Corrigendum:

Level-set approaches of L_2-type for recovering shape and contrast in ill-posed problems

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In what follows, we correct the pcLS approach introduced in our article [2]. The main modifications are: (i), we correct the proof of [2, Lemma 9]; (ii) a slight modification in the definition of admissible pairs is necessary. The convergence analysis results and the numerical experiments presented in the article do not change.

Corrections to the pcLS approach

Differently from [2], we assume that $\phi(x) = i - 1$, $x \in D_i$, $i = 1, 2$, i.e., $\phi(x) \in \{0, 1\}$ a.e. in Ω. Thus, defining the auxiliary functions $\psi_1(t) := 1 - t$ and $\psi_2(t) := t$, the characteristic functions of the subdomains D_i can be written in the form $\chi_{D_i}(x) = \psi_i(\phi(x))$, $i = 1, 2$. Moreover, a solution $u \in X := L_2(\Omega)$ of the operator equation (1) in [2] can be parameterized by the operator

$$u \;=\; c^1\psi_1(\phi) + c^2\psi_2(\phi) \;=:\; P_{pc}(\phi, c^j)\,. \tag{1}$$

Note that, the piecewise constant assumption on ϕ corresponds to the constraint $\mathcal{K}(\phi) = 0$, where $\mathcal{K}(\phi) := (\phi)(\phi - 1)$ is a smooth nonlinear operator.

Main changes and corrections

Let $\tilde{D} \subset \Omega$ be a open and bounded subset, with the Lebesgue measure $|\tilde{D}| > \gamma > 0$ for a fix γ. We define the following subset of $\mathtt{BV}$:

$$\mathtt{BV}_0(\Omega) := \{\phi \in \mathtt{BV}(\Omega) \,:\, \phi(x) = 0\,, \quad \text{a.e. } x \in \tilde{D}\}\,. \tag{2}$$

We redefine the admissible pairs as follows:

Definition 2 (Changed). *Let the operator P_{pc} defined as in (1) and $\tau > 0$. A vector $(\phi, c^j) \in L^2(\Omega) \times \mathbb{R}^2$ is called admissible when $\phi \in \mathtt{BV}_0(\Omega)$ and $|c^2 - c^1| \geq \tau$.*

It is worth noticing that, the modification in the definition of the operator $\mathcal{K}$ do not alter the conclusions of [2, Lemma 8].

Lemma 9 (Corrected). *Let P_{pc} be defined by (1) and $\mathcal{K}$ as above. For $1 \leq p < 2$, the following assertions holds true:*

(i) For every admissible vector (ϕ, c^j) we have $|P_{pc}(\phi, c^j)|_{\mathtt{BV}} \geq \tau|\phi|_{\mathtt{BV}}$.

(ii) $\mathtt{BV}_0(\Omega)$ *is a closed subset of* $\mathtt{BV}(\Omega)$ *with respect to the* $L_p(\Omega)$ *convergence. In other words, if* $\phi_k \in \mathtt{BV}_0(\Omega)$ *is a sequence converging to* $\phi \in \mathtt{BV}(\Omega)$ *with respect to the* $L_p(\Omega)$*-topology, then* $\phi \in \mathtt{BV}_0(\Omega)$.

(iii) *For every admissible vector* (ϕ, c^j)*, there exist a constant* $c > 0$ *such that* $|P_{pc}(\phi, c^j)|_{\mathtt{BV}} \geq c\|\phi\|_{L_2(\Omega)}$.

(iv) *The functional* $\|\mathcal{K}(\cdot)\|_{L_1(\Omega)}$ *is weak lower semi-continuous.*

Proof. The proof of assertion (i) does not need any corrections. Assertion (ii) follows from the inequality

$$\left(\int_{\tilde{D}} |\phi|^p dx\right)^{\frac{1}{p}} \leq \left(\int_{\tilde{D}} |\phi - \phi_k|^p dx\right)^{\frac{1}{p}} + \left(\int_{\tilde{D}} |\phi_k|^p dx\right)^{\frac{1}{p}} \leq \|\phi - \phi_k\|_{L_p(\Omega)}$$

(to obtain this inequality one uses the Minkowski inequality). Assertion (iii) follows from Assertion (i) and the Poincaré inequality for BV functions [3, Theorem 1 (ii), pg. 189]. To verify the assertion of item (iv), notice that the equation $\mathcal{K}(t) = 0$ is equivalent to $\tilde{\mathcal{K}}(t) = \frac{1}{4}$, where $\tilde{\mathcal{K}}(t) := \mathcal{K}(t) + \frac{1}{4}$. Thus, it is enough to prove that the functional $\left\|\tilde{\mathcal{K}}(\cdot)\right\|_{L_1(\Omega)}$ is weakly l.s.c. Since the real function $t \mapsto \tilde{\mathcal{K}}(t)$ is convex, this property follows from [1, Theorem 1.1, pg. 7; and subsequent remark, pg. 8]. □

This lemma is enough to guarantee weak lower semi-continuity of the functional $\|\mathcal{K}(\cdot)\|_{L_1}$. Notice that Lemma 9 (corrected) provides the essential tools needed to derive the main convergence analysis results for the (pcLS) approach in [2][Theorems 10 and 11].

References

[1] B. Dacorogna, *Weak continuity and weak lower continuity of nonlinear functionals*, Lecture Notes in Mathematics, vol. 922, Springer, New York, 1982.

[2] A. De Cezaro and A. Leitão, *Level-set approaches of L_2–type for recovering shape and contrast in ill-posed problems*, Inverse Problems in Science and Enginnering **20** (2012), no. 4, 571–587.

[3] L.C. Evans and R.F. Gariepy, *Measure theory and fine properties of functions*, Studies in Advanced Mathematics, CRC Press, Boca Raton, FL, 1992.