4	5	6
240	9	16	25	36
241	3	4	5	6
242	3	8	5	12

	R3	R4	R5	R6
243	9	4	5	18
244	3	16	5	12
245	3	4	25	6
246	9	8	5	36
247	3	4	5	6
248	3	16	5	12
249	9	4	5	18
250	3	8	25	12
251	3	4	5	6
252	9	16	5	36
253	3	4	5	6
254	3	8	5	12
255	9	4	25	18
256	3	16	5	12
257	3	4	5	6
258	9	8	5	36
259	3	4	5	6
260	3	16	25	12
261	9	4	5	18
262	3	8	5	12
263	3	4	5	6
264	9	16	5	36
265	3	4	25	6
266	3	8	5	12
267	9	4	5	18
268	3	16	5	12
269	3	4	5	6
270	9	8	25	36
271	3	4	5	6
272	3	16	5	12
273	9	4	5	18
274	3	8	5	12
275	3	4	25	6
276	9	16	5	36
277	3	4	5	6
278	3	8	5	12
279	9	4	5	18
280	3	16	25	12
281	3	4	5	6
282	9	8	5	36
283	3	4	5	6
284	3	16	5	12
285	9	4	25	18
286	3	8	5	12
287	3	4	5	6
288	9	16	5	36
289	3	4	5	6
290	3	8	25	12
291	9	4	5	18
292	3	16	5	12
293	3	4	5	6
294	9	8	5	36
295	3	4	25	6
296	3	16	5	12
297	9	4	5	18
298	3	8	5	12
299	3	4	5	6
300	9	16	25	36
301	3	4	5	6
302	3	8	5	12
303	9	4	5	18
304	3	16	5	12
305	3	4	25	6
306	9	8	5	36
307	3	4	5	6
308	3	16	5	12
309	9	4	5	18
310	3	8	25	12
311	3	4	5	6
312	9	16	5	36
313	3	4	5	6
314	3	8	5	12
315	9	4	25	18
316	3	16	5	12
317	3	4	5	6

	R3	R4	R5	R6
318	9	8	5	36
319	3	4	5	6
320	3	16	25	12
321	9	4	5	18
322	3	8	5	12
323	3	4	5	6
324	9	16	5	36
325	3	4	25	6
326	3	8	5	12
327	9	4	5	18
328	3	16	5	12
329	3	4	5	6
330	9	8	25	36
331	3	4	5	6
332	3	16	5	12
333	9	4	5	18
334	3	8	5	12
335	3	4	25	6
336	9	16	5	36
337	3	4	5	6
338	3	8	5	12
339	9	4	5	18
340	3	16	25	12
341	3	4	5	6
342	9	8	5	36
343	3	4	5	6
344	3	16	5	12
345	9	4	25	18
346	3	8	5	12
347	3	4	5	6
348	9	16	5	36
349	3	4	5	6
350	3	8	25	12
351	9	4	5	18
352	3	16	5	12
353	3	4	5	6
354	9	8	5	36
355	3	4	25	6
356	3	16	5	12
357	9	4	5	18
358	3	8	5	12
359	3	4	5	6
360	9	16	25	36
361	3	4	5	6
362	3	8	5	12
363	9	4	5	18
364	3	16	5	12
365	3	4	25	6
366	9	8	5	36
367	3	4	5	6
368	3	16	5	12
369	9	4	5	18
370	3	8	25	12
371	3	4	5	6
372	9	16	5	36
373	3	4	5	6
374	3	8	5	12
375	9	4	25	18
376	3	16	5	12
377	3	4	5	6
378	9	8	5	36
379	3	4	5	6
380	3	16	25	12
381	9	4	5	18
382	3	8	5	12
383	3	4	5	6
384	9	16	5	36
385	3	4	25	6
386	3	8	5	12
387	9	4	5	18
388	3	16	5	12
389	3	4	5	6
390	9	8	25	36
391	3	4	5	6
392	3	16	5	12

Table C.3: 2-Twist-spun $T(2,m)$ $(168 \leq m \leq 392)$ vs. Quandles R3-R6: Numbers of Colorings

	R3	R4	R5	R6
393	9	4	5	18
394	3	8	5	12
395	3	4	25	6
396	9	16	5	36
397	3	4	5	6
398	3	8	5	12
399	9	4	5	18
400	3	16	25	12
401	3	4	5	6
402	9	8	5	36
403	3	4	5	6
404	3	16	5	12
405	9	4	25	18
406	3	8	5	12
407	3	4	5	6
408	9	16	5	36
409	3	4	5	6
410	3	8	25	12
411	9	4	5	18
412	3	16	5	12
413	3	4	5	6
414	9	8	5	36
415	3	4	25	6
416	3	16	5	12
417	9	4	5	18
418	3	8	5	12
419	3	4	5	6
420	9	16	25	36
421	3	4	5	6
422	3	8	5	12
423	9	4	5	18
424	3	16	5	12
425	3	4	25	6
426	9	8	5	36
427	3	4	5	6
428	3	16	5	12
429	9	4	5	18
430	3	8	25	12
431	3	4	5	6
432	9	16	5	36
433	3	4	5	6
434	3	8	5	12
435	9	4	25	18
436	3	16	5	12
437	3	4	5	6
438	9	8	5	36
439	3	4	5	6
440	3	16	25	12
441	9	4	5	18
442	3	8	5	12
443	3	4	5	6
444	9	16	5	36
445	3	4	25	6
446	3	8	5	12
447	9	4	5	18
448	3	16	5	12
449	3	4	5	6
450	9	8	25	36
451	3	4	5	6
452	3	16	5	12
453	9	4	5	18
454	3	8	5	12
455	3	4	25	6
456	9	16	5	36
457	3	4	5	6
458	3	8	5	12
459	9	4	5	18
460	3	16	25	12
461	3	4	5	6
462	9	8	5	36
463	3	4	5	6
464	3	16	5	12
465	9	4	25	18
466	3	8	5	12
467	3	4	5	6
468	9	16	5	36
469	3	4	5	6
470	3	8	25	12
471	9	4	5	18
472	3	16	5	12
473	3	4	5	6
474	9	8	5	36
475	3	4	25	6
476	3	16	5	12
477	9	4	5	18
478	3	8	5	12
479	3	4	5	6
480	9	16	25	36
481	3	4	5	6
482	3	8	5	12
483	9	4	5	18
484	3	16	5	12
485	3	4	25	6
486	9	8	5	36
487	3	4	5	6
488	3	16	5	12
489	9	4	5	18
490	3	8	25	12
491	3	4	5	6
492	9	16	5	36
493	3	4	5	6
494	3	8	5	12
495	9	4	25	18
496	3	16	5	12
497	3	4	5	6
498	9	8	5	36
499	3	4	5	6
500	3	16	25	12
501	9	4	5	18
502	3	8	5	12
503	3	4	5	6
504	9	16	5	36
505	3	4	25	6
506	3	8	5	12
507	9	4	5	18
508	3	16	5	12
509	3	4	5	6
510	9	8	25	36
511	3	4	5	6
512	3	16	5	12
513	9	4	5	18
514	3	8	5	12
515	3	4	25	6
516	9	16	5	36
517	3	4	5	6
518	3	8	5	12
519	9	4	5	18
520	3	16	25	12
521	3	4	5	6
522	9	8	5	36
523	3	4	5	6
524	3	16	5	12
525	9	4	25	18
526	3	8	5	12
527	3	4	5	6
528	9	16	5	36
529	3	4	5	6
530	3	8	25	12
531	9	4	5	18
532	3	16	5	12
533	3	4	5	6
534	9	8	5	36
535	3	4	25	6
536	3	16	5	12
537	9	4	5	18
538	3	8	5	12
539	3	4	5	6
540	9	16	25	36
541	3	4	5	6
542	3	8	5	12
543	9	4	5	18
544	3	16	5	12
545	3	4	25	6
546	9	8	5	36
547	3	4	5	6
548	3	16	5	12
549	9	4	5	18
550	3	8	25	12
551	3	4	5	6
552	9	16	5	36
553	3	4	5	6
554	3	8	5	12
555	9	4	25	18
556	3	16	5	12
557	3	4	5	6
558	9	8	5	36
559	3	4	5	6
560	3	16	25	12
561	9	4	5	18
562	3	8	5	12
563	3	4	5	6
564	9	16	5	36
565	3	4	25	6
566	3	8	5	12
567	9	4	5	18
568	3	16	5	12
569	3	4	5	6
570	9	8	25	36
571	3	4	5	6
572	3	16	5	12
573	9	4	5	18
574	3	8	5	12
575	3	4	25	6
576	9	16	5	36
577	3	4	5	6
578	3	8	5	12
579	9	4	5	18
580	3	16	25	12
581	3	4	5	6
582	9	8	5	36
583	3	4	5	6
584	3	16	5	12
585	9	4	25	18
586	3	8	5	12
587	3	4	5	6
588	9	16	5	36
589	3	4	5	6
590	3	8	25	12
591	9	4	5	18
592	3	16	5	12
593	3	4	5	6
594	9	8	5	36
595	3	4	25	6
596	3	16	5	12
597	9	4	5	18
598	3	8	5	12
599	3	4	5	6
600	9	16	25	36
601	3	4	5	6
602	3	8	5	12
603	9	4	5	18
604	3	16	5	12
605	3	4	25	6
606	9	8	5	36
607	3	4	5	6
608	3	16	5	12
609	9	4	5	18
610	3	8	25	12
611	3	4	5	6
612	9	16	5	36
613	3	4	5	6
614	3	8	5	12
615	9	4	25	18
616	3	16	5	12
617	3	4	5	6

Table C.4: 2-Twist-spun $T(2, m)$ ($393 \leq m \leq 617$) vs. Quandles R3-R6: Numbers of Colorings

	R3	R4	R5	R6
618	9	8	5	36
619	3	4	5	6
620	3	16	25	12
621	9	4	5	18
622	3	8	5	12
623	3	4	5	6
624	9	16	5	36
625	3	4	25	6
626	3	8	5	12
627	9	4	5	18
628	3	16	5	12
629	3	4	5	6
630	9	8	25	36
631	3	4	5	6
632	3	16	5	12
633	9	4	5	18
634	3	8	5	12
635	3	4	25	6
636	9	16	5	36
637	3	4	5	6
638	3	8	5	12
639	9	4	5	18
640	3	16	25	12
641	3	4	5	6
642	9	8	5	36
643	3	4	5	6
644	3	16	5	12
645	9	4	25	18
646	3	8	5	12
647	3	4	5	6
648	9	16	5	36
649	3	4	5	6
650	3	8	25	12
651	9	4	5	18
652	3	16	5	12
653	3	4	5	6
654	9	8	5	36
655	3	4	25	6
656	3	16	5	12
657	9	4	5	18
658	3	8	5	12
659	3	4	5	6
660	9	16	25	36
661	3	4	5	6
662	3	8	5	12
663	9	4	5	18
664	3	16	5	12
665	3	4	25	6
666	9	8	5	36
667	3	4	5	6
668	3	16	5	12
669	9	4	5	18
670	3	8	25	12
671	3	4	5	6
672	9	16	5	36
673	3	4	5	6
674	3	8	5	12
675	9	4	25	18
676	3	16	5	12
677	3	4	5	6
678	9	8	5	36
679	3	4	5	6
680	3	16	25	12
681	9	4	5	18
682	3	8	5	12
683	3	4	5	6
684	9	16	5	36
685	3	4	25	6
686	3	8	5	12
687	9	4	5	18
688	3	16	5	12
689	3	4	5	6
690	9	8	25	36
691	3	4	5	6
692	3	16	5	12

	R3	R4	R5	R6
693	9	4	5	18
694	3	8	5	12
695	3	4	25	6
696	9	16	5	36
697	3	4	5	6
698	3	8	5	12
699	9	4	5	18
700	3	16	25	12
701	3	4	5	6
702	9	8	5	36
703	3	4	5	6
704	3	16	5	12
705	9	4	25	18
706	3	8	5	12
707	3	4	5	6
708	9	16	5	36
709	3	4	5	6
710	3	8	25	12
711	9	4	5	18
712	3	16	5	12
713	3	4	5	6
714	9	8	5	36
715	3	4	25	6
716	3	16	5	12
717	9	4	5	18
718	3	8	5	12
719	3	4	5	6
720	9	16	25	36
721	3	4	5	6
722	3	8	5	12
723	9	4	5	18
724	3	16	5	12
725	3	4	25	6
726	9	8	5	36
727	3	4	5	6
728	3	16	5	12
729	9	4	5	18
730	3	8	25	12
731	3	4	5	6
732	9	16	5	36
733	3	4	5	6
734	3	8	5	12
735	9	4	25	18
736	3	16	5	12
737	3	4	5	6
738	9	8	5	36
739	3	4	5	6
740	3	16	25	12
741	9	4	5	18
742	3	8	5	12
743	3	4	5	6
744	9	16	5	36
745	3	4	25	6
746	3	8	5	12
747	9	4	5	18
748	3	16	5	12
749	3	4	5	6
750	9	8	25	36
751	3	4	5	6
752	3	16	5	12
753	9	4	5	18
754	3	8	5	12
755	3	4	25	6
756	9	16	5	36
757	3	4	5	6
758	3	8	5	12
759	9	4	5	18
760	3	16	25	12
761	3	4	5	6
762	9	8	5	36
763	3	4	5	6
764	3	16	5	12
765	9	4	25	18
766	3	8	5	12
767	3	4	5	6

	R3	R4	R5	R6
768	9	16	5	36
769	3	4	5	6
770	3	8	25	12
771	9	4	5	18
772	3	16	5	12
773	3	4	5	6
774	9	8	5	36
775	3	4	25	6
776	3	16	5	12
777	9	4	5	18
778	3	8	5	12
779	3	4	5	6
780	9	16	25	36
781	3	4	5	6
782	3	8	5	12
783	9	4	5	18
784	3	16	5	12
785	3	4	25	6
786	9	8	5	36
787	3	4	5	6
788	3	16	5	12
789	9	4	5	18
790	3	8	25	12
791	3	4	5	6
792	9	16	5	36
793	3	4	5	6
794	3	8	5	12
795	9	4	25	18
796	3	16	5	12
797	3	4	5	6
798	9	8	5	36
799	3	4	5	6
800	3	16	25	12
801	9	4	5	18
802	3	8	5	12
803	3	4	5	6
804	9	16	5	36
805	3	4	25	6
806	3	8	5	12
807	9	4	5	18
808	3	16	5	12
809	3	4	5	6
810	9	8	25	36
811	3	4	5	6
812	3	16	5	12
813	9	4	5	18
814	3	8	5	12
815	3	4	25	6
816	9	16	5	36
817	3	4	5	6
818	3	8	5	12
819	9	4	5	18
820	3	16	25	12
821	3	4	5	6
822	9	8	5	36
823	3	4	5	6
824	3	16	5	12
825	9	4	25	18
826	3	8	5	12
827	3	4	5	6
828	9	16	5	36
829	3	4	5	6
830	3	8	25	12
831	9	4	5	18
832	3	16	5	12
833	3	4	5	6
834	9	8	5	36
835	3	4	25	6
836	3	16	5	12
837	9	4	5	18
838	3	8	5	12
839	3	4	5	6
840	9	16	25	36
841	3	4	5	6
842	3	8	5	12

Table C.5: 2-Twist-spun $T(2, m)$ ($618 \leq m \leq 842$) vs. Quandles R3-R6: Numbers of Colorings

	R3	R4	R5	R6
843	9	4	5	18
844	3	16	5	12
845	3	4	25	6
846	9	8	5	36
847	3	4	5	6
848	3	16	5	12
849	9	4	5	18
850	3	8	25	12
851	3	4	5	6
852	9	16	5	36
853	3	4	5	6
854	3	8	5	12
855	9	4	25	18
856	3	16	5	12
857	3	4	5	6
858	9	8	5	36
859	3	4	5	6
860	3	16	25	12
861	9	4	5	18
862	3	8	5	12
863	3	4	5	6
864	9	16	5	36
865	3	4	25	6
866	3	8	5	12
867	9	4	5	18
868	3	16	5	12
869	3	4	5	6
870	9	8	25	36
871	3	4	5	6
872	3	16	5	12
873	9	4	5	18
874	3	8	5	12
875	3	4	25	6
876	9	16	5	36
877	3	4	5	6
878	3	8	5	12
879	9	4	5	18
880	3	16	25	12
881	3	4	5	6
882	9	8	5	36
883	3	4	5	6
884	3	16	5	12
885	9	4	25	18
886	3	8	5	12
887	3	4	5	6
888	9	16	5	36
889	3	4	5	6
890	3	8	25	12
891	9	4	5	18
892	3	16	5	12
893	3	4	5	6
894	9	8	5	36
895	3	4	25	6
896	3	16	5	12
897	9	4	5	18
898	3	8	5	12
899	3	4	5	6
900	9	16	25	36
901	3	4	5	6
902	3	8	5	12
903	9	4	5	18
904	3	16	5	12
905	3	4	25	6
906	9	8	5	36
907	3	4	5	6
908	3	16	5	12
909	9	4	5	18
910	3	8	25	12
911	3	4	5	6
912	9	16	5	36
913	3	4	5	6
914	3	8	5	12
915	9	4	25	18
916	3	16	5	12
917	3	4	5	6
918	9	8	5	36
919	3	4	5	6
920	3	16	25	12
921	9	4	5	18
922	3	8	5	12
923	3	4	5	6
924	9	16	5	36
925	3	4	25	6
926	3	8	5	12
927	9	4	5	18
928	3	16	5	12
929	3	4	5	6
930	9	8	25	36
931	3	4	5	6
932	3	16	5	12
933	9	4	5	18
934	3	8	5	12
935	3	4	25	6
936	9	16	5	36
937	3	4	5	6
938	3	8	5	12
939	9	4	5	18
940	3	16	25	12
941	3	4	5	6
942	9	8	5	36
943	3	4	5	6
944	3	16	5	12
945	9	4	25	18
946	3	8	5	12
947	3	4	5	6
948	9	16	5	36
949	3	4	5	6
950	3	8	25	12
951	9	4	5	18
952	3	16	5	12
953	3	4	5	6
954	9	8	5	36
955	3	4	25	6
956	3	16	5	12
957	9	4	5	18
958	3	8	5	12
959	3	4	5	6
960	9	16	25	36
961	3	4	5	6
962	3	8	5	12
963	9	4	5	18
964	3	16	5	12
965	3	4	25	6
966	9	8	5	36
967	3	4	5	6
968	3	16	5	12
969	9	4	5	18
970	3	8	25	12
971	3	4	5	6
972	9	16	5	36
973	3	4	5	6
974	3	8	5	12
975	9	4	25	18
976	3	16	5	12
977	3	4	5	6
978	9	8	5	36
979	3	4	5	6
980	3	16	25	12
981	9	4	5	18
982	3	8	5	12
983	3	4	5	6
984	9	16	5	36
985	3	4	25	6
986	3	8	5	12
987	9	4	5	18
988	3	16	5	12
989	3	4	5	6
990	9	8	25	36
991	3	4	5	6
992	3	16	5	12
993	9	4	5	18
994	3	8	5	12
995	3	4	25	6
996	9	16	5	36
997	3	4	5	6
998	3	8	5	12
999	9	4	5	18
1000	3	16	25	12
1001	3	4	5	6
1002	9	8	5	36
1003	3	4	5	6
1004	3	16	5	12
1005	9	4	25	18
1006	3	8	5	12
1007	3	4	5	6
1008	9	16	5	36
1009	3	4	5	6
1010	3	8	25	12
1011	9	4	5	18
1012	3	16	5	12
1013	3	4	5	6
1014	9	8	5	36
1015	3	4	25	6
1016	3	16	5	12
1017	9	4	5	18
1018	3	8	5	12
1019	3	4	5	6
1020	9	16	25	36
1021	3	4	5	6
1022	3	8	5	12
1023	9	4	5	18
1024	3	16	5	12
1025	3	4	25	6
1026	9	8	5	36

Table C.6: 2-Twist-spun $T(2,m)$ ($843 \leq m \leq 1026$) vs. Quandles R3-R6: Numbers of Colorings

Quandle	Pairs Distinguished
R7	18146
R8	4710
R9	4643
R10	0
Undistinguished Pairs So Far: 46094	

Table C.7: Quandles R7-R10 and Their Performances

	R7	R8	R9	R10
3	7	8	27	10
4	7	32	9	20
5	7	8	9	50
6	7	16	27	20
7	49	8	9	10
8	7	64	9	20
9	7	8	81	10
10	7	16	9	100
11	7	8	9	10
12	7	32	27	20
13	7	8	9	10
14	49	16	9	20
15	7	8	27	50
16	7	64	9	20
17	7	8	9	10
18	7	16	81	20
19	7	8	9	10
20	7	32	9	100
21	49	8	27	10
22	7	16	9	20
23	7	8	9	10
24	7	64	27	20
25	7	8	9	50
26	7	16	9	20
27	7	8	81	10
28	49	32	9	20
29	7	8	9	10
30	7	16	27	100
31	7	8	9	10
32	7	64	9	20
33	7	8	27	10
34	7	16	9	20
35	49	8	9	50
36	7	32	81	20
37	7	8	9	10
38	7	16	9	20
39	7	8	27	10
40	7	64	9	100
41	7	8	9	10
42	49	16	27	20
43	7	8	9	10
44	7	32	9	20
45	7	8	81	50
46	7	16	9	20
47	7	8	9	10
48	7	64	27	20
49	49	8	9	10
50	7	16	9	100
51	7	8	27	10
52	7	32	9	20
53	7	8	9	10
54	7	16	81	20
55	7	8	9	50
56	49	64	9	20
57	7	8	27	10
58	7	16	9	20

	R7	R8	R9	R10
59	7	8	9	10
60	7	32	27	100
61	7	8	9	10
62	7	16	9	20
63	49	8	81	10
64	7	64	9	20
65	7	8	9	50
66	7	16	27	20
67	7	8	9	10
68	7	32	9	20
69	7	8	27	10
70	49	16	9	100
71	7	8	9	10
72	7	64	81	20
73	7	8	9	10
74	7	16	9	20
75	7	8	27	50
76	7	32	9	20
77	49	8	9	10
78	7	16	27	20
79	7	8	9	10
80	7	64	9	100
81	7	8	81	10
82	7	16	9	20
83	7	8	9	10
84	49	32	27	20
85	7	8	9	50
86	7	16	9	20
87	7	8	27	10
88	7	64	9	20
89	7	8	9	10
90	7	16	81	100
91	49	8	9	10
92	7	32	9	20
93	7	8	27	10
94	7	16	9	20
95	7	8	9	50
96	7	64	27	20
97	7	8	9	10
98	49	16	9	20
99	7	8	81	10
100	7	32	9	100
101	7	8	9	10
102	7	16	27	20
103	7	8	9	10
104	7	64	9	20
105	49	8	27	50
106	7	16	9	20
107	7	8	9	10
108	7	32	81	20
109	7	8	9	10
110	7	16	9	100
111	7	8	27	10
112	49	64	9	20
113	7	8	9	10
114	7	16	27	20

	R7	R8	R9	R10
115	7	8	9	50
116	7	32	9	20
117	7	8	81	10
118	7	16	9	20
119	49	8	9	10
120	7	64	27	100
121	7	8	9	10
122	7	16	9	20
123	7	8	27	10
124	7	32	9	20
125	7	8	9	50
126	49	16	81	20
127	7	8	9	10
128	7	64	9	20
129	7	8	27	10
130	7	16	9	100
131	7	8	9	10
132	7	32	27	20
133	49	8	9	10
134	7	16	9	20
135	7	8	81	50
136	7	64	9	20
137	7	8	9	10
138	7	16	27	20
139	7	8	9	10
140	49	32	9	100
141	7	8	27	10
142	7	16	9	20
143	7	8	9	10
144	7	64	81	20
145	7	8	9	50
146	7	16	9	20
147	49	8	27	10
148	7	32	9	20
149	7	8	9	10
150	7	16	27	100
151	7	8	9	10
152	7	64	9	20
153	7	8	81	10
154	49	16	9	20
155	7	8	9	50
156	7	32	27	20
157	7	8	9	10
158	7	16	9	20
159	7	8	27	10
160	7	64	9	100
161	49	8	9	10
162	7	16	81	20
163	7	8	9	10
164	7	32	9	20
165	7	8	27	50
166	7	16	9	20
167	7	8	9	10
168	49	64	27	20
169	7	8	9	10
170	7	16	9	100

Table C.8: 2-Twist-spun $T(2,m)$ $(3 \leq m \leq 170)$ vs. Quandles R7-R10: Numbers of Colorings

	R7	R8	R9	R10
171	7	8	81	10
172	7	32	9	20
173	7	8	9	10
174	7	16	27	20
175	49	8	9	50
176	7	64	9	20
177	7	8	27	10
178	7	16	9	20
179	7	8	9	10
180	7	32	81	100
181	7	8	9	10
182	49	16	9	20
183	7	8	27	10
184	7	64	9	20
185	7	8	9	50
186	7	16	27	20
187	7	8	9	10
188	7	32	9	20
189	49	8	81	10
190	7	16	9	100
191	7	8	9	10
192	7	64	27	20
193	7	8	9	10
194	7	16	9	20
195	7	8	27	50
196	49	32	9	20
197	7	8	9	10
198	7	16	81	20
199	7	8	9	10
200	7	64	9	100
201	7	8	27	10
202	7	16	9	20
203	49	8	9	10
204	7	32	27	20
205	7	8	9	50
206	7	16	9	20
207	7	8	81	10
208	7	64	9	20
209	7	8	9	10
210	49	16	27	100
211	7	8	9	10
212	7	32	9	20
213	7	8	27	10
214	7	16	9	20
215	7	8	9	50
216	7	64	81	20
217	49	8	9	10
218	7	16	9	20
219	7	8	27	10
220	7	32	9	100
221	7	8	9	10
222	7	16	27	20
223	7	8	9	10
224	49	64	9	20
225	7	8	81	50
226	7	16	9	20
227	7	8	9	10
228	7	32	27	20
229	7	8	9	10
230	7	16	9	100
231	49	8	27	10
232	7	64	9	20
233	7	8	9	10
234	7	16	81	20
235	7	8	9	50
236	7	32	9	20
237	7	8	27	10
238	49	16	9	20
239	7	8	9	10
240	7	64	27	100
241	7	8	9	10
242	7	16	9	20
243	7	8	81	10
244	7	32	9	20
245	49	8	9	50
246	7	16	27	20
247	7	8	9	10
248	7	64	9	20
249	7	8	27	10
250	7	16	9	100
251	7	8	9	10
252	49	32	81	20
253	7	8	9	10
254	7	16	9	20
255	7	8	27	50
256	7	64	9	20
257	7	8	9	10
258	7	16	27	20
259	49	8	9	10
260	7	32	9	100
261	7	8	81	10
262	7	16	9	20
263	7	8	9	10
264	7	64	27	20
265	7	8	9	50
266	49	16	9	20
267	7	8	27	10
268	7	32	9	20
269	7	8	9	10
270	7	16	81	100
271	7	8	9	10
272	7	64	9	20
273	49	8	27	10
274	7	16	9	20
275	7	8	9	50
276	7	32	27	20
277	7	8	9	10
278	7	16	9	20
279	7	8	81	10
280	49	64	9	100
281	7	8	9	10
282	7	16	27	20
283	7	8	9	10
284	7	32	9	20
285	7	8	27	50
286	7	16	9	20
287	49	8	9	10
288	7	64	81	20
289	7	8	9	10
290	7	16	9	100
291	7	8	27	10
292	7	32	9	20
293	7	8	9	10
294	49	16	27	20
295	7	8	9	50
296	7	64	9	20
297	7	8	81	10
298	7	16	9	20
299	7	8	9	10
300	7	32	27	100
301	49	8	9	10
302	7	16	9	20
303	7	8	27	10
304	7	64	9	20
305	7	8	9	50
306	7	16	81	20
307	7	8	9	10
308	49	32	9	20
309	7	8	27	10
310	7	16	9	100
311	7	8	9	10
312	7	64	27	20
313	7	8	9	10
314	7	16	9	20
315	49	8	81	50
316	7	32	9	20
317	7	8	9	10
318	7	16	27	20
319	7	8	9	10
320	7	64	9	100
321	7	8	27	10
322	49	16	9	20
323	7	8	9	10
324	7	32	81	20
325	7	8	9	50
326	7	16	9	20
327	7	8	27	10
328	7	64	9	20
329	49	8	9	10
330	7	16	27	100
331	7	8	9	10
332	7	32	9	20
333	7	8	81	10
334	7	16	9	20
335	7	8	9	50
336	49	64	27	20
337	7	8	9	10
338	7	16	9	20
339	7	8	27	10
340	7	32	9	100
341	7	8	9	10
342	7	16	81	20
343	49	8	9	10
344	7	64	9	20
345	7	8	27	50
346	7	16	9	20
347	7	8	9	10
348	7	32	27	20
349	7	8	9	10
350	49	16	9	100
351	7	8	81	10
352	7	64	9	20
353	7	8	9	10
354	7	16	27	20
355	7	8	9	50
356	7	32	9	20
357	49	8	27	10
358	7	16	9	20
359	7	8	9	10
360	7	64	81	100
361	7	8	9	10
362	7	16	9	20
363	7	8	27	10
364	49	32	9	20
365	7	8	9	50
366	7	16	27	20
367	7	8	9	10
368	7	64	9	20
369	7	8	81	10
370	7	16	9	100
371	49	8	9	10
372	7	32	27	20
373	7	8	9	10
374	7	16	9	20
375	7	8	27	50
376	7	64	9	20
377	7	8	9	10
378	49	16	81	20
379	7	8	9	10
380	7	32	9	100
381	7	8	27	10
382	7	16	9	20
383	7	8	9	10

Table C.9: 2-Twist-spun $T(2, m)$ $(171 \le m \le 383)$ vs. Quandles R7-R10: Numbers of Colorings

	R7	R8	R9	R10
384	7	64	27	20
385	49	8	9	50
386	7	16	9	20
387	7	8	81	10
388	7	32	9	20
389	7	8	9	10
390	7	16	27	100
391	7	8	9	10
392	49	64	9	20
393	7	8	27	10
394	7	16	9	20
395	7	8	9	50
396	7	32	81	20
397	7	8	9	10
398	7	16	9	20
399	49	8	27	10
400	7	64	9	100
401	7	8	9	10
402	7	16	27	20
403	7	8	9	10
404	7	32	9	20
405	7	8	81	50
406	49	16	9	20
407	7	8	9	10
408	7	64	27	20
409	7	8	9	10
410	7	16	9	100
411	7	8	27	10
412	7	32	9	20
413	49	8	9	10
414	7	16	81	20
415	7	8	9	50
416	7	64	9	20
417	7	8	27	10
418	7	16	9	20
419	7	8	9	10
420	49	32	27	100
421	7	8	9	10
422	7	16	9	20
423	7	8	81	10
424	7	64	9	20
425	7	8	9	50
426	7	16	27	20
427	49	8	9	10
428	7	32	9	20
429	7	8	27	10
430	7	16	9	100
431	7	8	9	10
432	7	64	81	20
433	7	8	9	10
434	49	16	9	20
435	7	8	27	50
436	7	32	9	20
437	7	8	9	10
438	7	16	27	20
439	7	8	9	10
440	7	64	9	100
441	49	8	81	10
442	7	16	9	20
443	7	8	9	10
444	7	32	27	20
445	7	8	9	50
446	7	16	9	20
447	7	8	27	10
448	49	64	9	20
449	7	8	9	10
450	7	16	81	100
451	7	8	9	10
452	7	32	9	20
453	7	8	27	10
454	7	16	9	20

	R7	R8	R9	R10
455	49	8	9	50
456	7	64	27	20
457	7	8	9	10
458	7	16	9	20
459	7	8	81	10
460	7	32	9	100
461	7	8	9	10
462	49	16	27	20
463	7	8	9	10
464	7	64	9	20
465	7	8	27	50
466	7	16	9	20
467	7	8	9	10
468	7	32	81	20
469	49	8	9	10
470	7	16	9	100
471	7	8	27	10
472	7	64	9	20
473	7	8	9	10
474	7	16	27	20
475	7	8	9	50
476	49	32	9	20
477	7	8	81	10
478	7	16	9	20
479	7	8	9	10
480	7	64	27	100
481	7	8	9	10
482	7	16	9	20
483	49	8	27	10
484	7	32	9	20
485	7	8	9	50
486	7	16	81	20
487	7	8	9	10
488	7	64	9	20
489	7	8	27	10
490	49	16	9	100
491	7	8	9	10
492	7	32	27	20
493	7	8	9	10
494	7	16	9	20
495	7	8	81	50
496	7	64	9	20
497	49	8	9	10
498	7	16	27	20
499	7	8	9	10
500	7	32	9	100
501	7	8	27	10
502	7	16	9	20
503	7	8	9	10
504	49	64	81	20
505	7	8	9	50
506	7	16	9	20
507	7	8	27	10
508	7	32	9	20
509	7	8	9	10
510	7	16	27	100
511	49	8	9	10
512	7	64	9	20
513	7	8	81	10
514	7	16	9	20
515	7	8	9	50
516	7	32	27	20
517	7	8	9	10
518	49	16	9	20
519	7	8	27	10
520	7	64	9	100
521	7	8	9	10
522	7	16	81	20
523	7	8	9	10
524	7	32	9	20
525	49	8	27	50

	R7	R8	R9	R10
526	7	16	9	20
527	7	8	9	10
528	7	64	27	20
529	7	8	9	10
530	7	16	9	100
531	7	8	81	10
532	49	32	9	20
533	7	8	9	10
534	7	16	27	20
535	7	8	9	50
536	7	64	9	20
537	7	8	27	10
538	7	16	9	20
539	49	8	9	10
540	7	32	81	100
541	7	8	9	10
542	7	16	9	20
543	7	8	27	10
544	7	64	9	20
545	7	8	9	50
546	49	16	27	20
547	7	8	9	10
548	7	32	9	20
549	7	8	81	10
550	7	16	9	100
551	7	8	9	10
552	7	64	27	20
553	49	8	9	10
554	7	16	9	20
555	7	8	27	50
556	7	32	9	20
557	7	8	9	10
558	7	16	81	20
559	7	8	9	10
560	49	64	9	100
561	7	8	27	10
562	7	16	9	20
563	7	8	9	10
564	7	32	27	20
565	7	8	9	50
566	7	16	9	20
567	49	8	81	10
568	7	64	9	20
569	7	8	9	10
570	7	16	27	100
571	7	8	9	10
572	7	32	9	20
573	7	8	27	10
574	49	16	9	20
575	7	8	9	50
576	7	64	81	20
577	7	8	9	10
578	7	16	9	20
579	7	8	27	10
580	7	32	9	100
581	49	8	9	10
582	7	16	27	20
583	7	8	9	10
584	7	64	9	20
585	7	8	81	50
586	7	16	9	20
587	7	8	9	10
588	49	32	27	20
589	7	8	9	10
590	7	16	9	100
591	7	8	27	10
592	7	64	9	20
593	7	8	9	10
594	7	16	81	20
595	49	8	9	50
596	7	32	9	20

Table C.10: 2-Twist-spun $T(2, m)$ ($384 \leq m \leq 596$) vs. Quandles R7-R10: Numbers of Colorings

	R7	R8	R9	R10
597	7	8	27	10
598	7	16	9	20
599	7	8	9	10
600	7	64	27	100
601	7	8	9	10
602	49	16	9	20
603	7	8	81	10
604	7	32	9	20
605	7	8	9	50
606	7	16	27	20
607	7	8	9	10
608	7	64	9	20
609	49	8	27	10
610	7	16	9	100
611	7	8	9	10
612	7	32	81	20
613	7	8	9	10
614	7	16	9	20
615	7	8	27	50
616	49	64	9	20
617	7	8	9	10
618	7	16	27	20
619	7	8	9	10
620	7	32	9	100
621	7	8	81	10
622	7	16	9	20
623	49	8	9	10
624	7	64	27	20
625	7	8	9	50
626	7	16	9	20
627	7	8	27	10
628	7	32	9	20
629	7	8	9	10
630	49	16	81	100
631	7	8	9	10
632	7	64	9	20
633	7	8	27	10
634	7	16	9	20
635	7	8	9	50
636	7	32	27	20
637	49	8	9	10
638	7	16	9	20
639	7	8	81	10
640	7	64	9	100
641	7	8	9	10
642	7	16	27	20
643	7	8	9	10
644	49	32	9	20
645	7	8	27	50
646	7	16	9	20
647	7	8	9	10
648	7	64	81	20
649	7	8	9	10
650	7	16	9	100
651	49	8	27	10
652	7	32	9	20
653	7	8	9	10
654	7	16	27	20
655	7	8	9	50
656	7	64	9	20
657	7	8	81	10
658	49	16	9	20
659	7	8	9	10
660	7	32	27	100
661	7	8	9	10
662	7	16	9	20
663	7	8	27	10
664	7	64	9	20
665	49	8	9	50
666	7	16	81	20
667	7	8	9	10
668	7	32	9	20

	R7	R8	R9	R10
669	7	8	27	10
670	7	16	9	100
671	7	8	9	10
672	49	64	27	20
673	7	8	9	10
674	7	16	9	20
675	7	8	81	50
676	7	32	9	20
677	7	8	9	10
678	7	16	27	20
679	49	8	9	10
680	7	64	9	100
681	7	8	27	10
682	7	16	9	20
683	7	8	9	10
684	7	32	81	20
685	7	8	9	50
686	49	16	9	20
687	7	8	27	10
688	7	64	9	20
689	7	8	9	10
690	7	16	27	100
691	7	8	9	10
692	7	32	9	20
693	49	8	81	10
694	7	16	9	20
695	7	8	9	50
696	7	64	27	20
697	7	8	9	10
698	7	16	9	20
699	7	8	27	10
700	49	32	9	100
701	7	8	9	10
702	7	16	81	20
703	7	8	9	10
704	7	64	9	20
705	7	8	27	50
706	7	16	9	20
707	49	8	9	10
708	7	32	27	20
709	7	8	9	10
710	7	16	9	100
711	7	8	81	10
712	7	64	9	20
713	7	8	9	10
714	49	16	27	20
715	7	8	9	50
716	7	32	9	20
717	7	8	27	10
718	7	16	9	20
719	7	8	9	10
720	7	64	81	100
721	49	8	9	10
722	7	16	9	20
723	7	8	27	10
724	7	32	9	20
725	7	8	9	50
726	7	16	27	20
727	7	8	9	10
728	49	64	9	20
729	7	8	81	10
730	7	16	9	100
731	7	8	9	10
732	7	32	27	20
733	7	8	9	10
734	7	16	9	20
735	49	8	27	50
736	7	64	9	20
737	7	8	9	10
738	7	16	81	20
739	7	8	9	10
740	7	32	9	100

	R7	R8	R9	R10
741	7	8	27	10
742	49	16	9	20
743	7	8	9	10
744	7	64	27	20
745	7	8	9	50
746	7	16	9	20
747	7	8	81	10
748	7	32	9	20
749	49	8	9	10
750	7	16	27	100
751	7	8	9	10
752	7	64	9	20
753	7	8	27	10
754	7	16	9	20
755	7	8	9	50
756	49	32	81	20
757	7	8	9	10
758	7	16	9	20
759	7	8	27	10
760	7	64	9	100
761	7	8	9	10
762	7	16	27	20
763	49	8	9	10
764	7	32	9	20
765	7	8	81	50
766	7	16	9	20
767	7	8	9	10
768	7	64	27	20
769	7	8	9	10
770	49	16	9	100
771	7	8	27	10
772	7	32	9	20
773	7	8	9	10
774	7	16	81	20
775	7	8	9	50
776	7	64	9	20
777	49	8	27	10
778	7	16	9	20
779	7	8	9	10
780	7	32	27	100
781	7	8	9	10
782	7	16	9	20
783	7	8	81	10
784	49	64	9	20
785	7	8	9	50
786	7	16	27	20
787	7	8	9	10
788	7	32	9	20
789	7	8	27	10
790	7	16	9	100
791	49	8	9	10
792	7	64	81	20
793	7	8	9	10
794	7	16	9	20
795	7	8	27	50
796	7	32	9	20
797	7	8	9	10
798	49	16	27	20
799	7	8	9	10
800	7	64	9	100
801	7	8	81	10
802	7	16	9	20
803	7	8	9	10
804	7	32	27	20
805	49	8	9	50
806	7	16	9	20
807	7	8	27	10
808	7	64	9	20
809	7	8	9	10
810	7	16	81	100
811	7	8	9	10
812	49	32	9	20

Table C.11: 2-Twist-spun $T(2, m)$ ($597 \leq m \leq 812$) vs. Quandles R7-R10: Numbers of Colorings

	R7	R8	R9	R10
813	7	8	27	10
814	7	16	9	20
815	7	8	9	50
816	7	64	27	20
817	7	8	9	10
818	7	16	9	20
819	49	8	81	10
820	7	32	9	100
821	7	8	9	10
822	7	16	27	20
823	7	8	9	10
824	7	64	9	20
825	7	8	27	50
826	49	16	9	20
827	7	8	9	10
828	7	32	81	20
829	7	8	9	10
830	7	16	9	100
831	7	8	27	10
832	7	64	9	20
833	49	8	9	10
834	7	16	27	20
835	7	8	9	50
836	7	32	9	20
837	7	8	81	10
838	7	16	9	20
839	7	8	9	10
840	49	64	27	100
841	7	8	9	10
842	7	16	9	20
843	7	8	27	10
844	7	32	9	20
845	7	8	9	50
846	7	16	81	20
847	49	8	9	10
848	7	64	9	20
849	7	8	27	10
850	7	16	9	100
851	7	8	9	10
852	7	32	27	20
853	7	8	9	10
854	49	16	9	20
855	7	8	81	50
856	7	64	9	20
857	7	8	9	10
858	7	16	27	20
859	7	8	9	10
860	7	32	9	100
861	49	8	27	10
862	7	16	9	20
863	7	8	9	10
864	7	64	81	20
865	7	8	9	50
866	7	16	9	20
867	7	8	27	10
868	49	32	9	20
869	7	8	9	10
870	7	16	27	100
871	7	8	9	10
872	7	64	9	20
873	7	8	81	10
874	7	16	9	20
875	49	8	9	50
876	7	32	27	20
877	7	8	9	10
878	7	16	9	20
879	7	8	27	10
880	7	64	9	100
881	7	8	9	10
882	49	16	81	20
883	7	8	9	10
884	7	32	9	20
885	7	8	27	50
886	7	16	9	20
887	7	8	9	10
888	7	64	27	20
889	49	8	9	10
890	7	16	9	100
891	7	8	81	10
892	7	32	9	20
893	7	8	9	10
894	7	16	27	20
895	7	8	9	50
896	49	64	9	20
897	7	8	27	10
898	7	16	9	20
899	7	8	9	10
900	7	32	81	100
901	7	8	9	10
902	7	16	9	20
903	49	8	27	10
904	7	64	9	20
905	7	8	9	50
906	7	16	27	20
907	7	8	9	10
908	7	32	9	20
909	7	8	81	10
910	49	16	9	100
911	7	8	9	10
912	7	64	27	20
913	7	8	9	10
914	7	16	9	20
915	7	8	27	50
916	7	32	9	20
917	49	8	9	10
918	7	16	81	20
919	7	8	9	10
920	7	64	9	100
921	7	8	27	10
922	7	16	9	20
923	7	8	9	10
924	49	32	27	20
925	7	8	9	50
926	7	16	9	20
927	7	8	81	10
928	7	64	9	20
929	7	8	9	10
930	7	16	27	100
931	49	8	9	10
932	7	32	9	20
933	7	8	27	10
934	7	16	9	20
935	7	8	9	50
936	7	64	81	20
937	7	8	9	10
938	49	16	9	20
939	7	8	27	10
940	7	32	9	100
941	7	8	9	10
942	7	16	27	20
943	7	8	9	10
944	7	64	9	20
945	49	8	81	50
946	7	16	9	20
947	7	8	9	10
948	7	32	27	20
949	7	8	9	10
950	7	16	9	100
951	7	8	27	10
952	49	64	9	20
953	7	8	9	10
954	7	16	81	20
955	7	8	9	50
956	7	32	9	20
957	7	8	27	10
958	7	16	9	20
959	49	8	9	10
960	7	64	27	100
961	7	8	9	10
962	7	16	9	20
963	7	8	81	10
964	7	32	9	20
965	7	8	9	50
966	49	16	27	20
967	7	8	9	10
968	7	64	9	20
969	7	8	27	10
970	7	16	9	100
971	7	8	9	10
972	7	32	81	20
973	49	8	9	10
974	7	16	9	20
975	7	8	27	50
976	7	64	9	20
977	7	8	9	10
978	7	16	27	20
979	7	8	9	10
980	49	32	9	100
981	7	8	81	10
982	7	16	9	20
983	7	8	9	10
984	7	64	27	20
985	7	8	9	50
986	7	16	9	20
987	49	8	27	10
988	7	32	9	20
989	7	8	9	10
990	7	16	81	100
991	7	8	9	10
992	7	64	9	20
993	7	8	27	10
994	49	16	9	20
995	7	8	9	50
996	7	32	27	20
997	7	8	9	10
998	7	16	9	20
999	7	8	81	10
1000	7	64	9	100
1001	49	8	9	10
1002	7	16	27	20
1003	7	8	9	10
1004	7	32	9	20
1005	7	8	27	50
1006	7	16	9	20
1007	7	8	9	10
1008	49	64	81	20
1009	7	8	9	10
1010	7	16	9	100
1011	7	8	27	10
1012	7	32	9	20
1013	7	8	9	10
1014	7	16	27	20
1015	49	8	9	50
1016	7	64	9	20
1017	7	8	81	10
1018	7	16	9	20
1019	7	8	9	10
1020	7	32	27	100
1021	7	8	9	10
1022	49	16	9	20
1023	7	8	27	10
1024	7	64	9	20
1025	7	8	9	50
1026	7	16	81	20

Table C.12: 2-Twist-spun $T(2, m)$ ($813 \leq m \leq 1026$) vs. Quandles R7-R10: Numbers of Colorings

Quandle	Pairs Distinguished
R11	7718
R12	0
R13	5503
R14	0
Undistinguished Pairs So Far: 32873	

Table C.13: Quandles R11-R14 and Their Performances

	R11	R12	R13	R14
3	11	36	13	14
4	11	48	13	28
5	11	12	13	14
6	11	72	13	28
7	11	12	13	98
8	11	48	13	28
9	11	36	13	14
10	11	24	13	28
11	121	12	13	14
12	11	144	13	28
13	11	12	169	14
14	11	24	13	196
15	11	36	13	14
16	11	48	13	28
17	11	12	13	14
18	11	72	13	28
19	11	12	13	14
20	11	48	13	28
21	11	36	13	98
22	121	24	13	28
23	11	12	13	14
24	11	144	13	28
25	11	12	13	14
26	11	24	169	28
27	11	36	13	14
28	11	48	13	196
29	11	12	13	14
30	11	72	13	28
31	11	12	13	14
32	11	48	13	28
33	121	36	13	14
34	11	24	13	28
35	11	12	13	98
36	11	144	13	28
37	11	12	13	14
38	11	24	13	28
39	11	36	169	14
40	11	48	13	28
41	11	12	13	14
42	11	72	13	196
43	11	12	13	14
44	121	48	13	28
45	11	36	13	14
46	11	24	13	28
47	11	12	13	14
48	11	144	13	28
49	11	12	13	98
50	11	24	13	28
51	11	36	13	14
52	11	48	169	28
53	11	12	13	14
54	11	72	13	28
55	121	12	13	14
56	11	48	13	196
57	11	36	13	14
58	11	24	13	28
59	11	12	13	14
60	11	144	13	28
61	11	12	13	14
62	11	24	13	28
63	11	36	13	98
64	11	48	13	28
65	11	12	169	14
66	121	72	13	28
67	11	12	13	14
68	11	48	13	28
69	11	36	13	14
70	11	24	13	196
71	11	12	13	14
72	11	144	13	28
73	11	12	13	14
74	11	24	13	28
75	11	36	13	14
76	11	48	13	28
77	121	12	13	98
78	11	72	169	28
79	11	12	13	14
80	11	48	13	28
81	11	36	13	14
82	11	24	13	28
83	11	12	13	14
84	11	144	13	196
85	11	12	13	14
86	11	24	13	28
87	11	36	13	14
88	121	48	13	28
89	11	12	13	14
90	11	72	13	28
91	11	12	169	98
92	11	48	13	28
93	11	36	13	14
94	11	24	13	28
95	11	12	13	14
96	11	144	13	28
97	11	12	13	14
98	11	24	13	196
99	121	36	13	14
100	11	48	13	28
101	11	12	13	14
102	11	72	13	28
103	11	12	13	14
104	11	48	169	28
105	11	36	13	98
106	11	24	13	28
107	11	12	13	14
108	11	144	13	28
109	11	12	13	14
110	121	24	13	28
111	11	36	13	14
112	11	48	13	196
113	11	12	13	14
114	11	72	13	28
115	11	12	13	14
116	11	48	13	28
117	11	36	169	14
118	11	24	13	28
119	11	12	13	98
120	11	144	13	28
121	121	12	13	14
122	11	24	13	28
123	11	36	13	14
124	11	48	13	28
125	11	12	13	14
126	11	72	13	196
127	11	12	13	14
128	11	48	13	28
129	11	36	13	14
130	11	24	169	28
131	11	12	13	14
132	121	144	13	28
133	11	12	13	98
134	11	24	13	28
135	11	36	13	14
136	11	48	13	28
137	11	12	13	14
138	11	72	13	28
139	11	12	13	14
140	11	48	13	196
141	11	36	13	14
142	11	24	13	28
143	121	12	169	14
144	11	144	13	28
145	11	12	13	14
146	11	24	13	28
147	11	36	13	98
148	11	48	13	28
149	11	12	13	14
150	11	72	13	28
151	11	12	13	14
152	11	48	13	28
153	11	36	13	14
154	121	24	13	196
155	11	12	13	14
156	11	144	169	28
157	11	12	13	14
158	11	24	13	28
159	11	36	13	14
160	11	48	13	28
161	11	12	13	98
162	11	72	13	28
163	11	12	13	14
164	11	48	13	28
165	121	36	13	14
166	11	24	13	28
167	11	12	13	14
168	11	144	13	196
169	11	12	169	14
170	11	24	13	28

Table C.14: 2-Twist-spun $T(2, m)$ $(3 \leq m \leq 170)$ vs. Quandles R11-R14: Numbers of Colorings

	R11	R12	R13	R14
171	11	36	13	14
172	11	48	13	28
173	11	12	13	14
174	11	72	13	28
175	11	12	13	98
176	121	48	13	28
177	11	36	13	14
178	11	24	13	28
179	11	12	13	14
180	11	144	13	28
181	11	12	13	14
182	11	24	169	196
183	11	36	13	14
184	11	48	13	28
185	11	12	13	14
186	11	72	13	28
187	121	12	13	14
188	11	48	13	28
189	11	36	13	98
190	11	24	13	28
191	11	12	13	14
192	11	144	13	28
193	11	12	13	14
194	11	24	13	28
195	11	36	169	14
196	11	48	13	196
197	11	12	13	14
198	121	72	13	28
199	11	12	13	14
200	11	48	13	28
201	11	36	13	14
202	11	24	13	28
203	11	12	13	98
204	11	144	13	28
205	11	12	13	14
206	11	24	13	28
207	11	36	13	14
208	11	48	169	28
209	121	12	13	14
211	11	12	13	14
212	11	48	13	28
213	11	36	13	14
214	11	24	13	28
215	11	12	13	14
216	11	144	13	28
217	11	12	13	98
218	11	24	13	28
219	11	36	13	14
220	121	48	13	28
221	11	12	169	14
222	11	72	13	28
223	11	12	13	14
224	11	48	13	196
225	11	36	13	14
226	11	24	13	28
227	11	12	13	14
228	11	144	13	28
229	11	12	13	14
230	11	24	13	28
231	121	36	13	98
232	11	48	13	28
233	11	12	13	14
234	11	72	169	28
235	11	12	13	14
236	11	48	13	28
237	11	36	13	14
238	11	24	13	196
239	11	12	13	14
240	11	144	13	28
241	11	12	13	14
242	121	24	13	28
243	11	36	13	14

	R11	R12	R13	R14
244	11	48	13	28
245	11	12	13	98
246	11	72	13	28
247	11	12	169	14
248	11	48	13	28
249	11	36	13	14
250	11	24	13	28
251	11	12	13	14
252	11	144	13	196
253	121	12	13	14
254	11	24	13	28
255	11	36	13	14
256	11	48	13	28
257	11	12	13	14
258	11	72	13	28
259	11	12	13	98
260	11	48	169	28
261	11	36	13	14
262	11	24	13	28
263	11	12	13	14
264	121	144	13	28
265	11	12	13	14
266	11	24	13	196
267	11	36	13	14
268	11	48	13	28
269	11	12	13	14
270	11	72	13	28
271	11	12	13	14
272	11	48	13	28
273	11	36	169	98
274	11	24	13	28
275	121	12	13	14
276	11	144	13	28
277	11	12	13	14
278	11	24	13	28
279	11	36	13	14
280	11	48	13	196
281	11	12	13	14
282	11	72	13	28
283	11	12	13	14
284	11	48	13	28
285	11	36	13	14
286	121	24	169	28
287	11	12	13	98
288	11	144	13	28
289	11	12	13	14
290	11	24	13	28
291	11	36	13	14
292	11	48	13	28
293	11	12	13	14
294	11	72	13	196
295	11	12	13	14
296	11	48	13	28
297	121	36	13	14
298	11	24	13	28
299	11	12	169	14
300	11	144	13	28
301	11	12	13	98
302	11	24	13	28
303	11	36	13	14
304	11	48	13	28
305	11	12	13	14
306	11	72	13	28
307	11	12	13	14
308	121	48	13	196
309	11	36	13	14
310	11	24	13	28
311	11	12	13	14
312	11	144	169	28
313	11	12	13	14
314	11	24	13	28
315	11	36	13	98

	R11	R12	R13	R14
316	11	48	13	28
317	11	12	13	14
318	11	72	13	28
319	121	12	13	14
320	11	48	13	28
321	11	36	13	14
322	11	24	13	196
323	11	12	13	14
324	11	144	13	28
325	11	12	169	14
326	11	24	13	28
327	11	36	13	14
328	11	48	13	28
329	11	12	13	98
330	121	72	13	28
331	11	12	13	14
332	11	48	13	28
333	11	36	13	14
334	11	24	13	28
335	11	12	13	14
336	11	144	13	196
337	11	12	13	14
338	11	24	169	28
339	11	36	13	14
340	11	48	13	28
341	121	12	13	14
342	11	72	13	28
343	11	12	13	98
344	11	48	13	28
345	11	36	13	14
346	11	24	13	28
347	11	12	13	14
348	11	144	13	28
349	11	12	13	14
350	11	24	13	196
351	11	36	169	14
352	121	48	13	28
353	11	12	13	14
354	11	72	13	28
355	11	12	13	14
356	11	48	13	28
357	11	36	13	98
358	11	24	13	28
359	11	12	13	14
360	11	144	13	28
361	11	12	13	14
362	11	24	13	28
363	121	36	13	14
364	11	48	169	196
365	11	12	13	14
366	11	72	13	28
367	11	12	13	14
368	11	48	13	28
369	11	36	13	14
370	11	24	13	28
371	11	12	13	98
372	11	144	13	28
373	11	12	13	14
374	121	24	13	28
375	11	36	13	14
376	11	48	13	28
377	11	12	169	14
378	11	72	13	196
379	11	12	13	14
380	11	48	13	28
381	11	36	13	14
382	11	24	13	28
383	11	12	13	14
384	11	144	13	28
385	121	12	13	98
386	11	24	13	28
387	11	36	13	14

Table C.15: 2-Twist-spun $T(2,m)$ $(171 \leq m \leq 387)$ vs. Quandles R11-R14: Numbers of Colorings

	R11	R12	R13	R14
388	11	48	13	28
389	11	12	13	14
390	11	72	169	28
391	11	12	13	14
392	11	48	13	196
393	11	36	13	14
394	11	24	13	28
395	11	12	13	14
396	121	144	13	28
397	11	12	13	14
398	11	24	13	28
399	11	36	13	98
400	11	48	13	28
401	11	12	13	14
402	11	72	13	28
403	11	12	169	14
404	11	48	13	28
405	11	36	13	14
406	11	24	13	196
407	121	12	13	14
408	11	144	13	28
409	11	12	13	14
410	11	24	13	28
411	11	36	13	14
412	11	48	13	28
413	11	12	13	98
414	11	72	13	28
415	11	12	13	14
416	11	48	169	28
417	11	36	13	14
418	121	24	13	28
419	11	12	13	14
421	11	12	13	14
422	11	24	13	28
423	11	36	13	14
424	11	48	13	28
425	11	12	13	14
426	11	72	13	28
427	11	12	13	98
428	11	48	13	28
429	121	36	169	14
430	11	24	13	28
431	11	12	13	14
432	11	144	13	28
433	11	12	13	14
434	11	24	13	196
435	11	36	13	14
436	11	48	13	28
437	11	12	13	14
438	11	72	13	28
439	11	12	13	14
440	121	48	13	28
441	11	36	13	98
442	11	24	169	28
443	11	12	13	14
444	11	144	13	28
445	11	12	13	14
446	11	24	13	28
447	11	36	13	14
448	11	48	13	196
449	11	12	13	14
450	11	72	13	28
451	121	12	13	14
452	11	48	13	28
453	11	36	13	14
454	11	24	13	28
455	11	12	169	98
456	11	144	13	28
457	11	12	13	14
458	11	24	13	28
459	11	36	13	14

	R11	R12	R13	R14
460	11	48	13	28
461	11	12	13	14
462	121	72	13	196
463	11	12	13	14
464	11	48	13	28
465	11	36	13	14
466	11	24	13	28
467	11	12	13	14
468	11	144	169	28
469	11	12	13	98
470	11	24	13	28
471	11	36	13	14
472	11	48	13	28
473	121	12	13	14
474	11	72	13	28
475	11	12	13	14
476	11	48	13	196
477	11	36	13	14
478	11	24	13	28
479	11	12	13	14
480	11	144	13	28
481	11	12	169	14
482	11	24	13	28
483	11	36	13	98
484	121	48	13	28
485	11	12	13	14
486	11	72	13	28
487	11	12	13	14
488	11	48	13	28
489	11	36	13	14
490	11	24	13	196
491	11	12	13	14
492	11	144	13	28
493	11	12	13	14
494	11	24	169	28
495	121	36	13	14
496	11	48	13	28
497	11	12	13	98
498	11	72	13	28
499	11	12	13	14
500	11	48	13	28
501	11	36	13	14
502	11	24	13	28
503	11	12	13	14
504	11	144	13	196
505	11	12	13	14
506	121	24	13	28
507	11	36	169	14
508	11	48	13	28
509	11	12	13	14
510	11	72	13	28
511	11	12	13	98
512	11	48	13	28
513	11	36	13	14
514	11	24	13	28
515	11	12	13	14
516	11	144	13	28
517	121	12	13	14
518	11	24	13	196
519	11	36	13	14
520	11	48	169	28
521	11	12	13	14
522	11	72	13	28
523	11	12	13	14
524	11	48	13	28
525	11	36	13	98
526	11	24	13	28
527	11	12	13	14
528	121	144	13	28
529	11	12	13	14
530	11	24	13	28

	R11	R12	R13	R14
531	11	36	13	14
532	11	48	13	196
533	11	12	169	14
534	11	72	13	28
535	11	12	13	14
536	11	48	13	28
537	11	36	13	14
538	11	24	13	28
539	121	12	13	98
540	11	144	13	28
541	11	12	13	14
542	11	24	13	28
543	11	36	13	14
544	11	48	13	28
545	11	12	13	14
546	11	72	169	196
547	11	12	13	14
548	11	48	13	28
549	11	36	13	14
550	121	24	13	28
551	11	12	13	14
552	11	144	13	28
553	11	12	13	98
554	11	24	13	28
555	11	36	13	14
556	11	48	13	28
557	11	12	13	14
558	11	72	13	28
559	11	12	169	14
560	11	48	13	196
561	121	36	13	14
562	11	24	13	28
563	11	12	13	14
564	11	144	13	28
565	11	12	13	14
566	11	24	13	28
567	11	36	13	98
568	11	48	13	28
569	11	12	13	14
570	11	72	13	28
571	11	12	13	14
572	121	48	169	28
573	11	36	13	14
574	11	24	13	196
575	11	12	13	14
576	11	144	13	28
577	11	12	13	14
578	11	24	13	28
579	11	36	13	14
580	11	48	13	28
581	11	12	13	98
582	11	72	13	28
583	121	12	13	14
584	11	48	13	28
585	11	36	169	14
586	11	24	13	28
587	11	12	13	14
588	11	144	13	196
589	11	12	13	14
590	11	24	13	28
591	11	36	13	14
592	11	48	13	28
593	11	12	13	14
594	121	72	13	28
595	11	12	13	98
596	11	48	13	28
597	11	36	13	14
598	11	24	169	28
599	11	12	13	14
600	11	144	13	28
601	11	12	13	14

Table C.16: 2-Twist-spun $T(2,m)$ $(388 \leq m \leq 601)$ vs. Quandles R11-R14: Numbers of Colorings

	R11	R12	R13	R14
602	11	24	13	196
603	11	36	13	14
604	11	48	13	28
605	121	12	13	14
606	11	72	13	28
607	11	12	13	14
608	11	48	13	28
609	11	36	13	98
610	11	24	13	28
611	11	12	169	14
612	11	144	13	28
613	11	12	13	14
614	11	24	13	28
615	11	36	13	14
616	121	48	13	196
617	11	12	13	14
618	11	72	13	28
619	11	12	13	14
620	11	48	13	28
621	11	36	13	14
622	11	24	13	28
623	11	12	13	98
624	11	144	169	28
625	11	12	13	14
626	11	24	13	28
627	121	36	13	14
628	11	48	13	28
629	11	12	13	14
631	11	12	13	14
632	11	48	13	28
633	11	36	13	14
634	11	24	13	28
635	11	12	13	14
636	11	144	13	28
637	11	12	169	98
638	121	24	13	28
639	11	36	13	14
640	11	48	13	28
641	11	12	13	14
642	11	72	13	28
643	11	12	13	14
644	11	48	13	196
645	11	36	13	14
646	11	24	13	28
647	11	12	13	14
648	11	144	13	28
649	121	12	13	14
650	11	24	169	28
651	11	36	13	98
652	11	48	13	28
653	11	12	13	14
654	11	72	13	28
655	11	12	13	14
656	11	48	13	28
657	11	36	13	14
658	11	24	13	196
659	11	12	13	14
660	121	144	13	28
661	11	12	13	14
662	11	24	13	28
663	11	36	169	14
664	11	48	13	28
665	11	12	13	98
666	11	72	13	28
667	11	12	13	14
668	11	48	13	28
669	11	36	13	14
670	11	24	13	28
671	121	12	13	14
672	11	144	13	196
673	11	12	13	14

	R11	R12	R13	R14
674	11	24	13	28
675	11	36	13	14
676	11	48	169	28
677	11	12	13	14
678	11	72	13	28
679	11	12	13	98
680	11	48	13	28
681	11	36	13	14
682	121	24	13	28
683	11	12	13	14
684	11	144	13	28
685	11	12	13	14
686	11	24	13	196
687	11	36	13	14
688	11	48	13	28
689	11	12	169	14
690	11	72	13	28
691	11	12	13	14
692	11	48	13	28
693	121	36	13	98
694	11	24	13	28
695	11	12	13	14
696	11	144	13	28
697	11	12	13	14
698	11	24	13	28
699	11	36	13	14
700	11	48	13	196
701	11	12	13	14
702	11	72	169	28
703	11	12	13	14
704	121	48	13	28
705	11	36	13	14
706	11	24	13	28
707	11	12	13	98
708	11	144	13	28
709	11	12	13	14
710	11	24	13	28
711	11	36	13	14
712	11	48	13	28
713	11	12	13	14
714	11	72	13	196
715	121	12	169	14
716	11	48	13	28
717	11	36	13	14
718	11	24	13	28
719	11	12	13	14
720	11	144	13	28
721	11	12	13	98
722	11	24	13	28
723	11	36	13	14
724	11	48	13	28
725	11	12	13	14
726	121	72	13	28
727	11	12	13	14
728	11	48	169	196
729	11	36	13	14
730	11	24	13	28
731	11	12	13	14
732	11	144	13	28
733	11	12	13	14
734	11	24	13	28
735	11	36	13	98
736	11	48	13	28
737	121	12	13	14
738	11	72	13	28
739	11	12	13	14
740	11	48	13	28
741	11	36	169	14
742	11	24	13	196
743	11	12	13	14
744	11	144	13	28

	R11	R12	R13	R14
745	11	12	13	14
746	11	24	13	28
747	11	36	13	14
748	121	48	13	28
749	11	12	13	98
750	11	72	13	28
751	11	12	13	14
752	11	48	13	28
753	11	36	13	14
754	11	24	169	28
755	11	12	13	14
756	11	144	13	196
757	11	12	13	14
758	11	24	13	28
759	121	36	13	14
760	11	48	13	28
761	11	12	13	14
762	11	72	13	28
763	11	12	13	98
764	11	48	13	28
765	11	36	13	14
766	11	24	13	28
767	11	12	169	14
768	11	144	13	28
769	11	12	13	14
770	121	24	13	196
771	11	36	13	14
772	11	48	13	28
773	11	12	13	14
774	11	72	13	28
775	11	12	13	14
776	11	48	13	28
777	11	36	13	98
778	11	24	13	28
779	11	12	13	14
780	11	144	169	28
781	121	12	13	14
782	11	24	13	28
783	11	36	13	14
784	11	48	13	196
785	11	12	13	14
786	11	72	13	28
787	11	12	13	14
788	11	48	13	28
789	11	36	13	14
790	11	24	13	28
791	11	12	13	98
792	121	144	13	28
793	11	12	169	14
794	11	24	13	28
795	11	36	13	14
796	11	48	13	28
797	11	12	13	14
798	11	72	13	196
799	11	12	13	14
800	11	48	13	28
801	11	36	13	14
802	11	24	13	28
803	121	12	13	14
804	11	144	13	28
805	11	12	13	98
806	11	24	169	28
807	11	36	13	14
808	11	48	13	28
809	11	12	13	14
810	11	72	13	28
811	11	12	13	14
812	11	48	13	196
813	11	36	13	14
814	121	24	13	28
815	11	12	13	14

Table C.17: 2-Twist-spun $T(2,m)$ ($602 \leq m \leq 815$) vs. Quandles R11-R14: Numbers of Colorings

	R11	R12	R13	R14
816	11	144	13	28
817	11	12	13	14
818	11	24	13	28
819	11	36	169	98
820	11	48	13	28
821	11	12	13	14
822	11	72	13	28
823	11	12	13	14
824	11	48	13	28
825	121	36	13	14
826	11	24	13	196
827	11	12	13	14
828	11	144	13	28
829	11	12	13	14
830	11	24	13	28
831	11	36	13	14
832	11	48	169	28
833	11	12	13	98
834	11	72	13	28
835	11	12	13	14
836	121	48	13	28
837	11	36	13	14
838	11	24	13	28
839	11	12	13	14
841	11	12	13	14
842	11	24	13	28
843	11	36	13	14
844	11	48	13	28
845	11	12	169	14
846	11	72	13	28
847	121	12	13	98
848	11	48	13	28
849	11	36	13	14
850	11	24	13	28
851	11	12	13	14
852	11	144	13	28
853	11	12	13	14
854	11	24	13	196
855	11	36	13	14
856	11	48	13	28
857	11	12	13	14
858	121	72	169	28
859	11	12	13	14
860	11	48	13	28
861	11	36	13	98
862	11	24	13	28
863	11	12	13	14
864	11	144	13	28
865	11	12	13	14
866	11	24	13	28
867	11	36	13	14
868	11	48	13	196
869	121	12	13	14
870	11	72	13	28
871	11	12	169	14
872	11	48	13	28
873	11	36	13	14
874	11	24	13	28
875	11	12	13	98
876	11	144	13	28
877	11	12	13	14
878	11	24	13	28
879	11	36	13	14
880	121	48	13	28
881	11	12	13	14
882	11	72	13	196
883	11	12	13	14
884	11	48	169	28
885	11	36	13	14
886	11	24	13	28

	R11	R12	R13	R14
887	11	12	13	14
888	11	144	13	28
889	11	12	13	98
890	11	24	13	28
891	121	36	13	14
892	11	48	13	28
893	11	12	13	14
894	11	72	13	28
895	11	12	13	14
896	11	48	13	196
897	11	36	169	14
898	11	24	13	28
899	11	12	13	14
900	11	144	13	28
901	11	12	13	14
902	121	24	13	28
903	11	36	13	98
904	11	48	13	28
905	11	12	13	14
906	11	72	13	28
907	11	12	13	14
908	11	48	13	28
909	11	36	13	14
910	11	24	169	196
911	11	12	13	14
912	11	144	13	28
913	121	12	13	14
914	11	24	13	28
915	11	36	13	14
916	11	48	13	28
917	11	12	13	98
918	11	72	13	28
919	11	12	13	14
920	11	48	13	28
921	11	36	13	14
922	11	24	13	28
923	11	12	169	14
924	121	144	13	196
925	11	12	13	14
926	11	24	13	28
927	11	36	13	14
928	11	48	13	28
929	11	12	13	14
930	11	72	13	28
931	11	12	13	98
932	11	48	13	28
933	11	36	13	14
934	11	24	13	28
935	121	12	13	14
936	11	144	169	28
937	11	12	13	14
938	11	24	13	196
939	11	36	13	14
940	11	48	13	28
941	11	12	13	14
942	11	72	13	28
943	11	12	13	14
944	11	48	13	28
945	11	36	13	98
946	121	24	13	28
947	11	12	13	14
948	11	144	13	28
949	11	12	169	14
950	11	24	13	28
951	11	36	13	14
952	11	48	13	196
953	11	12	13	14
954	11	72	13	28
955	11	12	13	14
956	11	48	13	28

	R11	R12	R13	R14
957	121	36	13	14
958	11	24	13	28
959	11	12	13	98
960	11	144	13	28
961	11	12	13	14
962	11	24	169	28
963	11	36	13	14
964	11	48	13	28
965	11	12	13	14
966	11	72	13	196
967	11	12	13	14
968	121	48	13	28
969	11	36	13	14
970	11	24	13	28
971	11	12	13	14
972	11	144	13	28
973	11	12	13	98
974	11	24	13	28
975	11	36	169	14
976	11	48	13	28
977	11	12	13	14
978	11	72	13	28
979	121	12	13	14
980	11	48	13	196
981	11	36	13	14
982	11	24	13	28
983	11	12	13	14
984	11	144	13	28
985	11	12	13	14
986	11	24	13	28
987	11	36	13	98
988	11	48	169	28
989	11	12	13	14
990	121	72	13	28
991	11	12	13	14
992	11	48	13	28
993	11	36	13	14
994	11	24	13	196
995	11	12	13	14
996	11	144	13	28
997	11	12	13	14
998	11	24	13	28
999	11	36	13	14
1000	11	48	13	28
1001	121	12	169	98
1002	11	72	13	28
1003	11	12	13	14
1004	11	48	13	28
1005	11	36	13	14
1006	11	24	13	28
1007	11	12	13	14
1008	11	144	13	196
1009	11	12	13	14
1010	11	24	13	28
1011	11	36	13	14
1012	121	48	13	28
1013	11	12	13	14
1014	11	72	169	28
1015	11	12	13	98
1016	11	48	13	28
1017	11	36	13	14
1018	11	24	13	28
1019	11	12	13	14
1020	11	144	13	28
1021	11	12	13	14
1022	11	24	13	196
1023	121	36	13	14
1024	11	48	13	28
1025	11	12	13	14
1026	11	72	13	28

Table C.18: 2-Twist-spun $T(2, m)$ ($816 \leq m \leq 1026$) vs. Quandles R11-R14: Numbers of Colorings

Quandle	Pairs Distinguished
R15	0
R16	769
R17	3491
R18	0
Undistinguished Pairs So Far: 28613	

Table C.19: Quandles R15-R18 and Their Performances

	R15	R16	R17	R18
3	45	16	17	54
4	15	64	17	36
5	75	16	17	18
6	45	32	17	108
7	15	16	17	18
8	15	128	17	36
9	45	16	17	162
10	75	32	17	36
11	15	16	17	18
12	45	64	17	108
13	15	16	17	18
14	15	32	17	36
15	225	16	17	54
16	15	256	17	36
17	15	16	289	18
18	45	32	17	324
19	15	16	17	18
20	75	64	17	36
21	45	16	17	54
22	15	32	17	36
23	15	16	17	18
24	45	128	17	108
25	75	16	17	18
26	15	32	17	36
27	45	16	17	162
28	15	64	17	36
29	15	16	17	18
30	225	32	17	108
31	15	16	17	18
32	15	256	17	36
33	45	16	17	54
34	15	32	289	36
35	75	16	17	18
36	45	64	17	324
37	15	16	17	18
38	15	32	17	36
39	45	16	17	54
40	75	128	17	36
41	15	16	17	18
42	45	32	17	108
43	15	16	17	18
44	15	64	17	36
45	225	16	17	162
46	15	32	17	36
47	15	16	17	18
48	45	256	17	108
49	15	16	17	18
50	75	32	17	36
51	45	16	289	54
52	15	64	17	36
53	15	16	17	18
54	45	32	17	324
55	75	16	17	18
56	15	128	17	36
57	45	16	17	54
58	15	32	17	36

	R15	R16	R17	R18
59	15	16	17	18
60	225	64	17	108
61	15	16	17	18
62	15	32	17	36
63	45	16	17	162
64	15	256	17	36
65	75	16	17	18
66	45	32	17	108
67	15	16	17	18
68	15	64	289	36
69	45	16	17	54
70	75	32	17	36
71	15	16	17	18
72	45	128	17	324
73	15	16	17	18
74	15	32	17	36
75	225	16	17	54
76	15	64	17	36
77	15	16	17	18
78	45	32	17	108
79	15	16	17	18
80	75	256	17	36
81	45	16	17	162
82	15	32	17	36
83	15	16	17	18
84	45	64	17	108
85	75	16	289	18
86	15	32	17	36
87	45	16	17	54
88	15	128	17	36
89	15	16	17	18
90	225	32	17	324
91	15	16	17	18
92	15	64	17	36
93	45	16	17	54
94	15	32	17	36
95	75	16	17	18
96	45	256	17	108
97	15	16	17	18
98	15	32	17	36
99	45	16	17	162
100	75	64	17	36
101	15	16	17	18
102	45	32	289	108
103	15	16	17	18
104	15	128	17	36
105	225	16	17	54
106	15	32	17	36
107	15	16	17	18
108	45	64	17	324
109	15	16	17	18
110	75	32	17	36
111	45	16	17	54
112	15	256	17	36
113	15	16	17	18
114	45	32	17	108

	R15	R16	R17	R18
115	75	16	17	18
116	15	64	17	36
117	45	16	17	162
118	15	32	17	36
119	15	16	289	18
120	225	128	17	108
121	15	16	17	18
122	15	32	17	36
123	45	16	17	54
124	15	64	17	36
125	75	16	17	18
126	45	32	17	324
127	15	16	17	18
128	15	256	17	36
129	45	16	17	54
130	75	32	17	36
131	15	16	17	18
133	15	16	17	18
134	15	32	17	36
135	225	16	17	162
136	15	128	289	36
137	15	16	17	18
138	45	32	17	108
139	15	16	17	18
140	75	64	17	36
141	45	16	17	54
142	15	32	17	36
144	45	256	17	324
145	75	16	17	18
146	15	32	17	36
147	45	16	17	54
148	15	64	17	36
149	15	16	17	18
150	225	32	17	108
151	15	16	17	18
152	15	128	17	36
153	45	16	289	162
155	75	16	17	18
157	15	16	17	18
158	15	32	17	36
159	45	16	17	54
160	75	256	17	36
161	15	16	17	18
162	45	32	17	324
163	15	16	17	18
164	15	64	17	36
165	225	16	17	54
166	15	32	17	36
167	15	16	17	18
168	45	128	17	108
169	15	16	17	18
170	75	32	289	36
171	45	16	17	162
172	15	64	17	36
173	15	16	17	18
174	45	32	17	108

Table C.20: 2-Twist-spun $T(2,m)$ $(3 \leq m \leq 174)$ vs. Quandles R15-R18: Numbers of Colorings

	R15	R16	R17	R18
175	75	16	17	18
176	15	256	17	36
177	45	16	17	54
178	15	32	17	36
179	15	16	17	18
180	225	64	17	324
181	15	16	17	18
183	45	16	17	54
184	15	128	17	36
185	75	16	17	18
186	45	32	17	108
187	15	16	289	18
188	15	64	17	36
189	45	16	17	162
190	75	32	17	36
191	15	16	17	18
192	45	256	17	108
193	15	16	17	18
194	15	32	17	36
195	225	16	17	54
196	15	64	17	36
197	15	16	17	18
198	45	32	17	324
199	15	16	17	18
200	75	128	17	36
201	45	16	17	54
202	15	32	17	36
203	15	16	17	18
204	45	64	289	108
205	75	16	17	18
206	15	32	17	36
207	45	16	17	162
208	15	256	17	36
209	15	16	17	18
211	15	16	17	18
212	15	64	17	36
213	45	16	17	54
214	15	32	17	36
215	75	16	17	18
216	45	128	17	324
217	15	16	17	18
218	15	32	17	36
219	45	16	17	54
221	15	16	289	18
222	45	32	17	108
223	15	16	17	18
224	15	256	17	36
225	225	16	17	162
226	15	32	17	36
227	15	16	17	18
228	45	64	17	108
229	15	16	17	18
230	75	32	17	36
232	15	128	17	36
233	15	16	17	18
234	45	32	17	324
235	75	16	17	18
236	15	64	17	36
237	45	16	17	54
238	15	32	289	36
239	15	16	17	18
240	225	256	17	108
241	15	16	17	18
242	15	32	17	36
243	45	16	17	162
244	15	64	17	36
245	75	16	17	18
246	45	32	17	108
247	15	16	17	18

	R15	R16	R17	R18
248	15	128	17	36
249	45	16	17	54
250	75	32	17	36
251	15	16	17	18
252	45	64	17	324
253	15	16	17	18
254	15	32	17	36
255	225	16	289	54
256	15	256	17	36
257	15	16	17	18
258	45	32	17	108
259	15	16	17	18
261	45	16	17	162
262	15	32	17	36
263	15	16	17	18
264	45	128	17	108
265	75	16	17	18
266	15	32	17	36
267	45	16	17	54
268	15	64	17	36
269	15	16	17	18
270	225	32	17	324
271	15	16	17	18
272	15	256	289	36
274	15	32	17	36
275	75	16	17	18
276	45	64	17	108
277	15	16	17	18
278	15	32	17	36
279	45	16	17	162
280	75	128	17	36
281	15	16	17	18
282	45	32	17	108
283	15	16	17	18
284	15	64	17	36
285	225	16	17	54
287	15	16	17	18
288	45	256	17	324
289	15	16	289	18
290	75	32	17	36
291	45	16	17	54
292	15	64	17	36
293	15	16	17	18
294	45	32	17	108
295	75	16	17	18
296	15	128	17	36
297	45	16	17	162
298	15	32	17	36
299	15	16	17	18
300	225	64	17	108
301	15	16	17	18
302	15	32	17	36
303	45	16	17	54
304	15	256	17	36
305	75	16	17	18
306	45	32	289	324
307	15	16	17	18
309	45	16	17	54
310	75	32	17	36
311	15	16	17	18
312	45	128	17	108
313	15	16	17	18
314	15	32	17	36
315	225	16	17	162
316	15	64	17	36
317	15	16	17	18
318	45	32	17	108
319	15	16	17	18
320	75	256	17	36

	R15	R16	R17	R18
321	45	16	17	54
322	15	32	17	36
323	15	16	289	18
324	45	64	17	324
325	75	16	17	18
326	15	32	17	36
327	45	16	17	54
328	15	128	17	36
329	15	16	17	18
331	15	16	17	18
332	15	64	17	36
333	45	16	17	162
334	15	32	17	36
335	75	16	17	18
336	45	256	17	108
337	15	16	17	18
338	15	32	17	36
339	45	16	17	54
340	75	64	289	36
341	15	16	17	18
342	45	32	17	324
343	15	16	17	18
344	15	128	17	36
345	225	16	17	54
346	15	32	17	36
347	15	16	17	18
348	45	64	17	108
349	15	16	17	18
350	75	32	17	36
351	45	16	17	162
352	15	256	17	36
353	15	16	17	18
354	45	32	17	108
355	75	16	17	18
356	15	64	17	36
357	45	16	289	54
358	15	32	17	36
359	15	16	17	18
360	225	128	17	324
361	15	16	17	18
362	15	32	17	36
363	45	16	17	54
365	75	16	17	18
366	45	32	17	108
367	15	16	17	18
368	15	256	17	36
369	45	16	17	162
370	75	32	17	36
371	15	16	17	18
372	45	64	17	108
373	15	16	17	18
374	15	32	289	36
375	225	16	17	54
376	15	128	17	36
377	15	16	17	18
378	45	32	17	324
379	15	16	17	18
380	75	64	17	36
381	45	16	17	54
382	15	32	17	36
383	15	16	17	18
384	45	256	17	108
386	15	32	17	36
387	45	16	17	162
388	15	64	17	36
389	15	16	17	18
391	15	16	289	18
392	15	128	17	36
393	45	16	17	54

Table C.21: 2-Twist-spun $T(2,m)$ $(175 \leq m \leq 393)$ vs. Quandles R15-R18: Numbers of Colorings

	R15	R16	R17	R18
394	15	32	17	36
395	75	16	17	18
397	15	16	17	18
398	15	32	17	36
399	45	16	17	54
400	75	256	17	36
401	15	16	17	18
402	45	32	17	108
403	15	16	17	18
404	15	64	17	36
405	225	16	17	162
406	15	32	17	36
407	15	16	17	18
408	45	128	289	108
409	15	16	17	18
410	75	32	17	36
411	45	16	17	54
412	15	64	17	36
413	15	16	17	18
414	45	32	17	324
415	75	16	17	18
416	15	256	17	36
417	45	16	17	54
418	15	32	17	36
419	15	16	17	18
421	15	16	17	18
422	15	32	17	36
423	45	16	17	162
424	15	128	17	36
425	75	16	289	18
426	45	32	17	108
427	15	16	17	18
428	15	64	17	36
430	75	32	17	36
431	15	16	17	18
432	45	256	17	324
433	15	16	17	18
434	15	32	17	36
435	225	16	17	54
436	15	64	17	36
437	15	16	17	18
438	45	32	17	108
439	15	16	17	18
440	75	128	17	36
441	45	16	17	162
442	15	32	289	36
443	15	16	17	18
444	45	64	17	108
445	75	16	17	18
446	15	32	17	36
447	45	16	17	54
448	15	256	17	36
449	15	16	17	18
450	225	32	17	324
451	15	16	17	18
452	15	64	17	36
453	45	16	17	54
454	15	32	17	36
456	45	128	17	108
457	15	16	17	18
458	15	32	17	36
459	45	16	289	162
460	75	64	17	36
461	15	16	17	18
463	15	16	17	18
464	15	256	17	36
465	225	16	17	54

	R15	R16	R17	R18
466	15	32	17	36
467	15	16	17	18
469	15	16	17	18
470	75	32	17	36
471	45	16	17	54
472	15	128	17	36
473	15	16	17	18
474	45	32	17	108
475	75	16	17	18
476	15	64	289	36
477	45	16	17	162
478	15	32	17	36
479	15	16	17	18
480	225	256	17	108
481	15	16	17	18
482	15	32	17	36
483	45	16	17	54
484	15	64	17	36
485	75	16	17	18
486	45	32	17	324
487	15	16	17	18
488	15	128	17	36
489	45	16	17	54
490	75	32	17	36
491	15	16	17	18
492	45	64	17	108
493	15	16	289	18
494	15	32	17	36
496	15	256	17	36
497	15	16	17	18
498	45	32	17	108
499	15	16	17	18
500	75	64	17	36
501	45	16	17	54
502	15	32	17	36
503	15	16	17	18
504	45	128	17	324
505	75	16	17	18
506	15	32	17	36
507	45	16	17	54
508	15	64	17	36
509	15	16	17	18
510	225	32	289	108
511	15	16	17	18
512	15	256	17	36
513	45	16	17	162
514	15	32	17	36
515	75	16	17	18
516	45	64	17	108
517	15	16	17	18
518	15	32	17	36
519	45	16	17	54
521	15	16	17	18
522	45	32	17	324
523	15	16	17	18
524	15	64	17	36
525	225	16	17	54
526	15	32	17	36
527	15	16	289	18
528	45	256	17	108
529	15	16	17	18
530	75	32	17	36
531	45	16	17	162
532	15	64	17	36
533	15	16	17	18
534	45	32	17	108
535	75	16	17	18

	R15	R16	R17	R18
536	15	128	17	36
537	45	16	17	54
538	15	32	17	36
539	15	16	17	18
540	225	64	17	324
541	15	16	17	18
542	15	32	17	36
543	45	16	17	54
544	15	256	289	36
545	75	16	17	18
547	15	16	17	18
548	15	64	17	36
549	45	16	17	162
550	75	32	17	36
551	15	16	17	18
552	45	128	17	108
553	15	16	17	18
554	15	32	17	36
555	225	16	17	54
556	15	64	17	36
557	15	16	17	18
558	45	32	17	324
559	15	16	17	18
560	75	256	17	36
561	45	16	289	54
562	15	32	17	36
563	15	16	17	18
564	45	64	17	108
565	75	16	17	18
566	15	32	17	36
567	45	16	17	162
568	15	128	17	36
569	15	16	17	18
570	225	32	17	108
571	15	16	17	18
573	45	16	17	54
574	15	32	17	36
575	75	16	17	18
576	45	256	17	324
577	15	16	17	18
578	15	32	289	36
579	45	16	17	54
580	75	64	17	36
581	15	16	17	18
582	45	32	17	108
583	15	16	17	18
584	15	128	17	36
586	15	32	17	36
587	15	16	17	18
588	45	64	17	108
589	15	16	17	18
590	75	32	17	36
591	45	16	17	54
592	15	256	17	36
593	15	16	17	18
594	45	32	17	324
595	75	16	289	18
596	15	64	17	36
597	45	16	17	54
598	15	32	17	36
599	15	16	17	18
600	225	128	17	108
601	15	16	17	18
602	15	32	17	36
603	45	16	17	162
604	15	64	17	36
605	75	16	17	18

Table C.22: 2-Twist-spun $T(2,m)$ ($394 \leq m \leq 605$) vs. Quandles R15-R18: Numbers of Colorings

	R15	R16	R17	R18
606	45	32	17	108
607	15	16	17	18
608	15	256	17	36
609	45	16	17	54
610	75	32	17	36
611	15	16	17	18
612	45	64	289	324
613	15	16	17	18
614	15	32	17	36
615	225	16	17	54
617	15	16	17	18
618	45	32	17	108
619	15	16	17	18
620	75	64	17	36
621	45	16	17	162
622	15	32	17	36
623	15	16	17	18
624	45	256	17	108
625	75	16	17	18
626	15	32	17	36
627	45	16	17	54
628	15	64	17	36
629	15	16	289	18
631	15	16	17	18
632	15	128	17	36
633	45	16	17	54
634	15	32	17	36
635	75	16	17	18
636	45	64	17	108
637	15	16	17	18
638	15	32	17	36
639	45	16	17	162
640	75	256	17	36
641	15	16	17	18
642	45	32	17	108
643	15	16	17	18
644	15	64	17	36
645	225	16	17	54
646	15	32	289	36
647	15	16	17	18
648	45	128	17	324
649	15	16	17	18
650	75	32	17	36
651	45	16	17	54
652	15	64	17	36
653	15	16	17	18
654	45	32	17	108
655	75	16	17	18
656	15	256	17	36
657	45	16	17	162
658	15	32	17	36
659	15	16	17	18
661	15	16	17	18
662	15	32	17	36
663	45	16	289	54
664	15	128	17	36
665	75	16	17	18
666	45	32	17	324
667	15	16	17	18
668	15	64	17	36
669	45	16	17	54
670	75	32	17	36
671	15	16	17	18
672	45	256	17	108
673	15	16	17	18
674	15	32	17	36
675	225	16	17	162
676	15	64	17	36

	R15	R16	R17	R18
677	15	16	17	18
678	45	32	17	108
679	15	16	17	18
680	75	128	289	36
681	45	16	17	54
682	15	32	17	36
683	15	16	17	18
684	45	64	17	324
685	75	16	17	18
686	15	32	17	36
687	45	16	17	54
688	15	256	17	36
689	15	16	17	18
690	225	32	17	108
691	15	16	17	18
692	15	64	17	36
694	15	32	17	36
695	75	16	17	18
696	45	128	17	108
697	15	16	289	18
698	15	32	17	36
699	45	16	17	54
700	75	64	17	36
701	15	16	17	18
702	45	32	17	324
703	15	16	17	18
704	15	256	17	36
705	225	16	17	54
706	15	32	17	36
707	15	16	17	18
708	45	64	17	108
709	15	16	17	18
710	75	32	17	36
711	45	16	17	162
712	15	128	17	36
713	15	16	17	18
714	45	32	289	108
716	15	64	17	36
717	45	16	17	54
718	15	32	17	36
719	15	16	17	18
720	225	256	17	324
721	15	16	17	18
722	15	32	17	36
723	45	16	17	54
724	15	64	17	36
725	75	16	17	18
726	45	32	17	108
727	15	16	17	18
729	45	16	17	162
730	75	32	17	36
731	15	16	289	18
732	45	64	17	108
733	15	16	17	18
734	15	32	17	36
735	225	16	17	54
736	15	256	17	36
737	15	16	17	18
738	45	32	17	324
739	15	16	17	18
740	75	64	17	36
741	45	16	17	54
742	15	32	17	36
743	15	16	17	18
744	45	128	17	108
745	75	16	17	18
746	15	32	17	36
747	45	16	17	162

	R15	R16	R17	R18
748	15	64	289	36
749	15	16	17	18
750	225	32	17	108
751	15	16	17	18
752	15	256	17	36
753	45	16	17	54
754	15	32	17	36
755	75	16	17	18
756	45	64	17	324
757	15	16	17	18
758	15	32	17	36
759	45	16	17	54
760	75	128	17	36
761	15	16	17	18
762	45	32	17	108
763	15	16	17	18
764	15	64	17	36
765	225	16	289	162
766	15	32	17	36
767	15	16	17	18
768	45	256	17	108
769	15	16	17	18
771	45	16	17	54
772	15	64	17	36
773	15	16	17	18
774	45	32	17	324
775	75	16	17	18
776	15	128	17	36
777	45	16	17	54
778	15	32	17	36
779	15	16	17	18
781	15	16	17	18
782	15	32	289	36
783	45	16	17	162
784	15	256	17	36
785	75	16	17	18
786	45	32	17	108
787	15	16	17	18
788	15	64	17	36
789	45	16	17	54
790	75	32	17	36
791	15	16	17	18
793	15	16	17	18
794	15	32	17	36
795	225	16	17	54
796	15	64	17	36
797	15	16	17	18
798	45	32	17	108
799	15	16	289	18
800	75	256	17	36
801	45	16	17	162
802	15	32	17	36
803	15	16	17	18
804	45	64	17	108
805	75	16	17	18
806	15	32	17	36
807	45	16	17	54
808	15	128	17	36
809	15	16	17	18
810	225	32	17	324
811	15	16	17	18
812	15	64	17	36
813	45	16	17	54
814	15	32	17	36
815	75	16	17	18
816	45	256	289	108
817	15	16	17	18
818	15	32	17	36

Table C.23: 2-Twist-spun $T(2,m)$ $(606 \leq m \leq 818)$ vs. Quandles R15-R18: Numbers of Colorings

	R15	R16	R17	R18
820	75	64	17	36
821	15	16	17	18
822	45	32	17	108
823	15	16	17	18
824	15	128	17	36
825	225	16	17	54
826	15	32	17	36
827	15	16	17	18
828	45	64	17	324
829	15	16	17	18
830	75	32	17	36
831	45	16	17	54
832	15	256	17	36
833	15	16	289	18
834	45	32	17	108
835	75	16	17	18
836	15	64	17	36
837	45	16	17	162
838	15	32	17	36
839	15	16	17	18
841	15	16	17	18
842	15	32	17	36
843	45	16	17	54
844	15	64	17	36
845	75	16	17	18
846	45	32	17	324
847	15	16	17	18
848	15	256	17	36
849	45	16	17	54
850	75	32	289	36
851	15	16	17	18
852	45	64	17	108
853	15	16	17	18
854	15	32	17	36
855	225	16	17	162
856	15	128	17	36
857	15	16	17	18
859	15	16	17	18
860	75	64	17	36
861	45	16	17	54
862	15	32	17	36
863	15	16	17	18
864	45	256	17	324
865	75	16	17	18
866	15	32	17	36
867	45	16	289	54
868	15	64	17	36
869	15	16	17	18
870	225	32	17	108
871	15	16	17	18
872	15	128	17	36
873	45	16	17	162
874	15	32	17	36
875	75	16	17	18
876	45	64	17	108
877	15	16	17	18
878	15	32	17	36
879	45	16	17	54
880	75	256	17	36
881	15	16	17	18
882	45	32	17	324
883	15	16	17	18
884	15	64	289	36
885	225	16	17	54
886	15	32	17	36
887	15	16	17	18

	R15	R16	R17	R18
888	45	128	17	108
889	15	16	17	18
890	75	32	17	36
891	45	16	17	162
892	15	64	17	36
893	15	16	17	18
894	45	32	17	108
895	75	16	17	18
896	15	256	17	36
897	45	16	17	54
898	15	32	17	36
899	15	16	17	18
900	225	64	17	324
901	15	16	289	18
902	15	32	17	36
903	45	16	17	54
904	15	128	17	36
905	75	16	17	18
906	45	32	17	108
907	15	16	17	18
908	15	64	17	36
909	45	16	17	162
911	15	16	17	18
912	45	256	17	108
913	15	16	17	18
914	15	32	17	36
915	225	16	17	54
916	15	64	17	36
917	15	16	17	18
918	45	32	289	324
919	15	16	17	18
920	75	128	17	36
921	45	16	17	54
922	15	32	17	36
923	15	16	17	18
925	75	16	17	18
926	15	32	17	36
927	45	16	17	162
928	15	256	17	36
929	15	16	17	18
930	225	32	17	108
931	15	16	17	18
932	15	64	17	36
933	45	16	17	54
934	15	32	17	36
935	75	16	289	18
937	15	16	17	18
938	15	32	17	36
939	45	16	17	54
940	75	64	17	36
941	15	16	17	18
942	45	32	17	108
943	15	16	17	18
944	15	256	17	36
945	225	16	17	162
946	15	32	17	36
947	15	16	17	18
948	45	64	17	108
949	15	16	17	18
950	75	32	17	36
951	45	16	17	54
952	15	128	289	36
953	15	16	17	18
954	45	32	17	324
955	75	16	17	18
956	15	64	17	36

	R15	R16	R17	R18
957	45	16	17	54
958	15	32	17	36
959	15	16	17	18
960	225	256	17	108
961	15	16	17	18
962	15	32	17	36
963	45	16	17	162
964	15	64	17	36
965	75	16	17	18
966	45	32	17	108
967	15	16	17	18
968	15	128	17	36
969	45	16	289	54
970	75	32	17	36
971	15	16	17	18
972	45	64	17	324
973	15	16	17	18
974	15	32	17	36
975	225	16	17	54
976	15	256	17	36
977	15	16	17	18
978	45	32	17	108
979	15	16	17	18
980	75	64	17	36
981	45	16	17	162
982	15	32	17	36
983	15	16	17	18
984	45	128	17	108
985	75	16	17	18
986	15	32	289	36
987	45	16	17	54
988	15	64	17	36
989	15	16	17	18
991	15	16	17	18
992	15	256	17	36
993	45	16	17	54
994	15	32	17	36
995	75	16	17	18
996	45	64	17	108
997	15	16	17	18
998	15	32	17	36
999	45	16	17	162
1000	75	128	17	36
1002	45	32	17	108
1003	15	16	289	18
1004	15	64	17	36
1005	225	16	17	54
1006	15	32	17	36
1007	15	16	17	18
1008	45	256	17	324
1009	15	16	17	18
1010	75	32	17	36
1011	45	16	17	54
1012	15	64	17	36
1013	15	16	17	18
1014	45	32	17	108
1015	75	16	17	18
1016	15	128	17	36
1017	45	16	17	162
1018	15	32	17	36
1019	15	16	17	18
1020	225	64	289	108
1021	15	16	17	18
1022	15	32	17	36
1023	45	16	17	54
1024	15	256	17	36
1025	75	16	17	18
1026	45	32	17	324

Table C.24: 2-Twist-spun $T(2,m)$ $(819 \leq m \leq 1026)$ vs. Quandles R15-R18: Numbers of Colorings

Quandle	Pairs Distinguished
R19	2668
R20	0
R21	0
R22	0
Undistinguished Pairs So Far: 25945	

Table C.25: Quandles R19-R22 and Their Performances

	R19	R20	R21	R22
3	19	20	63	22
4	19	80	21	44
5	19	100	21	22
6	19	40	63	44
7	19	20	147	22
8	19	80	21	44
9	19	20	63	22
10	19	200	21	44
11	19	20	21	242
12	19	80	63	44
13	19	20	21	22
14	19	40	147	44
15	19	100	63	22
16	19	80	21	44
17	19	20	21	22
18	19	40	63	44
19	361	20	21	22
20	19	400	21	44
21	19	20	441	22
22	19	40	21	484
23	19	20	21	22
24	19	80	63	44
25	19	100	21	22
26	19	40	21	44
27	19	20	63	22
28	19	80	147	44
29	19	20	21	22
30	19	200	63	44
31	19	20	21	22
32	19	80	21	44
33	19	20	63	242
34	19	40	21	44
35	19	100	147	22
36	19	80	63	44
37	19	20	21	22
38	361	40	21	44
39	19	20	63	22
40	19	400	21	44
41	19	20	21	22
42	19	40	441	44
43	19	20	21	22
44	19	80	21	484
45	19	100	63	22
46	19	40	21	44
47	19	20	21	22
48	19	80	63	44
49	19	20	147	22
50	19	200	21	44
51	19	20	63	22
52	19	80	21	44
53	19	20	21	22
54	19	40	63	44
55	19	100	21	242
56	19	80	147	44
57	361	20	63	22
58	19	40	21	44
59	19	20	21	22
60	19	400	63	44
61	19	20	21	22
62	19	40	21	44
63	19	20	441	22
64	19	80	21	44
65	19	100	21	22
66	19	40	63	484
67	19	20	21	22
69	19	20	63	22
70	19	200	147	44
71	19	20	21	22
72	19	80	63	44
73	19	20	21	22
74	19	40	21	44
75	19	100	63	22
76	361	80	21	44
77	19	20	147	242
78	19	40	63	44
79	19	20	21	22
80	19	400	21	44
81	19	20	63	22
82	19	40	21	44
83	19	20	21	22
84	19	80	441	44
85	19	100	21	22
86	19	40	21	44
87	19	20	63	22
88	19	80	21	484
89	19	20	21	22
90	19	200	63	44
91	19	20	147	22
92	19	80	21	44
93	19	20	63	22
94	19	40	21	44
95	361	100	21	22
96	19	80	63	44
97	19	20	21	22
98	19	40	147	44
99	19	20	63	242
100	19	400	21	44
101	19	20	21	22
103	19	20	21	22
105	19	100	441	22
106	19	40	21	44
107	19	20	21	22
108	19	80	63	44
109	19	20	21	22
110	19	200	21	484
111	19	20	63	22
112	19	80	147	44
113	19	20	21	22
114	361	40	63	44
115	19	100	21	22
116	19	80	21	44
117	19	20	63	22
118	19	40	21	44
119	19	20	147	22
120	19	400	63	44
121	19	20	21	242
122	19	40	21	44
123	19	20	63	22
124	19	80	21	44
125	19	100	21	22
126	19	40	441	44
127	19	20	21	22
128	19	80	21	44
129	19	20	63	22
130	19	200	21	44
131	19	20	21	22
133	361	20	147	22
134	19	40	21	44
135	19	100	63	22
137	19	20	21	22
138	19	40	63	44
139	19	20	21	22
140	19	400	147	44
141	19	20	63	22
142	19	40	21	44
144	19	80	63	44
145	19	100	21	22
146	19	40	21	44
147	19	20	441	22
148	19	80	21	44
149	19	20	21	22
150	19	200	63	44
151	19	20	21	22
152	361	80	21	44
153	19	20	63	22
155	19	100	21	22
157	19	20	21	22
158	19	40	21	44
159	19	20	63	22
160	19	400	21	44
161	19	20	147	22
162	19	40	63	44
163	19	20	21	22
164	19	80	21	44
165	19	100	63	242
166	19	40	21	44
167	19	20	21	22
169	19	20	21	22
170	19	200	21	44
171	361	20	63	22
172	19	80	21	44
173	19	20	21	22
174	19	40	63	44
175	19	100	147	22
176	19	80	21	484
177	19	20	63	22
178	19	40	21	44
179	19	20	21	22

Table C.26: 2-Twist-spun $T(2, m)$ ($3 \leq m \leq 179$) vs. Quandles R19-R22: Numbers of Colorings

	R19	R20	R21	R22
180	19	400	63	44
181	19	20	21	22
183	19	20	63	22
184	19	80	21	44
185	19	100	21	22
186	19	40	63	44
188	19	80	21	44
189	19	20	441	22
190	361	200	21	44
191	19	20	21	22
192	19	80	63	44
193	19	20	21	22
194	19	40	21	44
195	19	100	63	22
196	19	80	147	44
197	19	20	21	22
198	19	40	63	484
199	19	20	21	22
200	19	400	21	44
201	19	20	63	22
202	19	40	21	44
203	19	20	147	22
205	19	100	21	22
206	19	40	21	44
207	19	20	63	22
208	19	80	21	44
209	361	20	21	242
211	19	20	21	22
212	19	80	21	44
213	19	20	63	22
214	19	40	21	44
215	19	100	21	22
216	19	80	63	44
217	19	20	147	22
218	19	40	21	44
219	19	20	63	22
222	19	40	63	44
223	19	20	21	22
224	19	80	147	44
225	19	100	63	22
226	19	40	21	44
227	19	20	21	22
228	361	80	63	44
229	19	20	21	22
230	19	200	21	44
232	19	80	21	44
233	19	20	21	22
234	19	40	63	44
235	19	100	21	22
236	19	80	21	44
237	19	20	63	22
239	19	20	21	22
240	19	400	63	44
241	19	20	21	22
242	19	40	21	484
243	19	20	63	22
244	19	80	21	44
245	19	100	147	22
246	19	40	63	44
247	361	20	21	22
248	19	80	21	44
249	19	20	63	22
250	19	200	21	44
251	19	20	21	22

	R19	R20	R21	R22
252	19	80	441	44
253	19	20	21	242
254	19	40	21	44
256	19	80	21	44
257	19	20	21	22
258	19	40	63	44
259	19	20	147	22
261	19	20	63	22
262	19	40	21	44
263	19	20	21	22
265	19	100	21	22
266	361	40	147	44
267	19	20	63	22
268	19	80	21	44
269	19	20	21	22
270	19	200	63	44
271	19	20	21	22
272	19	80	21	44
274	19	40	21	44
275	19	100	21	242
276	19	80	63	44
277	19	20	21	22
278	19	40	21	44
279	19	20	63	22
281	19	20	21	22
282	19	40	63	44
283	19	20	21	22
284	19	80	21	44
285	361	100	63	22
287	19	20	147	22
288	19	80	63	44
289	19	20	21	22
290	19	200	21	44
291	19	20	63	22
292	19	80	21	44
293	19	20	21	22
294	19	40	441	44
295	19	100	21	22
296	19	80	21	44
297	19	20	63	242
298	19	40	21	44
299	19	20	21	22
300	19	400	63	44
301	19	20	147	22
302	19	40	21	44
303	19	20	63	22
304	361	80	21	44
305	19	100	21	22
306	19	40	63	44
307	19	20	21	22
309	19	20	63	22
310	19	200	21	44
311	19	20	21	22
313	19	20	21	22
314	19	40	21	44
315	19	100	441	22
316	19	80	21	44
317	19	20	21	22
318	19	40	63	44
319	19	20	21	242
320	19	400	21	44
321	19	20	63	22
322	19	40	147	44
323	361	20	21	22

	R19	R20	R21	R22
324	19	80	63	44
325	19	100	21	22
326	19	40	21	44
327	19	20	63	22
328	19	80	21	44
329	19	20	147	22
331	19	20	21	22
332	19	80	21	44
333	19	20	63	22
334	19	40	21	44
335	19	100	21	22
336	19	80	441	44
337	19	20	21	22
338	19	40	21	44
339	19	20	63	22
341	19	20	21	242
342	361	40	63	44
343	19	20	147	22
344	19	80	21	44
345	19	100	63	22
346	19	40	21	44
347	19	20	21	22
348	19	80	63	44
349	19	20	21	22
350	19	200	147	44
351	19	20	63	22
352	19	80	21	484
353	19	20	21	22
354	19	40	63	44
355	19	100	21	22
356	19	80	21	44
358	19	40	21	44
359	19	20	21	22
361	361	20	21	22
362	19	40	21	44
363	19	20	63	242
365	19	100	21	22
366	19	40	63	44
367	19	20	21	22
368	19	80	21	44
369	19	20	63	22
370	19	200	21	44
371	19	20	147	22
372	19	80	63	44
373	19	20	21	22
375	19	100	63	22
376	19	80	21	44
377	19	20	21	22
378	19	40	441	44
379	19	20	21	22
380	361	400	21	44
381	19	20	63	22
382	19	40	21	44
383	19	20	21	22
384	19	80	63	44
386	19	40	21	44
387	19	20	63	22
388	19	80	21	44
389	19	20	21	22
391	19	20	21	22
392	19	80	147	44
393	19	20	63	22
394	19	40	21	44
395	19	100	21	22

Table C.27: 2-Twist-spun $T(2,m)$ $(180 \le m \le 395)$ vs. Quandles R19-R22: Numbers of Colorings

	R19	R20	R21	R22
397	19	20	21	22
398	19	40	21	44
399	361	20	441	22
400	19	400	21	44
401	19	20	21	22
402	19	40	63	44
403	19	20	21	22
404	19	80	21	44
405	19	100	63	22
406	19	40	147	44
407	19	20	21	242
409	19	20	21	22
410	19	200	21	44
411	19	20	63	22
412	19	80	21	44
413	19	20	147	22
414	19	40	63	44
415	19	100	21	22
416	19	80	21	44
417	19	20	63	22
418	361	40	21	484
419	19	20	21	22
421	19	20	21	22
422	19	40	21	44
423	19	20	63	22
424	19	80	21	44
425	19	100	21	22
426	19	40	63	44
427	19	20	147	22
428	19	80	21	44
430	19	200	21	44
431	19	20	21	22
432	19	80	63	44
433	19	20	21	22
434	19	40	147	44
435	19	100	63	22
436	19	80	21	44
437	361	20	21	22
438	19	40	63	44
439	19	20	21	22
441	19	20	441	22
443	19	20	21	22
444	19	80	63	44
445	19	100	21	22
446	19	40	21	44
447	19	20	63	22
448	19	80	147	44
449	19	20	21	22
450	19	200	63	44
451	19	20	21	242
452	19	80	21	44
453	19	20	63	22
454	19	40	21	44
456	361	80	63	44
457	19	20	21	22
458	19	40	21	44
459	19	20	63	22
460	19	400	21	44
461	19	20	21	22
463	19	20	21	22
464	19	80	21	44
465	19	100	63	22
466	19	40	21	44
467	19	20	21	22

	R19	R20	R21	R22
469	19	20	147	22
470	19	200	21	44
471	19	20	63	22
472	19	80	21	44
473	19	20	21	242
474	19	40	63	44
475	361	100	21	22
477	19	20	63	22
478	19	40	21	44
479	19	20	21	22
480	19	400	63	44
481	19	20	21	22
482	19	40	21	44
483	19	20	441	22
484	19	80	21	484
485	19	100	21	22
486	19	40	63	44
487	19	20	21	22
488	19	80	21	44
489	19	20	63	22
490	19	200	147	44
491	19	20	21	22
492	19	80	63	44
493	19	20	21	22
494	361	40	21	44
496	19	80	21	44
497	19	20	147	22
498	19	40	63	44
499	19	20	21	22
500	19	400	21	44
501	19	20	63	22
502	19	40	21	44
503	19	20	21	22
505	19	100	21	22
506	19	40	21	484
507	19	20	63	22
508	19	80	21	44
509	19	20	21	22
511	19	20	147	22
512	19	80	21	44
513	361	20	63	22
514	19	40	21	44
515	19	100	21	22
516	19	80	63	44
517	19	20	21	242
518	19	40	147	44
519	19	20	63	22
521	19	20	21	22
522	19	40	63	44
523	19	20	21	22
524	19	80	21	44
525	19	100	441	22
526	19	40	21	44
527	19	20	21	22
529	19	20	21	22
530	19	200	21	44
531	19	20	63	22
532	361	80	147	44
533	19	20	21	22
534	19	40	63	44
535	19	100	21	22
536	19	80	21	44
537	19	20	63	22
538	19	40	21	44

	R19	R20	R21	R22
539	19	20	147	242
540	19	400	63	44
541	19	20	21	22
542	19	40	21	44
543	19	20	63	22
544	19	80	21	44
545	19	100	21	22
547	19	20	21	22
548	19	80	21	44
549	19	20	63	22
550	19	200	21	484
551	361	20	21	22
552	19	80	63	44
553	19	20	147	22
554	19	40	21	44
555	19	100	63	22
556	19	80	21	44
557	19	20	21	22
558	19	40	63	44
559	19	20	21	22
562	19	40	21	44
563	19	20	21	22
564	19	80	63	44
565	19	100	21	22
566	19	40	21	44
567	19	20	441	22
568	19	80	21	44
569	19	20	21	22
570	361	200	63	44
571	19	20	21	22
573	19	20	63	22
574	19	40	147	44
575	19	100	21	22
576	19	80	63	44
577	19	20	21	22
578	19	40	21	44
579	19	20	63	22
580	19	400	21	44
581	19	20	147	22
582	19	40	63	44
583	19	20	21	242
584	19	80	21	44
586	19	40	21	44
587	19	20	21	22
588	19	80	441	44
589	361	20	21	22
590	19	200	21	44
591	19	20	63	22
592	19	80	21	44
593	19	20	21	22
594	19	40	63	484
596	19	80	21	44
597	19	20	63	22
598	19	40	21	44
599	19	20	21	22
600	19	400	63	44
601	19	20	21	22
602	19	40	147	44
603	19	20	63	22
604	19	80	21	44
605	19	100	21	242
606	19	40	63	44
607	19	20	21	22
608	361	80	21	44

Table C.28: 2-Twist-spun $T(2,m)$ ($397 \leq m \leq 608$) vs. Quandles R19-R22: Numbers of Colorings

	R19	R20	R21	R22
609	19	20	441	22
610	19	200	21	44
611	19	20	21	22
613	19	20	21	22
614	19	40	21	44
615	19	100	63	22
617	19	20	21	22
618	19	40	63	44
619	19	20	21	22
620	19	400	21	44
621	19	20	63	22
622	19	40	21	44
623	19	20	147	22
625	19	100	21	22
626	19	40	21	44
627	361	20	63	242
628	19	80	21	44
629	19	20	21	22
631	19	20	21	22
632	19	80	21	44
633	19	20	63	22
634	19	40	21	44
635	19	100	21	22
636	19	80	63	44
637	19	20	147	22
638	19	40	21	484
639	19	20	63	22
640	19	400	21	44
641	19	20	21	22
642	19	40	63	44
643	19	20	21	22
644	19	80	147	44
645	19	100	63	22
646	361	40	21	44
647	19	20	21	22
648	19	80	63	44
649	19	20	21	242
650	19	200	21	44
651	19	20	441	22
652	19	80	21	44
653	19	20	21	22
654	19	40	63	44
655	19	100	21	22
656	19	80	21	44
657	19	20	63	22
658	19	40	147	44
659	19	20	21	22
661	19	20	21	22
662	19	40	21	44
664	19	80	21	44
665	361	100	147	22
666	19	40	63	44
667	19	20	21	22
668	19	80	21	44
669	19	20	63	22
670	19	200	21	44
671	19	20	21	242
672	19	80	441	44
673	19	20	21	22
674	19	40	21	44
675	19	100	63	22
676	19	80	21	44
677	19	20	21	22
678	19	40	63	44
679	19	20	147	22
681	19	20	63	22
682	19	40	21	484
683	19	20	21	22
684	361	80	63	44
685	19	100	21	22
686	19	40	147	44
687	19	20	63	22
688	19	80	21	44
689	19	20	21	22
690	19	200	63	44
691	19	20	21	22
692	19	80	21	44
694	19	40	21	44
695	19	100	21	22
696	19	80	63	44
697	19	20	21	22
698	19	40	21	44
699	19	20	63	22
700	19	400	147	44
701	19	20	21	22
702	19	40	63	44
703	361	20	21	22
704	19	80	21	484
705	19	100	63	22
706	19	40	21	44
707	19	20	147	22
708	19	80	63	44
709	19	20	21	22
710	19	200	21	44
711	19	20	63	22
712	19	80	21	44
713	19	20	21	22
716	19	80	21	44
717	19	20	63	22
718	19	40	21	44
719	19	20	21	22
721	19	20	147	22
722	361	40	21	44
723	19	20	63	22
724	19	80	21	44
725	19	100	21	22
726	19	40	63	484
727	19	20	21	22
729	19	20	63	22
730	19	200	21	44
731	19	20	21	22
732	19	80	63	44
733	19	20	21	22
734	19	40	21	44
735	19	100	441	22
736	19	80	21	44
737	19	20	21	242
738	19	40	63	44
739	19	20	21	22
740	19	400	21	44
741	361	20	63	22
742	19	40	147	44
743	19	20	21	22
744	19	80	63	44
745	19	100	21	22
746	19	40	21	44
747	19	20	63	22
749	19	20	147	22
750	19	200	63	44
751	19	20	21	22
752	19	80	21	44
753	19	20	63	22
754	19	40	21	44
755	19	100	21	22
756	19	80	441	44
757	19	20	21	22
758	19	40	21	44
759	19	20	63	242
760	361	400	21	44
761	19	20	21	22
762	19	40	63	44
763	19	20	147	22
764	19	80	21	44
766	19	40	21	44
767	19	20	21	22
768	19	80	63	44
769	19	20	21	22
771	19	20	63	22
772	19	80	21	44
773	19	20	21	22
774	19	40	63	44
775	19	100	21	22
776	19	80	21	44
777	19	20	441	22
778	19	40	21	44
779	361	20	21	22
781	19	20	21	242
782	19	40	21	44
783	19	20	63	22
784	19	80	147	44
785	19	100	21	22
786	19	40	63	44
787	19	20	21	22
788	19	80	21	44
789	19	20	63	22
790	19	200	21	44
791	19	20	147	22
793	19	20	21	22
794	19	40	21	44
795	19	100	63	22
796	19	80	21	44
797	19	20	21	22
798	361	40	441	44
799	19	20	21	22
800	19	400	21	44
801	19	20	63	22
802	19	40	21	44
803	19	20	21	242
804	19	80	63	44
805	19	100	147	22
806	19	40	21	44
807	19	20	63	22
808	19	80	21	44
809	19	20	21	22
810	19	200	63	44
811	19	20	21	22
812	19	80	147	44
813	19	20	63	22
814	19	40	21	484
815	19	100	21	22
817	361	20	21	22
818	19	40	21	44
820	19	400	21	44
821	19	20	21	22
822	19	40	63	44
823	19	20	21	22
824	19	80	21	44
825	19	100	63	242

Table C.29: 2-Twist-spun $T(2,m)$ $(609 \leq m \leq 825)$ vs. Quandles R19-R22: Numbers of Colorings

	R19	R20	R21	R22
826	19	40	147	44
827	19	20	21	22
828	19	80	63	44
829	19	20	21	22
830	19	200	21	44
831	19	20	63	22
832	19	80	21	44
833	19	20	147	22
834	19	40	63	44
835	19	100	21	22
836	361	80	21	484
837	19	20	63	22
838	19	40	21	44
839	19	20	21	22
841	19	20	21	22
842	19	40	21	44
843	19	20	63	22
844	19	80	21	44
845	19	100	21	22
846	19	40	63	44
847	19	20	147	242
848	19	80	21	44
849	19	20	63	22
850	19	200	21	44
851	19	20	21	22
852	19	80	63	44
853	19	20	21	22
854	19	40	147	44
855	361	100	63	22
856	19	80	21	44
857	19	20	21	22
859	19	20	21	22
860	19	400	21	44
861	19	20	441	22
862	19	40	21	44
863	19	20	21	22
864	19	80	63	44
865	19	100	21	22
866	19	40	21	44
867	19	20	63	22
868	19	80	147	44
869	19	20	21	242
870	19	200	63	44
871	19	20	21	22
872	19	80	21	44
873	19	20	63	22
874	361	40	21	44
875	19	100	147	22
876	19	80	63	44
877	19	20	21	22
878	19	40	21	44
879	19	20	63	22
881	19	20	21	22
882	19	40	441	44
883	19	20	21	22
885	19	100	63	22
886	19	40	21	44
887	19	20	21	22
888	19	80	63	44
889	19	20	147	22
890	19	200	21	44
891	19	20	63	242

	R19	R20	R21	R22
892	19	80	21	44
893	361	20	21	22
894	19	40	63	44
895	19	100	21	22
896	19	80	147	44
897	19	20	63	22
898	19	40	21	44
899	19	20	21	22
900	19	400	63	44
901	19	20	21	22
902	19	40	21	484
903	19	20	441	22
904	19	80	21	44
905	19	100	21	22
906	19	40	63	44
907	19	20	21	22
908	19	80	21	44
909	19	20	63	22
911	19	20	21	22
912	361	80	63	44
913	19	20	21	242
914	19	40	21	44
915	19	100	63	22
916	19	80	21	44
917	19	20	147	22
918	19	40	63	44
919	19	20	21	22
920	19	400	21	44
921	19	20	63	22
922	19	40	21	44
923	19	20	21	22
925	19	100	21	22
926	19	40	21	44
927	19	20	63	22
928	19	80	21	44
929	19	20	21	22
930	19	200	63	44
931	361	20	147	22
932	19	80	21	44
933	19	20	63	22
934	19	40	21	44
937	19	20	21	22
938	19	40	147	44
939	19	20	63	22
940	19	400	21	44
941	19	20	21	22
942	19	40	63	44
943	19	20	21	22
944	19	80	21	44
945	19	100	441	22
946	19	40	21	484
947	19	20	21	22
948	19	80	63	44
949	19	20	21	22
950	361	200	21	44
951	19	20	63	22
953	19	20	21	22
954	19	40	63	44
955	19	100	21	22
956	19	80	21	44
957	19	20	63	242
958	19	40	21	44

	R19	R20	R21	R22
959	19	20	147	22
960	19	400	63	44
961	19	20	21	22
962	19	40	21	44
963	19	20	63	22
964	19	80	21	44
965	19	100	21	22
966	19	40	441	44
967	19	20	21	22
968	19	80	21	484
969	361	20	63	22
970	19	200	21	44
971	19	20	21	22
972	19	80	63	44
973	19	20	147	22
974	19	40	21	44
975	19	100	63	22
976	19	80	21	44
977	19	20	21	22
978	19	40	63	44
979	19	20	21	242
980	19	400	147	44
981	19	20	63	22
982	19	40	21	44
983	19	20	21	22
984	19	80	63	44
985	19	100	21	22
986	19	40	21	44
987	19	20	441	22
988	361	80	21	44
989	19	20	21	22
991	19	20	21	22
992	19	80	21	44
993	19	20	63	22
994	19	40	147	44
995	19	100	21	22
996	19	80	63	44
997	19	20	21	22
998	19	40	21	44
999	19	20	63	22
1000	19	400	21	44
1002	19	40	63	44
1003	19	20	21	22
1004	19	80	21	44
1005	19	100	63	22
1006	19	40	21	44
1007	361	20	21	22
1009	19	20	21	22
1010	19	200	21	44
1011	19	20	63	22
1012	19	80	21	484
1013	19	20	21	22
1014	19	40	63	44
1015	19	100	147	22
1016	19	80	21	44
1017	19	20	63	22
1018	19	40	21	44
1019	19	20	21	22
1021	19	20	21	22
1022	19	40	147	44
1023	19	20	63	242
1024	19	80	21	44
1025	19	100	21	22
1026	361	40	63	44

Table C.30: 2-Twist-spun $T(2, m)$ ($826 \leq m \leq 1026$) vs. Quandles R19-R22: Numbers of Colorings

Quandle	Pairs Distinguished
R23	1816
R24	0
R25	392
R26	0
Undistinguished Pairs So Far: 23737	

Table C.31: Quandles R23-R26 and Their Performances

	R23	R24	R25	R26
3	23	72	25	26
4	23	96	25	52
5	23	24	125	26
6	23	144	25	52
7	23	24	25	26
8	23	192	25	52
9	23	72	25	26
10	23	48	125	52
11	23	24	25	26
12	23	288	25	52
13	23	24	25	338
14	23	48	25	52
15	23	72	125	26
16	23	192	25	52
17	23	24	25	26
18	23	144	25	52
19	23	24	25	26
20	23	96	125	52
21	23	72	25	26
22	23	48	25	52
23	529	24	25	26
24	23	576	25	52
25	23	24	625	26
26	23	48	25	676
27	23	72	25	26
28	23	96	25	52
29	23	24	25	26
30	23	144	125	52
31	23	24	25	26
32	23	192	25	52
33	23	72	25	26
34	23	48	25	52
35	23	24	125	26
36	23	288	25	52
37	23	24	25	26
38	23	48	25	52
39	23	72	25	338
40	23	192	125	52
41	23	24	25	26
42	23	144	25	52
43	23	24	25	26
44	23	96	25	52
45	23	72	125	26
46	529	48	25	52
47	23	24	25	26
48	23	576	25	52
49	23	24	25	26
50	23	48	625	52
51	23	72	25	26
52	23	96	25	676
53	23	24	25	26
54	23	144	25	52
55	23	24	125	26
56	23	192	25	52
58	23	48	25	52

	R23	R24	R25	R26
59	23	24	25	26
60	23	288	125	52
61	23	24	25	26
62	23	48	25	52
63	23	72	25	26
64	23	192	25	52
65	23	24	125	338
66	23	144	25	52
67	23	24	25	26
69	529	72	25	26
70	23	48	125	52
71	23	24	25	26
72	23	576	25	52
73	23	24	25	26
74	23	48	25	52
75	23	72	625	26
77	23	24	25	26
78	23	144	25	676
79	23	24	25	26
80	23	192	125	52
81	23	72	25	26
82	23	48	25	52
83	23	24	25	26
84	23	288	25	52
85	23	24	125	26
86	23	48	25	52
87	23	72	25	26
88	23	192	25	52
89	23	24	25	26
90	23	144	125	52
91	23	24	25	338
92	529	96	25	52
93	23	72	25	26
94	23	48	25	52
95	23	24	125	26
96	23	576	25	52
97	23	24	25	26
98	23	48	25	52
99	23	72	25	26
100	23	96	625	52
101	23	24	25	26
103	23	24	25	26
105	23	72	125	26
106	23	48	25	52
107	23	24	25	26
108	23	288	25	52
109	23	24	25	26
110	23	48	125	52
111	23	72	25	26
112	23	192	25	52
113	23	24	25	26
115	529	24	125	26
116	23	96	25	52
117	23	72	25	338
118	23	48	25	52

	R23	R24	R25	R26
119	23	24	25	26
120	23	576	125	52
121	23	24	25	26
122	23	48	25	52
123	23	72	25	26
124	23	96	25	52
125	23	24	625	26
126	23	144	25	52
127	23	24	25	26
128	23	192	25	52
129	23	72	25	26
130	23	48	125	676
131	23	24	25	26
133	23	24	25	26
134	23	48	25	52
135	23	72	125	26
137	23	24	25	26
138	529	144	25	52
139	23	24	25	26
140	23	96	125	52
141	23	72	25	26
142	23	48	25	52
144	23	576	25	52
145	23	24	125	26
146	23	48	25	52
147	23	72	25	26
148	23	96	25	52
149	23	24	25	26
150	23	144	625	52
151	23	24	25	26
153	23	72	25	26
155	23	24	125	26
157	23	24	25	26
158	23	48	25	52
159	23	72	25	26
160	23	192	125	52
161	529	24	25	26
162	23	144	25	52
163	23	24	25	26
164	23	96	25	52
165	23	72	125	26
166	23	48	25	52
167	23	24	25	26
169	23	24	25	338
170	23	48	125	52
171	23	72	25	26
172	23	96	25	52
173	23	24	25	26
174	23	144	25	52
175	23	24	625	26
176	23	192	25	52
177	23	72	25	26
178	23	48	25	52
179	23	24	25	26
180	23	288	125	52

Table C.32: 2-Twist-spun $T(2,m)$ ($3 \leq m \leq 180$) vs. Quandles R23-R26: Numbers of Colorings

	R23	R24	R25	R26
181	23	24	25	26
183	23	72	25	26
184	529	192	25	52
185	23	24	125	26
186	23	144	25	52
188	23	96	25	52
189	23	72	25	26
190	23	48	125	52
191	23	24	25	26
192	23	576	25	52
193	23	24	25	26
194	23	48	25	52
195	23	72	125	338
196	23	96	25	52
197	23	24	25	26
198	23	144	25	52
199	23	24	25	26
200	23	192	625	52
201	23	72	25	26
202	23	48	25	52
203	23	24	25	26
205	23	24	125	26
206	23	48	25	52
207	529	72	25	26
208	23	192	25	676
211	23	24	25	26
212	23	96	25	52
213	23	72	25	26
214	23	48	25	52
215	23	24	125	26
216	23	576	25	52
217	23	24	25	26
218	23	48	25	52
219	23	72	25	26
222	23	144	25	52
223	23	24	25	26
224	23	192	25	52
225	23	72	625	26
226	23	48	25	52
227	23	24	25	26
229	23	24	25	26
230	529	48	125	52
232	23	192	25	52
233	23	24	25	26
234	23	144	25	676
235	23	24	125	26
236	23	96	25	52
237	23	72	25	26
239	23	24	25	26
240	23	576	125	52
241	23	24	25	26
242	23	48	25	52
243	23	72	25	26
244	23	96	25	52
245	23	24	125	26
246	23	144	25	52
248	23	192	25	52
249	23	72	25	26
250	23	48	625	52
251	23	24	25	26
252	23	288	25	52
253	529	24	25	26
254	23	48	25	52
256	23	192	25	52
257	23	24	25	26
258	23	144	25	52
259	23	24	25	26
261	23	72	25	26
262	23	48	25	52
263	23	24	25	26
265	23	24	125	26
267	23	72	25	26
268	23	96	25	52
269	23	24	25	26
270	23	144	125	52
271	23	24	25	26
272	23	192	25	52
274	23	48	25	52
275	23	24	625	26
276	529	288	25	52
277	23	24	25	26
278	23	48	25	52
279	23	72	25	26
281	23	24	25	26
282	23	144	25	52
283	23	24	25	26
284	23	96	25	52
287	23	24	25	26
288	23	576	25	52
289	23	24	25	26
290	23	48	125	52
291	23	72	25	26
292	23	96	25	52
293	23	24	25	26
294	23	144	25	52
295	23	24	125	26
296	23	192	25	52
297	23	72	25	26
298	23	48	25	52
299	529	24	25	338
300	23	288	625	52
301	23	24	25	26
302	23	48	25	52
303	23	72	25	26
304	23	192	25	52
305	23	24	125	26
306	23	144	25	52
307	23	24	25	26
309	23	72	25	26
310	23	48	125	52
311	23	24	25	26
313	23	24	25	26
314	23	48	25	52
315	23	72	125	26
316	23	96	25	52
317	23	24	25	26
318	23	144	25	52
319	23	24	25	26
320	23	192	125	52
321	23	72	25	26
322	529	48	25	52
324	23	288	25	52
325	23	24	625	338
326	23	48	25	52
327	23	72	25	26
328	23	192	25	52
329	23	24	25	26
331	23	24	25	26
332	23	96	25	52
333	23	72	25	26
334	23	48	25	52
335	23	24	125	26
336	23	576	25	52
337	23	24	25	26
338	23	48	25	676
339	23	72	25	26
341	23	24	25	26
342	23	144	25	52
343	23	24	25	26
344	23	192	25	52
345	529	72	125	26
346	23	48	25	52
347	23	24	25	26
348	23	288	25	52
349	23	24	25	26
350	23	48	625	52
351	23	72	25	338
352	23	192	25	52
353	23	24	25	26
354	23	144	25	52
355	23	24	125	26
356	23	96	25	52
358	23	48	25	52
359	23	24	25	26
361	23	24	25	26
362	23	48	25	52
363	23	72	25	26
365	23	24	125	26
366	23	144	25	52
367	23	24	25	26
368	529	192	25	52
369	23	72	25	26
370	23	48	125	52
371	23	24	25	26
372	23	288	25	52
373	23	24	25	26
375	23	72	625	26
376	23	192	25	52
377	23	24	25	338
378	23	144	25	52
379	23	24	25	26
381	23	72	25	26
382	23	48	25	52
383	23	24	25	26
384	23	576	25	52
386	23	48	25	52
387	23	72	25	26
388	23	96	25	52
389	23	24	25	26
391	529	24	25	26
392	23	192	25	52
393	23	72	25	26
394	23	48	25	52
395	23	24	125	26
397	23	24	25	26
398	23	48	25	52
400	23	192	625	52
401	23	24	25	26
402	23	144	25	52
403	23	24	25	338
404	23	96	25	52
405	23	72	125	26
406	23	48	25	52
407	23	24	25	26
409	23	24	25	26
410	23	48	125	52
411	23	72	25	26
412	23	96	25	52
413	23	24	25	26
414	529	144	25	52
415	23	24	125	26
416	23	192	25	676
417	23	72	25	26
419	23	24	25	26
421	23	24	25	26
422	23	48	25	52
423	23	72	25	26
424	23	192	25	52
425	23	24	625	26
426	23	144	25	52

Table C.33: 2-Twist-spun $T(2,m)$ $(181 \leq m \leq 426)$ vs. Quandles R23-R26: Numbers of Colorings

	R23	R24	R25	R26
427	23	24	25	26
428	23	96	25	52
430	23	48	125	52
431	23	24	25	26
432	23	576	25	52
433	23	24	25	26
434	23	48	25	52
435	23	72	125	26
436	23	96	25	52
437	529	24	25	26
438	23	144	25	52
439	23	24	25	26
441	23	72	25	26
443	23	24	25	26
444	23	288	25	52
445	23	24	125	26
446	23	48	25	52
447	23	72	25	26
448	23	192	25	52
449	23	24	25	26
450	23	144	625	52
451	23	24	25	26
452	23	96	25	52
453	23	72	25	26
454	23	48	25	52
457	23	24	25	26
458	23	48	25	52
459	23	72	25	26
460	529	96	125	52
461	23	24	25	26
463	23	24	25	26
464	23	192	25	52
465	23	72	125	26
466	23	48	25	52
467	23	24	25	26
469	23	24	25	26
470	23	48	125	52
471	23	72	25	26
472	23	192	25	52
473	23	24	25	26
474	23	144	25	52
475	23	24	625	26
477	23	72	25	26
478	23	48	25	52
479	23	24	25	26
480	23	576	125	52
481	23	24	25	338
482	23	48	25	52
483	529	72	25	26
484	23	96	25	52
485	23	24	125	26
486	23	144	25	52
487	23	24	25	26
488	23	192	25	52
489	23	72	25	26
490	23	48	125	52
491	23	24	25	26
492	23	288	25	52
493	23	24	25	26
496	23	192	25	52
497	23	24	25	26
498	23	144	25	52
499	23	24	25	26
500	23	96	625	52
501	23	72	25	26
502	23	48	25	52
503	23	24	25	26
505	23	24	125	26
506	529	48	25	52
507	23	72	25	338
508	23	96	25	52

	R23	R24	R25	R26
509	23	24	25	26
511	23	24	25	26
512	23	192	25	52
513	23	72	25	26
514	23	48	25	52
515	23	24	125	26
516	23	288	25	52
517	23	24	25	26
518	23	48	25	52
519	23	72	25	26
521	23	24	25	26
522	23	144	25	52
523	23	24	25	26
524	23	96	25	52
525	23	72	625	26
526	23	48	25	52
527	23	24	25	26
529	529	24	25	26
530	23	48	125	52
531	23	72	25	26
533	23	24	25	338
534	23	144	25	52
535	23	24	125	26
536	23	192	25	52
537	23	72	25	26
538	23	48	25	52
539	23	24	25	26
540	23	288	125	52
541	23	24	25	26
542	23	48	25	52
543	23	72	25	26
544	23	192	25	52
545	23	24	125	26
547	23	24	25	26
548	23	96	25	52
549	23	72	25	26
550	23	48	625	52
551	23	24	25	26
552	529	576	25	52
553	23	24	25	26
554	23	48	25	52
555	23	72	125	26
556	23	96	25	52
557	23	24	25	26
558	23	144	25	52
559	23	24	25	338
562	23	48	25	52
563	23	24	25	26
564	23	288	25	52
565	23	24	125	26
566	23	48	25	52
567	23	72	25	26
568	23	192	25	52
569	23	24	25	26
571	23	24	25	26
573	23	72	25	26
574	23	48	25	52
575	529	24	625	26
576	23	576	25	52
577	23	24	25	26
578	23	48	25	52
579	23	72	25	26
580	23	96	125	52
581	23	24	25	26
582	23	144	25	52
583	23	24	25	26
584	23	192	25	52
586	23	48	25	52
587	23	24	25	26
588	23	288	25	52
589	23	24	25	26

	R23	R24	R25	R26
590	23	48	125	52
591	23	72	25	26
592	23	192	25	52
593	23	24	25	26
594	23	144	25	52
596	23	96	25	52
597	23	72	25	26
598	529	48	25	676
599	23	24	25	26
600	23	576	625	52
601	23	24	25	26
602	23	48	25	52
603	23	72	25	26
604	23	96	25	52
605	23	24	125	26
606	23	144	25	52
607	23	24	25	26
608	23	192	25	52
609	23	72	25	26
610	23	48	125	52
611	23	24	25	338
613	23	24	25	26
614	23	48	25	52
615	23	72	125	26
617	23	24	25	26
618	23	144	25	52
619	23	24	25	26
620	23	96	125	52
621	529	72	25	26
622	23	48	25	52
623	23	24	25	26
625	23	24	625	26
626	23	48	25	52
628	23	96	25	52
629	23	24	25	26
631	23	24	25	26
632	23	192	25	52
633	23	72	25	26
634	23	48	25	52
635	23	24	125	26
636	23	288	25	52
637	23	24	25	338
638	23	48	25	52
639	23	72	25	26
640	23	192	125	52
641	23	24	25	26
642	23	144	25	52
643	23	24	25	26
644	529	96	25	52
645	23	72	125	26
647	23	24	25	26
648	23	576	25	52
649	23	24	25	26
650	23	48	625	676
651	23	72	25	26
652	23	96	25	52
653	23	24	25	26
654	23	144	25	52
655	23	24	125	26
656	23	192	25	52
657	23	72	25	26
658	23	48	25	52
659	23	24	25	26
661	23	24	25	26
662	23	48	25	52
664	23	192	25	52
666	23	144	25	52
667	529	24	25	26
668	23	96	25	52
669	23	72	25	26
670	23	48	125	52

Table C.34: 2-Twist-spun $T(2,m)$ $(427 \leq m \leq 670)$ vs. Quandles R23-R26: Numbers of Colorings

	R23	R24	R25	R26
671	23	24	25	26
672	23	576	25	52
673	23	24	25	26
674	23	48	25	52
675	23	72	625	26
676	23	96	25	676
677	23	24	25	26
678	23	144	25	52
679	23	24	25	26
681	23	72	25	26
682	23	48	25	52
683	23	24	25	26
685	23	24	125	26
686	23	48	25	52
687	23	72	25	26
688	23	192	25	52
689	23	24	25	338
690	529	144	125	52
691	23	24	25	26
692	23	96	25	52
694	23	48	25	52
695	23	24	125	26
696	23	576	25	52
697	23	24	25	26
698	23	48	25	52
699	23	72	25	26
700	23	96	625	52
701	23	24	25	26
702	23	144	25	676
703	23	24	25	26
704	23	192	25	52
705	23	72	125	26
706	23	48	25	52
707	23	24	25	26
708	23	288	25	52
709	23	24	25	26
710	23	48	125	52
711	23	72	25	26
712	23	192	25	52
713	529	24	25	26
716	23	96	25	52
717	23	72	25	26
718	23	48	25	52
719	23	24	25	26
721	23	24	25	26
722	23	48	25	52
723	23	72	25	26
724	23	96	25	52
725	23	24	625	26
726	23	144	25	52
727	23	24	25	26
729	23	72	25	26
730	23	48	125	52
731	23	24	25	26
732	23	288	25	52
733	23	24	25	26
734	23	48	25	52
735	23	72	125	26
736	529	192	25	52
737	23	24	25	26
738	23	144	25	52
739	23	24	25	26
740	23	96	125	52
742	23	48	25	52
743	23	24	25	26
744	23	576	25	52
745	23	24	125	26
746	23	48	25	52
747	23	72	25	26
749	23	24	25	26

	R23	R24	R25	R26
750	23	144	625	52
751	23	24	25	26
752	23	192	25	52
753	23	72	25	26
754	23	48	25	676
755	23	24	125	26
756	23	288	25	52
757	23	24	25	26
758	23	48	25	52
759	529	72	25	26
761	23	24	25	26
762	23	144	25	52
763	23	24	25	26
764	23	96	25	52
766	23	48	25	52
767	23	24	25	338
768	23	576	25	52
769	23	24	25	26
771	23	72	25	26
772	23	96	25	52
773	23	24	25	26
774	23	144	25	52
775	23	24	625	26
776	23	192	25	52
777	23	72	25	26
778	23	48	25	52
779	23	24	25	26
781	23	24	25	26
782	529	48	25	52
783	23	72	25	26
784	23	192	25	52
785	23	24	125	26
786	23	144	25	52
787	23	24	25	26
788	23	96	25	52
789	23	72	25	26
790	23	48	125	52
791	23	24	25	26
793	23	24	25	338
794	23	48	25	52
795	23	72	125	26
796	23	96	25	52
797	23	24	25	26
799	23	24	25	26
800	23	192	625	52
801	23	72	25	26
802	23	48	25	52
803	23	24	25	26
804	23	288	25	52
805	529	24	125	26
806	23	48	25	676
807	23	72	25	26
808	23	192	25	52
809	23	24	25	26
810	23	144	125	52
811	23	24	25	26
812	23	96	25	52
813	23	72	25	26
814	23	48	25	52
815	23	24	125	26
817	23	24	25	26
818	23	48	25	52
820	23	96	125	52
821	23	24	25	26
822	23	144	25	52
823	23	24	25	26
824	23	192	25	52
825	23	72	625	26
826	23	48	25	52
827	23	24	25	26

	R23	R24	R25	R26
828	529	288	25	52
829	23	24	25	26
830	23	48	125	52
831	23	72	25	26
832	23	192	25	676
833	23	24	25	26
834	23	144	25	52
835	23	24	125	26
837	23	72	25	26
838	23	48	25	52
839	23	24	25	26
841	23	24	25	26
842	23	48	25	52
843	23	72	25	26
844	23	96	25	52
845	23	24	125	338
846	23	144	25	52
847	23	24	25	26
848	23	192	25	52
849	23	72	25	26
850	23	48	625	52
851	529	24	25	26
852	23	288	25	52
853	23	24	25	26
854	23	48	25	52
856	23	192	25	52
857	23	24	25	26
859	23	24	25	26
860	23	96	125	52
861	23	72	25	26
862	23	48	25	52
863	23	24	25	26
864	23	576	25	52
865	23	24	125	26
866	23	48	25	52
867	23	72	25	26
868	23	96	25	52
869	23	24	25	26
870	23	144	125	52
871	23	24	25	338
872	23	192	25	52
873	23	72	25	26
874	529	48	25	52
875	23	24	625	26
876	23	288	25	52
877	23	24	25	26
878	23	48	25	52
879	23	72	25	26
881	23	24	25	26
882	23	144	25	52
883	23	24	25	26
885	23	72	125	26
886	23	48	25	52
887	23	24	25	26
888	23	576	25	52
889	23	24	25	26
890	23	48	125	52
891	23	72	25	26
892	23	96	25	52
893	23	24	25	26
894	23	144	25	52
895	23	24	125	26
896	23	192	25	52
897	529	72	25	338
898	23	48	25	52
899	23	24	25	26
900	23	288	625	52
901	23	24	25	26
902	23	48	25	52
903	23	72	25	26

Table C.35: 2-Twist-spun $T(2,m)$ $(671 \leq m \leq 903)$ vs. Quandles R23-R26: Numbers of Colorings

	R23	R24	R25	R26
904	23	192	25	52
905	23	24	125	26
906	23	144	25	52
907	23	24	25	26
908	23	96	25	52
909	23	72	25	26
911	23	24	25	26
913	23	24	25	26
914	23	48	25	52
915	23	72	125	26
916	23	96	25	52
917	23	24	25	26
918	23	144	25	52
919	23	24	25	26
920	529	192	125	52
921	23	72	25	26
922	23	48	25	52
923	23	24	25	338
925	23	24	625	26
926	23	48	25	52
927	23	72	25	26
928	23	192	25	52
929	23	24	25	26
930	23	144	125	52
931	23	24	25	26
932	23	96	25	52
933	23	72	25	26
934	23	48	25	52
937	23	24	25	26
938	23	48	25	52
939	23	72	25	26
940	23	96	125	52
941	23	24	25	26
942	23	144	25	52
943	529	24	25	26
944	23	192	25	52
945	23	72	125	26
946	23	48	25	52
947	23	24	25	26
948	23	288	25	52
949	23	24	25	338
950	23	48	625	52
951	23	72	25	26
953	23	24	25	26
954	23	144	25	52
955	23	24	125	26
956	23	96	25	52
957	23	72	25	26
958	23	48	25	52
959	23	24	25	26
960	23	576	125	52
961	23	24	25	26
962	23	48	25	676
963	23	72	25	26
964	23	96	25	52
965	23	24	125	26

	R23	R24	R25	R26
966	529	144	25	52
967	23	24	25	26
968	23	192	25	52
970	23	48	125	52
971	23	24	25	26
972	23	288	25	52
973	23	24	25	26
974	23	48	25	52
975	23	72	625	338
976	23	192	25	52
977	23	24	25	26
978	23	144	25	52
979	23	24	25	26
980	23	96	125	52
981	23	72	25	26
982	23	48	25	52
983	23	24	25	26
984	23	576	25	52
985	23	24	125	26
986	23	48	25	52
987	23	72	25	26
989	529	24	25	26
991	23	24	25	26
992	23	192	25	52
993	23	72	25	26
994	23	48	25	52
995	23	24	125	26
996	23	288	25	52
997	23	24	25	26
998	23	48	25	52
999	23	72	25	26
1000	23	192	625	52
1002	23	144	25	52
1003	23	24	25	26
1004	23	96	25	52
1005	23	72	125	26
1006	23	48	25	52
1007	23	24	25	26
1009	23	24	25	26
1010	23	48	125	52
1011	23	72	25	26
1012	529	96	25	52
1013	23	24	25	26
1014	23	144	25	676
1015	23	24	125	26
1016	23	192	25	52
1017	23	72	25	26
1018	23	48	25	52
1019	23	24	25	26
1021	23	24	25	26
1022	23	48	25	52
1023	23	72	25	26
1024	23	192	25	52
1025	23	24	625	26
1026	23	144	25	52

Table C.36: 2-Twist-spun $T(2,m)$ ($904 \leq m \leq 1026$) vs. Quandles R23-R26: Numbers of Colorings